Abstract — We define a notion of ‘sensing capacity’ that characterizes the ability of a sensor network to successfully distinguish among a discrete set of targets. Sensing capacity is defined as the maximum ratio of target positions to sensors for which inference of targets within a certain distortion is achievable. We demonstrate a lower bound on this capacity. Unlike previous work on ‘sensor network capacity’, our notion of sensing capacity is defined by the sensing task itself, as opposed to external resource constraints such as power, communications and processing.

I. INTRODUCTION

Sensor networks consist of a set of sensors that cooperatively sense an environment. Previous research on limits of performance of such networks concentrated on network channel capacity, under various resource constraints. In this paper, we consider a target detection problem and prove the existence of a ‘sensing capacity’ $C(D)$, such that, for a given tolerable distortion $D$, if the ratio of target positions to sensors is smaller than $C(D)$, the maximal average error probability converges to zero as the number of target positions (and sensors) goes to infinity.

Previous work on ‘sensor network capacity’ seeks to characterize the capacity by exploring the constraints imposed by power, communications, and computation. [2], [3] extend the results in [1] to a sensor network scenario and show communication-based limits on the amount of data that a sensor network can transport. [4] has considered the interaction between transmission rates and power constraints to obtain statements of capacity. Interestingly, [5] combines the notion of transport capacity with knowledge about the nature of the sensing task, which in his case is to sense an underlying continuous random process to within a given distortion. [6] extends this work by accounting for compression at each node.

In contrast, we provide a statement of sensing capacity inherent to the sensing task of detecting discrete targets to within a given distortion, rather than using external resource constraints. Section II introduces the sensor network model. Section III states the main theorem. Illustrative calculations of the sensing capacity are presented in Section IV. Section V extends the result to more general cases and Section VI concludes the paper.

II. SENSOR NETWORK MODEL

We denote random variables by upper-case letters and specific instantiations or constants by lower-case letters. Bold-font denotes vectors, whose length is clear from context. Bold-font upper-case letters denote random vectors. $\log(\cdot)$ has base-2.

We consider the problem of detecting discrete targets. An example of such work includes a target counting protocol for a sensor network consisting of seismic sensors implemented by [7]. In another example, a sensor network consisting of multiple cameras was designed to count the number of people in a crowd [8]. [9] proposed an abstract sensor network model for discrete target location. A coding-based approach was demonstrated to bound the minimum number of sensors required for discrimination, but no notion of sensing capacity was considered.

Our sensor network model is motivated by the following specific scenarios. In a seismic sensor network, each sensor can count the number of targets it senses (based on the intensity of vibration). Each sensor is affected by targets in several locations within a region, randomly distributed due to variations in soil composition. In a camera-based people-counting scenario, the view is broken into a target-sized grid, where each grid square may contain at most one person. In this scenario, each camera is affected by several grid squares randomly, due to random occlusions in its view. In large scale image processing to detect sparsely distributed targets, instead of searching over the entire image, one can break up the image into a grid and process random combinations of the grid squares to save computation. Element analysis provides another scenario of sensor cooperation. Rather than separately analyzing several substances for a constituent element, one can view the set of substances as a bit vector, where ‘1’ indicates the presence of the element. Combinations of substances can be analyzed (each analysis corresponds to one sensor) to detect the element.

The model we present here is a first-cut attempt to abstractly characterize the essence of various discrete target detection applications for sensor networks, as motivated by the above scenarios. Figure 1 shows our sensor network model. There are $k$ spatial positions that need to be sensed. Each position may represent an actual region in space, or may have other interpretations, such as a substance, in the elemental analysis example. Each position may contain no target or one target. A $k$-bit ‘target vector’ $v$ represents the target configuration in these $k$ positions. The Figure shows $v = (0,0,1,0,1,1)$ indicating 4 targets among the 7 positions. The possible target vectors are denoted $v_i$, $i \in \{1, \ldots, 2^k\}$. We say that ‘a certain $v$ has occurred’ if that vector represents the true target configuration in the spatial positions. The sensor network has $n$ identical sensors. Sensor $\ell$ is connected to (i.e., senses) exactly $c$ out of the $k$ spatial positions. Its function is to indicate the number of positions (among the $c$ positions it senses) $x \in \mathcal{X} = \{0, 1, \ldots, c\}$
that contain a target. For example, a seismic sensor can sense
the intensity of vibration to detect the number of targets. Thus,
the ‘ideal output vector’ of the sensor network \( \mathbf{x} \) depends on
the sensor connections, and on the target vector \( \mathbf{v} \) that occurs.
However, we assume that each sensor output \( y \in \mathbb{Y} \) is corrupted by
noise, so that the conditional p.m.f. \( P_{Y|X}(y|x) \) determines the
output. Since the sensors are identical, \( P_{Y|X} \) is the same for all
the sensors. Further, we assume that the noise is independent
in the sensors, so that the ‘sensor output vector’ \( y \) relates to the
ideal output \( x \) as \( P_{Y|X}(y|x) = \prod_{i=1}^{n} P_{Y_i|X_i}(y_i|x_i) \). Observing
the output \( y \), a decoder (described in detail below) must determine
which of the \( 2^k \) target vectors \( \mathbf{v}_i \) have actually occurred.

