Range Sidelobe Suppression in a Desired Doppler Interval

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Summary

We present a novel method of constructing a Doppler resilient pulse train of Golay complementary waveforms, for which the range sidelobes of the pulse train ambiguity function vanish inside a desired Doppler interval. This is accomplished by coordinating the transmission of a Golay pair of phase coded waveforms in time according to the 1’s and −1’s in a biphase sequence. The magnitude of the range sidelobes of the pulse train ambiguity function is shown to be proportional to the magnitude spectrum of the biphase sequence. Therefore, range sidelobes inside a desired Doppler interval can be suppressed by selecting a biphase sequence whose spectrum has a high-order null at a Doppler frequency inside the desired interval. We show that the spectrum of the biphase sequence obtained by oversampling the length-$2^M$ Prouhet-Thue-Morse sequence by a factor $m$ has an $M$th-order null at all rational Doppler shifts $\theta_0 = 2\pi\ell/m$, where $\ell$ and $m \neq 1$ are co-prime integers. This spectrum also has an $(M-h-1)$th-order null at zero Doppler and $(M-h-1)$th-order nulls at all Doppler shifts $\theta_0 = 2\pi\ell/(2^hm)$, where $\ell$ and $m \neq 1$ are again co-prime and $1 \leq h \leq M-1$.

Motivation

A key issue in phase coding is the presence of range sidelobes in the ambiguity function of the coded waveforms. The effective ambiguity function of a Golay pair of phase coded waveforms is free of range sidelobes along the zero-Doppler axis. However, off the zero-Doppler axis the ambiguity function has large range sidelobes. (see Fig. 1). It is recently shown that range sidelobes along modest Doppler shifts can be significantly suppressed if the transmission of the two waveforms in pair is coordinated according to a Prouhet-Thue-Morse (PTM) sequence. The ambiguity function of the resulting pulse train has a high-order null along the zero-Doppler axis. It is natural to ask if it is possible to suppress the range sidelobes in a Doppler interval away from zero.

Results

Let $\mathcal{P} = \{p_n\}_{n=0}^{2^M-1}$ be the $(2^M, m)$-PTM sequence, that is to say that $\{p_{rm+i}\}_{r=0}^{2^M-1}$, $i = 0, \ldots, m-1$ is a PTM sequence of length $2^M$. Then the spectrum $S_{\mathcal{P}}(\theta)$ of $\mathcal{P}$ has $M$th-order nulls at all $\theta_0 = 2\pi l/m$ where $l$ and $m \neq 1$ are co-prime integers. Moreover, it has an $(M-h)$th-order null at $\theta_0 = 0$, and $(M-h-1)$th-order nulls at all $\theta_0 = 2\pi l/(2^hm)$, where $l$ and $m \neq 1$ are co-prime, and $1 \leq h \leq M-1$. Using the $(2^M, m)$-PTM sequence to coordinate the transmission of Golay complementary waveforms can efficiently suppress the range sidelobes.

Figure 2(a) shows the ambiguity function of a $(2^8, 3)$-PTM sequence of Golay complementary waveforms. The color bar values are in dB. This ambiguity function has an eighth-order null at $\theta_0 = \pm 2\pi/3$, a seventh-order null at zero Doppler, sixth-order nulls at $\theta_0 = \pm \pi/3$, and so on. A zoom in around

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$\theta_0 = 2\pi/3$ is provided in Fig. 2(b) to demonstrate that range sidelobes in this Doppler region are significantly suppressed. The range sidelobes in this region are at least 80 dB below the peak of the ambiguity function.

**Figures**

Figure 1: (a) The perfect autocorrelation property of a Golay pair of phase coded waveforms (b) The ambiguity function of a Golay pair of phase coded waveforms seperated in time

Figure 2: Ambiguity function of the $(2^8, 3)$-PTM pulse train of Golay complementary waveforms: (a) the entire Doppler band (b) Zoom in around $\theta_0 = 2\pi/3$. The range sidelobes are cleared out around $\theta_0 = 2\pi/3$. 