Coping with Heterogeneity and Privacy in Communication-Efficient Federated Optimization

Yuejie Chi

Carnegie Mellon University

Lehigh University
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Acknowledgements

Zhize Li
CMU

Boyue Li
CMU

Haoyu Zhao
Princeton

Peter Richtarik
KAUST
Empirical Risk Minimization (ERM)

Given a set of data $\mathcal{M}$,

$$\text{minimize}_{\mathbf{x}} \quad f(\mathbf{x}) = \frac{1}{N} \sum_{\mathbf{z} \in \mathcal{M}} \ell(\mathbf{x}; \mathbf{z})$$

Here, $N =$ number of total samples.

- **convex:** least squares, logistic regression
- **non-convex:** PCA, training neural networks (focus of this talk)
Distributed ERM

**Distributed/Federated learning:** due to privacy and scalability, data are distributed at multiple locations / workers / agents.

Let $\mathcal{M} = \bigcup_i \mathcal{M}_i$ be a data partition with equal splitting:

$$f(x) := \frac{1}{n} \sum_{i=1}^{n} f_i(x), \quad \text{where} \quad f_i(x) := \frac{1}{(N/n)} \sum_{z \in \mathcal{M}_i} \ell(x; z).$$

- $n = \text{number of agents}$
- $N/n = \text{number of local samples}$
- $m = \ldots$
Challenges in federated/decentralized learning

- **Communication efficiency**: limited bandwidth, stragglers, ...

- **Heterogeneity**: non-iid data and systems across the agents

- **Privacy**: does not come for free without sharing data
Two distributed schemes
Two distributed schemes

Server/client model

PS coordinates *global* information sharing
Two distributed schemes

Server/client model

PS coordinates *global* information sharing

Network/decentralized model

agents share *local* information over a graph topology
Communication efficiency

Communication cost = Communication rounds × Cost per round
Communication efficiency

Communication cost = Communication rounds \times \text{Cost per round}

- **Local method**: perform more local computation to reduce communication rounds, e.g. FedAvg (McMahan et al., 2016).
Communication efficiency

\[
\text{Communication cost} = \text{Communication rounds} \times \text{Cost per round}
\]

- **Local method**: perform more local computation to reduce communication rounds, e.g. FedAvg \(\text{(McMahan et al., 2016)}\).

- **Communication compression**: compress the message into fewer bits, e.g. sparsification or quantization \(\text{(Alistarh et al., 2017)}\).
Communication efficiency

Communication cost = Communication rounds × Cost per round

- **Local method**: perform more local computation to reduce communication rounds, e.g. FedAvg (McMahan et al., 2016).

- **Communication compression**: compress the message into fewer bits, e.g. sparsification or quantization (Alistarh et al., 2017).

We will focus on the latter, which are particularly suitable for bandwidth-limited environments.
Communication compression is a popular approach to reduce communication cost (e.g., (Alistarh et al., 2017); (Koloskova et al., 2019)).

\[ \mathbb{E}\|C(x) - x\|^2 \leq (1 - \alpha)\|x\|^2 \]
Communication compression is a popular approach to reduce communication cost (e.g., (Alistarh et al., 2017); (Koloskova et al., 2019)).

\[ \mathbb{E}\|C(x) - x\|^2 \leq (1 - \alpha)\|x\|^2 \]

- random sparsification: \( \alpha = k/d \) measures the compression ratio.
- Other examples: random quantization, top quantization, etc....
A prelude: what should we compress?

What about $x_{t+1} = x_t - \eta \sum_{i=1}^{n} C(\nabla f_i(x_t))$?

Somewhat surprisingly, direct compression may not work!
A prelude: what should we compress?

What about

$$\mathbf{x}^{t+1} = \mathbf{x}^t - \eta \frac{1}{n} \sum_{i=1}^{n} C(\nabla f_i(\mathbf{x}^t))$$?
A prelude: what should we compress?

What about

$$\mathbf{x}^{t+1} = \mathbf{x}^t - \eta \frac{1}{n} \sum_{i=1}^{n} C(\nabla f_i(\mathbf{x}^t))$$

Somewhat surprisingly, *direct compression* may not work!
A counter-example

Consider $n = 3$ and let $f_i(x) = (a_i^\top x)^2 + \frac{1}{2} \|x\|^2$, where $a_1 = (-4, 3, 3)^\top$, $a_2 = (3, -4, 3)^\top$ and $a_3 = (3, 3, -4)^\top$. 