We define the sensor network \( S(k,n) \) as the bipartite graph
showing the connections of the sensors to the \( k \) spatial positions.
We assume a simple sensor network model, where each of the \( c \)
connections of each sensor is independently made to a spatial position,
chosen equi-probable among the \( k \) positions. Although this model is a simplification of general sensor networks, it accurately
describes the specific sensing scenarios described above. Furthermore, its analysis will, hopefully, motivate the analysis of
more complicated models in the future.

III. SENSOR NETWORK CAPACITY THEOREM

For such a randomly generated sensor network, the ideal output
\( \mathbf{x} \) is a function of the sensor network instantiation \( s(k,n) \) and the occurring target vector \( \mathbf{v} \). Considering \( X_i \) as the
random vector which occurs when \( \mathbf{v}_i \) is the target vector (i.e.,
\( X_i \) is random because of the random generation of \( S(k,n) \)), we can obtain the p.m.f. of \( X_i \) very simply. Since each sensor counts
the number of targets it senses, and its connections are formed independently.
\( P_{X_i}(x_i) = \prod_{i=1}^{n} P_{X_i}(x_{i,t}) \). However, it is important to note that the random vectors \( X_i \) and \( X_j \), associated with a pair of target vectors \( \mathbf{v}_i \) and \( \mathbf{v}_j \) respectively, are not independent, since the sensor connections produce a dependency
detected by them. However, the sensors are independent, given the
target vector, so that \( P_{X_i,X_j}(x_i,x_j) = \prod_{i=1}^{n} P_{X_i}(x_{i,t},x_{j,t}) \). Thus, the ‘codewords’ \( \{X_i, i = 1, 2, \ldots, 2^k \} \) of the sensor network are non-identical and dependent on each other, unlike channel codes in classical information theory.

Given the noise corrupted output \( y \) of the sensor network, we estimate the target vector \( \mathbf{v} \) which generated this noisy output
by using a decoder \( g(y) \). We allow the decoder a distortion of
\( D \in [0,1] \), i.e., if \( d_0(v_i, v_j) \) is the Hamming distance between
two target vectors and if we define the tolerable distortion region of
\( \mathbf{v}_i \) as \( D_i = \{ j : \frac{1}{c} d_0(v_i, v_j) < D \} \), then given that \( \mathbf{v}_i \) occurred,
the probability of error is \( P_{e,i} = \Pr[\text{error}|i, s, X_i, Y] \) \( = \Pr[Y|Y] \notin D_i \) \( i, s, X_i, Y_i \). Averaging this probability over all
sensor networks, we write the average error probability, given that \( \mathbf{v}_i \) occurred, as \( P_{e,i} = E[P_{e,i}|\mathbf{v}_i] \). We use the maximum
average error probability \( P_{e,max} = \max_i P_{e,i} \) as our error metric.

We define the ‘rate’ of the sensor network as the ratio of target
positions to sensors, \( R = \frac{k}{n} \). Given a tolerable distortion
\( D \), we call \( R \) achievable if the sequence of sensor networks
\( S([nR], n) \) satisfies \( P_{e,max} \to 0 \) as \( n \to \infty \). The sensing capacity
of the sensor network is defined as \( C(D) = \max R \) over achievable \( R \).

