A Wavelet Balance Approach for Steady-State Analysis of Nonlinear Circuits*

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ABSTRACT

In this paper, a novel wavelet balance method is proposed for steady-state analysis of nonlinear circuits. The proposed method presents several merits compared with those conventional frequency domain techniques. First, it has a high convergence rate. Second, it works in time domain so that many critical problems in frequency domain can be handled efficiently. Third, an adaptive scheme exists to automatically select the wavelet basis functions needed at a given accuracy.

1. INTRODUCTION

A major difficulty in time domain simulation of nonlinear circuits, such as power supplies, high-Q amplifiers, modulator and oscillators, etc., is that the transient response may stand for quite a long time before the steady-state is reached. This problem makes it infeasible to calculate the steady-state response by conventional transient simulation algorithms because direct integration of the circuit equations throughout the transients consumes unbearable computing time.

During the past decades, a great number of techniques have been developed to solve the periodic steady-state problem [1]-[6], which can be categorized into three classes: shooting methods [1]-[2], harmonic balance methods [3]-[5] and sample balance methods [6]. The shooting methods attempt to find a set of initial conditions satisfying the two-point boundary constraint, such that the circuit starts in periodic steady state directly. However, the shooting methods consume expensive computing time since they require to numerically integrate the system equations time after time. The harmonic balance methods assume the circuit solutions in the form of Fourier series. Moreover, they divide the circuit into a linear and a nonlinear part so that the linear subnetwork can be solved efficiently in frequency domain. Unfortunately, the harmonic balance methods need to repeatedly execute DFT and IDFT operations during the solution process, and employ a large number of harmonic components to achieve an accurate simulation result. Therefore, they also expend substantial computing time. The sample balance methods directly approximate the time-domain state-variable waveforms by suitable basis functions, such as periodic cubic splines, and use time-domain samples as problem unknowns. Nevertheless, there doesn't exist a strong theory to ensure the convergence features of these methods and it is often determined by experience that how many basis functions are needed at a given accuracy.

Recently, the wavelet theory has been well developed [7], [8] and widely used in many applications, such as solving partial different equations [9] and performing high-speed circuit simulations [10], [11]. However, the wavelet-based method for steady-state analysis has never been explored. In this paper, we propose a novel wavelet-based algorithm, named the *wavelet balance method*, to access the periodic steady-state problem. Taking advantage of the wavelets, the proposed method works more efficiently in time domain than those frequency domain techniques, and has a solid theoretical background based on wavelet approximation theory.

The rest of the paper is organized as follows. In Section 2, we first introduce the basic principle of the wavelet balance method, then compare the proposed algorithm with the conventional Fourier-based algorithms [3]-[5]. In Section 3 we present numerical experiments to demonstrate the computational efficiency of the proposed algorithm, and draw conclusions in Section 4.

2. WAVELET BALANCE APPROACH FOR STEADY-STATE ANALYSIS

2.1 The Wavelet Balanced Steady-State Analysis

Without loss of generality, we assume that a circuit is described by an ordinary differential equation of the type

$$\frac{dX}{dt} = f(X,t) \qquad (1)$$

where $X(t)=[X_1(t) \;\; X_2(t) \;\; \cdots \;\; X_N(t)]^T$ are the N unknown state variables, and $f(X,t)$ is a given nonlinear vector function.

The basic idea of the wavelet balance method in this paper is to expand the unknown state variables by wavelet series, but not the Fourier series that have been used in the conventional techniques [3]-[5]. Assume the steady-state response period of the circuit is T, and the Sobolev space $H^2[0,L]$ is studied for wavelet expansion. Usually, because the period T doesn't satisfy some specific conditions needed for wavelets, for example, $L \geq 4$ corresponding to the wavelet basis functions in [10], [11], a scaling operation should be applied to map the interval $[0,T]$ to $[0,L]$,

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i.e.

$$l = K \cdot t \tag{2}$$

where $t \in [0,T]$, $l \in [0,L]$, and K is a constant which equals to L/T. Substituting (2) into (1), we obtain

$$K \cdot \frac{dX}{dl} = f\left(X, \frac{l}{K}\right) \tag{3}$$

In order to find the steady-state solution, the state variables $X(l)$ is expanded by wavelets

$$X(l) = \begin{bmatrix} X_1(l) \\ \vdots \\ X_N(l) \end{bmatrix} = \begin{bmatrix} C_{11} & \cdots & C_{1M} \\ \vdots & \vdots & \vdots \\ C_{N1} & \cdots & C_{NM} \end{bmatrix} \cdot \begin{bmatrix} B_1(l) \\ \vdots \\ B_M(l) \end{bmatrix} = C \cdot B(l) \tag{4}$$

where $C \in R^{N \times M}$ is *the coefficient matrix*, $\{B_i(l), i = 1,2,\cdots,M\}$ are wavelet basis functions, and M is the total number of basis functions that have been employed. The wavelet basis functions can be constructed by many means [8], but in this paper, we prefer to use the basis functions in [10], [11] because they are proved to have a high convergence rate $O(h^4)$, where h is the step length [10], [11].