Zhize Li
A counter-example

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- Let $x^0 = (b, b, b)$, and the compressor be $\text{top}_1$,
  \[
  \nabla f_1(x^0) = b(-15, 13, 13)^\top \quad \rightarrow \quad C(\nabla f_1(x^0)) = b(-15, 0, 0)^\top \\
  \nabla f_2(x^0) = b(13, -15, 13)^\top \quad \rightarrow \quad C(\nabla f_2(x^0)) = b(0, -15, 0)^\top \\
  \nabla f_3(x^0) = b(13, 13, -15)^\top \quad \rightarrow \quad C(\nabla f_3(x^0)) = b(0, 0, -15)^\top 
  \]
A counter-example

Consider $n = 3$ and let $f_i(x) = (a_i^\top x)^2 + \frac{1}{2} \|x\|^2$, where $a_1 = (-4, 3, 3)^\top$, $a_2 = (3, -4, 3)^\top$ and $a_3 = (3, 3, -4)^\top$.

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  \nabla f_3(x^0) = b(13, 13, -15)^\top \quad\rightarrow\quad C(\nabla f_3(x^0)) = b(0, 0, -15)^\top \\
\]

- The next iteration
  \[
  x^1 = x^0 - \eta \frac{1}{3} \sum_{i=1}^{3} C(\nabla f_i(x^0)) = (1 + 5\eta)x^0,
  \]
  and then $x^t = (1 + 5\eta)^t x^0$ diverges exponentially.
A better scheme: shift compression

(Stich et al., 2018; Richtárik et al., 2021)

- The PS updates the model:

\[ x^{t+1} = x^t - \frac{\eta}{n} \sum_{i=1}^{n} g_i^t \]

— \( g_i^t \) is the compressed surrogate of \( \nabla f_i(x^t) \)
A better scheme: shift compression

(Stich et al., 2018; Richtárik et al., 2021)

- The PS updates the model:

\[ x^{t+1} = x^t - \eta \sum_{i=1}^{n} g^t_i \]

- \( g^t_i \) is the compressed surrogate of \( \nabla f_i(x^t) \)

- Clients update \( g^t_i \) with a shift compression:

\[ g^{t+1}_i = g^t_i + C(\nabla f_i(x^{t+1}) - g^t_i) \]

- \( g^t_i \) is constructed accumulatively over time
Let’s revisit the example

- Let $x^0 = (b, b, b)$, and the compressor be $\text{top}_1$, $g^0_i = C(\nabla f_i(x^0))$, and the first iteration is still $x^1 = (1 + 5\eta)x^0$.
Let’s revisit the example

- Let $x^0 = (b, b, b)$, and the compressor be $\text{top}_1$, $g^0_i = C(\nabla f_i(x^0))$, and the first iteration is still $x^1 = (1 + 5\eta)x^0$.

- Error feedback:

$$\nabla f_1(x^1) - g^0_1 = b \begin{bmatrix} -75\eta \\ 13(1 + 5\eta) \\ 13(1 + 5\eta) \end{bmatrix}$$
Let’s revisit the example

- Let \( x^0 = (b, b, b) \), and the compressor be \( \text{top}_1 \), \( \mathbf{g}^0_i = \mathcal{C}(\nabla f_i(x^0)) \), and the first iteration is still \( x^1 = (1 + 5\eta)x^0 \).

- Error feedback:

\[
\nabla f_1(x^1) - \mathbf{g}^0_1 = b \begin{bmatrix}
-75\eta \\
13(1 + 5\eta) \\
13(1 + 5\eta)
\end{bmatrix}
\]

and as long as \( \eta < 13/30 \):

\[
\mathcal{C} \left( \nabla f_1(x^1) - \mathbf{g}^0_1 \right) = b \begin{bmatrix}
0 \\
13(1 + 5\eta) \\
0
\end{bmatrix}
\]

receiving information from coordinates other than the first one, leading to a better compressed gradient!
Let’s revisit the example

- Let $x^0 = (b, b, b)$, and the compressor be $\text{top}_1$, $g^0_i = C(\nabla f_i(x^0))$, and the first iteration is still $x^1 = (1 + 5\eta)x^0$.

- **Error feedback:**

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  receiving information from coordinates other than the first one, leading to a better compressed gradient!