The main result of this paper is to show that the sensing capacity \( C(D) \) of the sensor network is non-zero, and to characterize it as a function of noise \( P_{Y|X} \) and sensor connections \( c \). The proof of this result broadly follows the proof of channel capacity provided by Gallager [10], by analyzing a union bound of pair-wise error probabilities, averaged over randomly generated sensor networks. However, it differs from [10] in several important ways. In our sensor network model, the distribution of the ‘encoder’ (i.e., sensor network generation) is fixed. Given the encoder (sensor network), the codewords are dependent on each other. Further, the ‘codebook’ \( \{x_i\} \) obtained is non-linear, so that techniques used to analyze linear random codes [11], which use the parity check matrix for analysis, are not applicable. However, since each sensor in our network counts the number of targets, our model is symmetric with respect to permutations of the target vector \( \mathbf{v} \). This allows us to use the method of types to group the exponential number of pair-wise error probability terms into a polynomial number of (joint) types in order to prove convergence of error probability.

The statement of the main result requires an explanation of joint types. Since each sensor counts the number of targets it observes, and the sensor makes each of its \( c \) connections to the spatial positions independently, therefore for each \( i \), the distribution of its ideal output \( X_i \) depends only on the type \( \gamma = (\gamma_0, \gamma_1) \) of the \( i \)th target vector \( v_i \), i.e., only on the number of \( 0 \)’s and \( 1 \)’s in \( v_i \). Here, \( \gamma_0 \) denotes the fraction of zeros in \( v_i \). Due to this permutation symmetry, \( P_{X_i}(x_i) = P^{\gamma_0}(x_i) = \prod_{t=1}^{n} P^{\gamma_0}(x_{i,t}) \) for all \( x_i \) of the same type \( \gamma \).

Next, we note that the conditional probability \( P_{X_j|X_i} \) depends on the joint type of the \( i \)th and \( j \)th target vectors, i.e., Let \( \lambda_{ij} \) be the fraction of positions in \( v_i, v_j \) where \( v_i \) has bit ‘0’ while \( v_j \) has bit ‘1’. Similarly, define \( \lambda_{00}, \lambda_{10}, \lambda_{11} \) and define \( \lambda = (\lambda_{00}, \lambda_{01}, \lambda_{10}, \lambda_{11}) \). We call \( \lambda \) the joint type of \( v_i, v_j \). Following the notation introduced in [12], \( \lambda \in \mathcal{P}_h \{0,1\}^2 \), indicating that \( \lambda \) is in the set of joint types of \( h \)-bit binary vector pairs. Again, since each sensor exhibits permutation symmetry, \( P_{X_j|X_i} \) depends only on the joint type \( \lambda \), i.e., \( P_{X_j|X_i}(x_j|x_i) = P^{\lambda n}(x_j|x_i) = \prod_{t=1}^{n} P^{\lambda}(x_{jt}|x_{it}) \) for all \( i, j \) of the same joint type \( \lambda \). Since the joint type \( \lambda \) also defines the type \( \gamma \) of \( v_i \), we must have \( \lambda_{00} + \lambda_{01} = \gamma_0, \lambda_{10} + \lambda_{11} = \gamma_1 \).
To illustrate, Table 1 lists the joint type of 4 vectors \( \mathbf{v}_j \) with \( i = 1 \) (Thus, \( \gamma = (5/8, 3/8) \) here.) As an example, consider a sensor network where each sensor is connected randomly to \( c = 2 \) spatial positions. Thus, each sensor has an ideal output alphabet \( \mathcal{X} = \{0, 1, 2\} \). Given two target vectors \( \mathbf{v}_i, \mathbf{v}_j \) of joint type \( \lambda \), a sensor will output ‘0’ for both target vectors only if both its connections are connected to spatial positions that have a ‘0’ bit in both these target vectors. This happens with probability \((\lambda_{00})^2\)

Table 2 lists the joint p.m.f.

<table>
<thead>
<tr>
<th>( \mathbf{v}_j )</th>
<th>((5/8,0,3/8))</th>
<th>((2/8,3/8,2/8,1/8))</th>
<th>((2/8,3/8,2/8,1/8))</th>
<th>((5/8,0,3/8))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{v}_1 )</td>
<td>011101000</td>
<td>010111100</td>
<td>010001111</td>
<td>000000000</td>
</tr>
<tr>
<td>( \mathbf{v}_2 )</td>
<td>000011110</td>
<td>000000000</td>
<td>000000000</td>
<td>000000000</td>
</tr>
<tr>
<td>( \mathbf{v}_3 )</td>
<td>001000111</td>
<td>001000111</td>
<td>001000111</td>
<td>001000111</td>
</tr>
<tr>
<td>( \mathbf{v}_4 )</td>
<td>000000000</td>
<td>000000000</td>
<td>000000000</td>
<td>000000000</td>
</tr>
</tbody>
</table>