Substituting (4) into (3), we get

$$K \cdot C \cdot \frac{dB(l)}{dl} = f\left(CB(l), \frac{l}{K}\right) \tag{5}$$

Then, discretizate the state equation (5) at some interior collocation points $\{l_1, l_2, \cdots l_M\}$ [10], [11]

$$K \cdot C \cdot \left[\frac{dB(l_1)}{dl} \cdots \frac{dB(l_M)}{dl}\right] = \left[f\left(CB(l_1), \frac{l_1}{K}\right) \cdots f\left(CB(l_M), \frac{l_M}{K}\right)\right] \tag{6}$$

The above equations, from (1) to (6), are similar to those illustrated in [10] and [11]. But for steady-state response, the state variables $X(l)$ should further satisfy the two-point boundary constraint

$$X(0) = X(L) \tag{7}$$

Thus, we get one more equation for the coefficient matrix C

$$C \cdot B(0) = C \cdot B(L) \tag{8}$$

From (6) and (8), we define the merit function

$$Q = \left\| CB(0) - CB(L) \quad KC\frac{dB(l_1)}{dl} - f\left(CB(l_1), \frac{l_1}{K}\right) \quad \cdots \quad KC\frac{dB(l_M)}{dl} - f\left(CB(l_M), \frac{l_M}{K}\right) \right\|^2 \tag{9}$$

where $\|\bullet\|$ denotes the Frobenius norm. It is obvious that both (6) and (8) will hold as long as the merit function Q reaches its minimum value, i.e. zero.

For nonautonomous circuits, the steady-state response period T is determined by the input excitations, and the constant K in (9) is known in advance. Therefore, some optimization algorithm, such as the Levenberg-Marquardt method in [12], can be employed to find out the optimal coefficient matrix C, as well as the steady-state solutions

$$X(l) = C \cdot B(l) \qquad X(t) = C \cdot B(Kt) \tag{10}$$

On the other hand, in the autonomous cases, the nonlinear vector function $f(X,t)$ in (1) only depends on the state variables so that the merit function in (9) may be simplified to

$$Q = \left\| CB(0) - CB(L) \quad K \cdot C\frac{dB(l_1)}{dl} - f(CB(l_1)) \quad \cdots \quad K \cdot C\frac{dB(l_M)}{dl} - f(CB(l_M)) \right\|^2 \tag{11}$$

Here, because the oscillation period T is unknown, the constant K in (11) needs also to be determined by optimization process.

In summary, the essence of the proposed wavelet balance method is to approximate the state variables by wavelet basis functions, but not the Fourier bases $e^{j2\pi k/T}$ as in many conventional algorithms. Moreover, the wavelets are forced to keep "balance" at the two boundary points in (8), so that the steady-state solution is guaranteed when the state equation is solved.

2.2 Adaptive Technique

One of the main advantages of the wavelet balance method is that there exists an adaptive scheme, which relies on the multiresolution analysis in wavelet theory [9]-[11]. Using adaptive techniques, the number of the wavelet basis functions, which is needed for approximating state variables, can be determined automatically, and this improves the computational efficiency significantly.

A multiresolution approximation of $H^2[0,L]$ is a sequence $\{V_J, J = \cdots, -1, 0, 1, \cdots\}$ of closed subspaces of $H^2[0,L]$ such that:

1. $\cdots \subset V_{-1} \subset V_0 \subset V_1 \subset \cdots$
2. $\bigcap_{J=-\infty}^{+\infty} V_J = \{0\}$
3. $\bigcup_{J=-\infty}^{+\infty} V_J = H^2[0,L]$
4. $V_J = V_{J-1} \oplus W_J$

where the notation $\oplus$ stands for the direct sum. It is clear that the approximation accuracy depends on the wavelet space order J. The higher the space order is, the less the error will be. Furthermore, it is pointed out in [9]-[11] that the wavelet coefficients of an approximation to a function reflect the singularity of the function. Thus, the magnitude of the wavelet coefficients in W_J will indicate whether a refinement, by increasing the wavelet space order, is needed or not. For example, define the maximum relative magnitude of the wavelet coefficients in W_J as

$$R_J = \frac{MAX\left|C_i^J\right|}{MAX\left|C_i\right|} \tag{12}$$

where $MAX\left|C_i^J\right|$ is the maximum magnitude of the wavelet coefficients in W_J, and $MAX\left|C_i\right|$ is the maximum magnitude of all the wavelet coefficients. If R_J is greater than the given error tolerance ε, then we increase the wavelet space order J to J', where $J' > J$ [11].

More importantly, because of compact support of the wavelet bases, not all wavelet basis functions in higher wavelet spaces $W_{J'}$ are needed in order to improve the accuracy. In fact, only basis functions, whose positions near the singularities (which can be determined by the magnitude of the wavelet coefficients on order J), shall be included [11].

2.3 Comparison with the Conventional Fourier-Based Techniques

The key feature of the wavelet balance method is the fact that wavelet bases have compact support or local support in time domain, whereas Fourier bases $e^{j2\pi k/T}$ have a global support. Hence, the proposed wavelet balance method presents following advantages.

First, the wavelet balance method has a high convergence rate, resulting in low computational complexity. It is demonstrated in [10], [11] that the wavelet basis functions presented there, which are also used in this paper, have a high convergence rate $O(h^4)$, where h is the step length, i.e. the distance between two adjacent collocation points.

Second, the wavelet balance method works in time domain, so that many critical problems in frequency domain, such as nonlinearity and high order harmonics, can be handled efficiently.

Third, an adaptive scheme exists to automatically select the wavelet basis functions and determine the wavelet space order needed at a given accuracy.

3. NUMERICAL EXPERIMENTS

In this section, two circuit examples are examined to demonstrate the effectiveness of our proposed wavelet balance method for steady-state analysis.

3.1 Van der Pol Oscillator

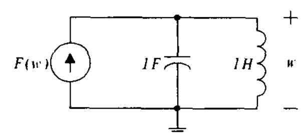

Fig. 1. Circuit schematic of Van der Pol oscillator where $F(w)=5\cdot(w-w^3/3)$.

Shown in Fig. 1 is a highly nonlinear Van der Pol oscillator, which has been served as a benchmark circuit for testing various steady-state analysis algorithms [2], [5] for autonomous circuits. The circuit solution by wavelet balance method is depicted in Fig. 2, where 63 basis functions are employed. Comparing Fig. 2 with that in [5], the waveform obtained in [5] with 60 samples still gives some small ringing, but our result in Fig. 2 shows great smoothness and almost achieves the same accuracy as that obtained in [5] with 240 samples.

By using wavelet balance method, the oscillator period T converges to $11.51s$ for 23 basis functions, $11.57s$ for 43 basis functions and $11.61s$ for 63 basis functions, respectively. The accuracy of T is also much better than that presented in [5], where the Fourier-based technique is used. Especially, a very accurate solution can be reached even at extremely low wavelet order, for example, only 23 basis functions are employed. This characteristic makes it possible to optimize a high order solution with initial values from the low order solution. It is obvious that such an optimization procedure is robust and may converge very fast.

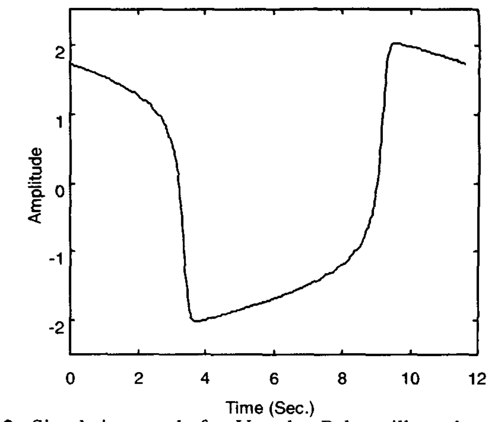

Fig. 2. Simulation result for Van der Pol oscillator by wavelet balance method.

3.2 MOS Amplifier

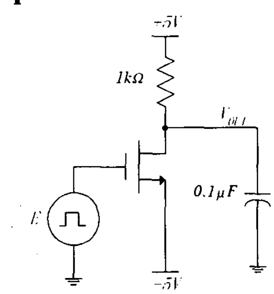

Fig. 3. Circuit schematic of MOS amplifier.