We’ll consider algorithms using shift compression!
This talk: communication-compressed algorithms

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Network/decentralized model
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This talk: communication-compressed algorithms

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*Coping with heterogeneity*
This talk: communication-compressed algorithms

Server/client model
PS coordinates global information sharing

Coping with privacy

Network/decentralized model
agents share local information over a graph topology

Coping with heterogeneity
BEER: Fast Decentralized Nonconvex Optimization with Communication Compression

Haoyu Zhao
Princeton

Boyue Li
CMU

Zhize Li
CMU

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KAUST
The mixing of information is characterized by a mixing matrix $W = [w_{ij}] \in \mathbb{R}^{n \times n}$ aligned with the network topology.
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The spectral quantity, which we call the spectral gap,

$$\rho \triangleq 1 - |\lambda_2(W)| \in (0, 1]$$

captures how fast information mixes over the network.

Goal: design fast-converging algorithms with communication compression
Data heterogeneity

Heterogeneity measure

\[ \mathbb{E}_i \| \nabla f_i(x) - \nabla f(x) \|^2 \leq G^2 \]

- local obj.
- global obj.
Data heterogeneity

Heterogeneity measure

$$\mathbb{E}_i \| \nabla f_i(x) - \nabla f(x) \|^2 \leq G^2$$

— $G$ can be unbounded!
Prior art

CHOCO-SGD (Koloskova et al., 2019) / DeepSqueeze (Tang et al., 2019):

- slow convergence rates (need more communication rounds)
- Incompatible with heterogeneity

Can we converge at the rate $O\left(\frac{G}{\varepsilon^{3/2}}\right)$ under arbitrary heterogeneity?

Yes, by using gradient tracking!
Prior art

CHOCO-SGD (Koloskova et al., 2019) / DeepSqueeze (Tang et al., 2019):

$O\left(\frac{G}{\varepsilon^{3/2}}\right)$

$O\left(\frac{1}{\varepsilon}\right)$
Prior art

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CHOCO-SGD (Koloskova et al., 2019) / DeepSqueeze (Tang et al., 2019):
- slow convergence rates (need more communication rounds) and
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Can we converge at the rate $O\left(\frac{1}{\varepsilon}\right)$ under arbitrary heterogeneity?

Yes, by using gradient tracking!
Detour: DGD with gradient tracking

Centralized Gradient Descent (GD):

\[ x^t = x^{t-1} - \eta \nabla f(x^{t-1}) \]
Detour: DGD with gradient tracking

Centralized Gradient Descent (GD):

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Decentralized Gradient Descent (DGD):

\[ x^t_i = \sum_j w_{ij} x^{t-1}_j - \eta \nabla f_i(x^{t-1}_i) \]

**mixing**  
**local gradient**
Detour: DGD with gradient tracking

**Centralized Gradient Descent (GD):**

\[
x^t = x^{t-1} - \eta \nabla f(x^{t-1})
\]

Constant step size, linear convergence for strongly convex problems.

**Decentralized Gradient Descent (DGD):**

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Constant step size, does not converge!

At optimal point \( x^* \): \( \nabla f(x^*) = 0 \), but \( \nabla f_i(x^*) \neq 0 \)

How do we fix this?
DGD with gradient tracking

Use dynamic average consensus (Zhu and Martinez, 2010) to track the global gradient $s^t_i$:

$$x^t_i = \sum_j w_{ij} x^t_j - \eta s^t_i$$

$$(\text{mixing})$$

$$s^t_i = \sum_j w_{ij} s^{t-1}_j + \nabla f_i(x^t_i) - \nabla f_i(x^{t-1}_i)$$

$$(\text{mixing}, \text{gradient tracking})$$

This trick, and other alternatives, have been used extensively to fix the non-convergence issue in decentralized optimization.

- EXTRA (Shi, Ling, Wu and Yin, 2015);
- NEXT (Di Lorenzo and Scutari, 2016);
- NIDS (Li, Shi, Yan, 2017);
- ADD-OPT (Xi, Xin, and Khan, 2017);
- DIGING (Nedic, Olshevsky, and Shi, 2017);
- DGD (Qu and Li, 2018);
- many, many more...
Use dynamic average consensus (Zhu and Martinez, 2010) to track the global gradient $s_i^t$:

$$x_i^t = \sum_j w_{ij} x_j^{t-1} - \eta s_i^t$$

$$s_i^t = \sum_j w_{ij} s_i^{t-1} + \nabla f_i(x_i^t) - \nabla f_i(x_i^{t-1})$$

This trick, and other alternatives, have been used extensively to fix the non-convergence issue in decentralized optimization.
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Use dynamic average consensus (Zhu and Martinez, 2010) to track the global gradient $s_i^t$:

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$\text{mixing}$

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s_i^t = \sum_j w_{ij} s_j^{t-1} + \nabla f_i(x_i^t) - \nabla f_i(x_i^{t-1})
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$\text{mixing}$ $\text{gradient tracking}$

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- EXTRA (Shi, Ling, Wu and Yin, 2015); NEXT (Di Lorenzo and Scutari, 2016); NIDS (Li, Shi, Yan, 2017); ADD-OPT (Xi, Xin, and Khan, 2017); DIGING (Nedic, Olshevsky, and Shi, 2017); DGD (Qu and Li, 2018);
- many, many more...
BEER: gradient tracking + shift compression

\(X = [x_1, x_2, \cdots, x_n]\): local models.
\(\nabla F(X) = [\nabla f_1(x_1), \nabla f_2(x_2), \cdots, \nabla f_n(x_n)]\): local gradients.
BEER: gradient tracking + shift compression

\[ X = [x_1, x_2, \cdots, x_n] \]: local models.
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- **model update:**

\[
X^{t+1} = X^t + \gamma H^t (W - I) - \eta V^t
\]

where \( H^t \) is the accumulated compressed surrogate of \( X^t \), and \( V^t \)
is the global gradient estimates across the agents.
BEER: gradient tracking + shift compression

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- **gradient tracking:**

\[
V^{t+1} = V^t + \gamma G^t (W - I) + \nabla F(X^{t+1}) - \nabla F(X^t),
\]

where \( G^t \) is the accumulated compressed surrogate of \( V^t \).
BEER: gradient tracking + shift compression

\[ X = [x_1, x_2, \cdots, x_n]: \text{local models.} \]
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\]

where \( G^t \) is the accumulated compressed surrogate of \( V^t \).

- Both \( H^t \) and \( G^t \) are updated using shift compression.
Theoretical convergence of BEER

Theorem (Zhao et al., 2022)

To achieve $\mathbb{E}\|\nabla f(x^{\text{output}})\|^2 \leq \varepsilon$, BEER requires at most

$$O\left(\frac{1}{\rho^3 \alpha \varepsilon}\right)$$

communication rounds, without the bounded heterogeneity assumption. Here, $\alpha$ is the compression ratio, $\beta$ is the spectral gap of the network.
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communication rounds, without the bounded heterogeneity assumption. Here, $\alpha$ is the compression ratio, $\beta$ is the spectral gap of the network.

- Assuming constant $\alpha$ and $\rho$, the convergence rate of BEER is

$$O\left(\frac{1}{\varepsilon}\right).$$

- Our result can also be extended to using stochastic gradients.
Theoretical convergence of BEER

BEER converges at the rate $O\left(\frac{1}{\varepsilon}\right)$ under arbitrary heterogeneity!
**BEER vs CHOCO-SGD**

![Graphs showing training gradient norm and testing accuracy against communication rounds for classification on the unshuffled MNIST dataset using a simple neural network. Both BEER and CHOCO-SGD employ the biased gsgd\(_b\) compression with \(b = 20\).](image)

**Figure:** Training gradient norm and testing accuracy against communication rounds for classification on the *unshuffled* MNIST dataset using a simple neural network. Both BEER and CHOCO-SGD employ the biased gsgd\(_b\) compression with \(b = 20\).
SoteriaFL: A Unified Framework for Private FL with Communication Compression

Zhize Li
CMU

Haoyu Zhao
Princeton

Boyue Li
CMU
A little privacy, please

“Before I write my name on the board, I’ll need to know how you’re planning to use that data.”

Privacy guarantees are becoming increasingly critical!
Protecting local privacy via differential privacy

Introducing local differential privacy to guarantee the client privacy
Protecting local privacy via differential privacy

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Protecting local privacy via differential privacy

Introducing local differential privacy to guarantee the client privacy

— used by Google, Apple, etc in products
Theorem (Li et al., 2022)

CDP-SGD achieves \((\epsilon, \delta)\)-LDP, and the utility

\[ E \| \nabla f(x_{\text{output}}) \|^2 \lesssim \frac{1}{m\epsilon} \sqrt{d \log(1/\delta)} \cdot \frac{\alpha n}{m\epsilon} \sqrt{d \log(1/\delta)} + \frac{\alpha n^2 \epsilon^2 \log(1/\delta)}{m\epsilon} . \]