The MOS amplifier shown in Fig. 3 is used to test the wavelet balance method for nonautonomous circuits. The input excitation is a square wave of amplitude $\pm 1V$ and frequency $1kHz$. First, we simulate the circuit by harmonic balance method and Table 1 gives the relative simulation errors as different basis function numbers are employed. The relative error is defined as

$$Err_R = \sqrt{\frac{\int [y_{SPICE}(t) - y_{HB}(t)]^2 dt}{\int [y_{SPICE}(t)]^2 dt}} \quad (12)$$

where $y_{SPICE}(t)$ represents the exact steady-state response obtained by SPICE and $y_{HB}(t)$ represents the simulation response by the harmonic balance method. Note that the state equation should be integrated by SPICE for quite a long time until the transients die out, so that an accurate steady-state response can be obtained.

Second, we expand the state variables by wavelets and apply the wavelet balance method to calculate the steady-state response. When the adaptive technique is not used, all basis functions in given wavelet spaces are employed for computation. Table 2 gives the relative simulation errors under different wavelet basis function numbers. Comparing Table 2 with Table 1, one would find that the wavelet balance method hasn't gained any point in this special example and the convergence rate of the harmonic balance method is even a little higher than that of the wavelet balance method.

Finally, we show that the wavelet balance method can improve its efficiency significantly after the adaptive algorithm is applied. Let the error tolerance $\varepsilon = 10^{-3}$ and employ the adaptive technique to select proper basis functions and determine the necessary wavelet space order at such accuracy. Seven wavelet spaces (from order 0 to order 6) in all are applied by the adaptive algorithm during the course of iteration. Table 3 displays the relative simulation error and the total number of selected wavelet basis functions as different wavelet spaces are included. Two comments can be made according to the data in Table 1, Table 2 and Table 3.

1. After the adaptive algorithm is applied, the number of wavelet basis functions which are used at a given space order is reduced since not all basis functions in that wavelet space are needed for computation. On the other hand, the simulation error doesn't change much as shown in Table 2 and Table 3. This demonstrates that the adaptive algorithm has the potential to pick up the most important basis functions and those ones neglected by the adaptive scheme don't contribute a lot to the simulation accuracy.

Table 1. Simulation result of the harmonic balance method

Basis Function Number	Relative Simulation Error
21	9.606031×10^{-2}
41	5.034534×10^{-2}
81	2.481495×10^{-2}
161	1.155528×10^{-2}
321	4.832508×10^{-3}
641	1.624008×10^{-3}

Table 2. Simulation result of the wavelet balance method (without adaptive scheme)

Wavelet Space Order Employed	Overall Basis Function Number	Relative Simulation Error
0	23	9.731733×10^{-2}
1	43	5.427637×10^{-2}
2	83	2.770902×10^{-2}
3	163	1.328406×10^{-2}
4	323	5.888023×10^{-3}
5	643	2.250459×10^{-3}

Table 3. Simulation result of the wavelet balance method (with adaptive scheme)

Wavelet Space Order Employed	Overall Basis Function Number	Relative Simulation Error
0	23	9.731733×10^{-2}
1	43	5.427637×10^{-2}
2	77	2.770964×10^{-2}
3	123	1.329575×10^{-2}
4	195	5.933110×10^{-3}
5	319	2.440557×10^{-3}
6	515	1.233289×10^{-3}

2. Taking advantage of the property of compact support, the adaptive algorithm can automatically select proper wavelet basis functions in time domain to expand the state variables. High order basis functions are only included near singularities, which improves the computational efficiency significantly. However, Fourier bases have global support and it is infeasible to realize such a selection in time domain as wavelets. Therefore, as displayed in Table 1 and Table 3, the wavelet balance method exhibits higher convergence rate than the harmonic balance method after the adaptive scheme is employed.

4. CONCLUSION REMARKS

We present in this paper a novel wavelet balance method for steady-state analysis of nonlinear circuits. As a counterpart of those Fourier-based techniques, the wavelet balance method works in time domain and has a solid theoretical background based on wavelet approximation theory. Taking advantage of the superior computational properties of wavelets, the proposed method presents greater efficiency than the conventional Fourier-based techniques in many practical problems, which is both analyzed theoretically and confirmed by numerical experiments in the paper. In this point of view, the wavelet balance method exploits a new approach to access the steady-state problem besides those frequency domain methods that have been used for a long time.

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