- Larger \( \sqrt{\log(1/\delta)}/\epsilon \) gives stronger privacy, worse accuracy, fewer communication.
- Caveat: the communication complexity is \( O(m^2) \) when the local data size \( m \) is dominating.
Warm-up: a direct compression approach (CDP-SGD)

Theorem (Li et al., 2022)

CDP-SGD achieves \((\epsilon, \delta)\)-LDP, and the utility

\[
\mathbb{E}\|\nabla f(x^{\text{output}})\|^2 \lesssim \frac{1}{m\epsilon} \sqrt{\frac{d \log(1/\delta)}{\alpha n}}
\]

within communication complexity on the order of

\[
\frac{n^{3/2} \alpha^{3/2} m \epsilon}{\sqrt{\log(1/\delta)}} + \frac{\alpha n m^2 \epsilon^2}{\log(1/\delta)}.
\]
**Theorem (Li et al., 2022)**

CDP-SGD achieves $(\epsilon, \delta)$-LDP, and the utility

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$$

within communication complexity on the order of

$$
n^{3/2} \alpha^{3/2} m\epsilon \sqrt{\frac{d}{\log(1/\delta)}} + \frac{\alpha nm^2 \epsilon^2}{\log(1/\delta)}.
$$

- Larger $\frac{\sqrt{\log(1/\delta)}}{\epsilon}$ gives stronger privacy, worse accuracy, fewer communication.
### Theorem (Li et al., 2022)

CDP-SGD achieves \((\epsilon, \delta)\)-LDP, and the utility

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\mathbb{E}\|\nabla f(x^{\text{output}})\|^2 \lesssim \frac{1}{m\epsilon} \sqrt{\frac{d \log(1/\delta)}{\alpha n}}
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within communication complexity on the order of

\[
\frac{n^{3/2} \alpha^{3/2} m \epsilon \sqrt{d}}{\log(1/\delta)} + \frac{\alpha nm^2 \epsilon^2}{\log(1/\delta)}.
\]

- Larger \(\frac{\sqrt{\log(1/\delta)}}{\epsilon}\) gives stronger privacy, worse accuracy, fewer communication.

- **Caveat:** the communication complexity is \(O(m^2)\) when the local data size \(m\) is dominating.
Better compression and compute: a unified framework?

- **Compression**: shift compression with many options, e.g. sparsification or quantization
- **Computation**: stochastic local gradient estimators with many options, e.g. SGD, SVRG or SAGA
Better compression and compute: a unified framework?

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- **Computation**: stochastic local gradient estimators with many options, e.g. SGD, SVRG or SAGA

Can we develop a unified framework for private FL with compression, with a characterization of the privacy-utility-communication trade-off?
SoteriaFL: a unified framework for compressed private FL

Highlights of SoteriaFL:

- Flexible local gradient estimators
- Protect local data privacy
- State-of-the-art shift compression scheme
- Privacy-utility-communication trade-offs
Theorem (Li et al., 2022)

When $n \geq 1/\alpha^3$, SoteriaFL—with SGD, GD, SVRG, SAGA—achieves $(\epsilon, \delta)$-LDP, and the utility

$$\mathbb{E}\|\nabla f(x^{\text{output}})\|^2 \lesssim \frac{1}{m\epsilon} \sqrt{\frac{d \log(1/\delta)}{\alpha n}}$$

with communication complexity on the order of

$$n^{3/2} \alpha^{3/2} m\epsilon \sqrt{\frac{d}{\log(1/\delta)}}.$$ 

- Communication complexity is linear in $m$, better than CDP-SGD!
- Our analysis applies to unbiased compressions, and adapts to other gradient estimators too.
Privacy-utility-communication trade-off

- Stronger privacy, worse accuracy, fewer communication
- More compression, worse accuracy, fewer communication
**Numerical experiments**

**Figure**: Shallow NN training on the MNIST dataset under $(1, 10^{-3})$-LDP.
Provably efficient communication-compressed FL algorithms for heterogeneous and private data!

Future work:
- Client-adaptive privacy-preserving decentralized algorithms under data heterogeneity.
1. **BEER: Fast $O(1/T)$ Rate for Decentralized Nonconvex Optimization with Communication Compression**

2. **SoteriaFL: A Unified Framework for Private Federated Learning with Communication Compression**