To specify two probability distributions which we will utilize in the main theorem. The first is the joint distribution of the ideal output \( \mathbf{x}_i \) when \( \mathbf{v}_i \) occurs and the noise corrupted output \( \mathbf{y} \) caused by it. i.e., \( P_{X,Y}(x_i,y) = \prod_{i=1}^n P_{X,Y}(x_{ii},y_i) = \prod_{i=1}^n P_{X}(x_{ii})P_{Y}(y_i|x_{ii}) \). The second distribution is the joint distribution of the ideal output \( x_i \) corresponding to \( v_i \) and the noise corrupted output \( y \) generated by the occurrence of a different target vector \( \mathbf{v}_j \). We can write this joint distribution as \( Q_{X,Y}^i(x_i,y) = \prod_{i=1}^n Q_{X,Y}(x_{ii},y_i) = \prod_{a\in\mathcal{X}} P_{X}(x_{ii})P_{Y}(y_i|x_{ii}) \).

Since the sensor network exhibits permutation symmetry, \( P_{X,Y}(x_i,y) \) depends only on the type \( \gamma \) of \( v_i \). Thus, we write \( P_{X,Y}(x_i,y) = \prod_{i=1}^n Q_{X,Y}^i(x_{ii},y_i) \) where \( P_{X,Y}(x_{ii},y_i) = P_{Y}(y_i|x_{ii}) \). Similarly, \( Q_{X,Y}^i(x_i,y) \) depends only on the joint type \( \lambda \) of \( v_i, v_j \) and can be written as \( \prod_{i=1}^n Q_{X,Y}^i(x_{ii},y_i) \) where \( Q_{X,Y}^i(x_{ii},y_i) = \sum_{a\in\mathcal{X}} P_{X}(x_{ii})P_{Y}(y_i|x_{ii}) \).

Denoting \( D(P||Q) \) as Kullback-Leibler distance and \( H(P) \) as entropy, the sensing capacity, at distortion \( D \) is bounded as follows:

### Sensing Capacity Theorem

\[
C(D) \geq C_{LB}(D) = \min_{\lambda} \min_{\gamma} \frac{D(P_{X,Y}||Q_{X,Y}^i)}{H(\lambda) - H(\gamma)}
\]

where \( \gamma = (\gamma_0, \gamma_1) \) and \( \lambda = (\lambda_{00}, \lambda_{01}, \lambda_{10}, \lambda_{11}) \) are two arbitrary probability mass functions.

From the definition of \( Q_{X,Y}^i \), we notice that if the 'codewords' \( \mathbf{X} \) were independent, the Kullback-Leibler distance in (1) would reduce to the mutual information between \( \mathbf{X} \) and its noisy version \( \mathbf{Y} \). Further, the denominator in (1) accounts for the non-identical distribution of the codewords. The minimization over the joint type \( \lambda \) appears, because the closest pair of code-words dominates the error probability. Thus, the 'sensing capacity' is similar to classical channel capacity, with differences arising due to non-identical, dependent codeword distribution.

**Proof.** We assume a maximum-likelihood decoder \( g_{ML}(y) = \arg\max_j P_{Y|X}(y|x_j) \). For this decoder, we consider \( P_{e_{max}} = \max_i P_{e_{i}} \), where \( P_{e_{i}} \) is averaged over the random sensor network. We seek to bound \( P_{e_{i}} \), which we write out below.

\[
P_{e_{i}} = \sum_{x_i \in \mathcal{X}^n} \sum_{y \in \mathcal{Y}^n} P_{X}(x_i)P_{Y|X}(x_i|y_i)P[\text{error}|i,x_i,y]
\]

We bound the expression \( P[\text{error}|i,x_i,y] \) by defining events \( A_{ij} = \{x_j : P_{X}(y|x_j) > P_{Y}(y|x_i) \} \) and using the union bound. Since decoding to a \( j \notin D \) results in error,

\[
Pr[\text{error}|i,x_i,y] \leq P\left( \bigcup_{j \notin D} A_{ij} \right) \leq \sum_{j \notin D} P(A_{ij})
\]

We proceed to bound the probability \( P(A_{ij}) \). For any \( s_{ij} \geq 0 \),

\[
P(A_{ij}) = \sum_{x_j \in \mathcal{X}^n} P_{X}(x_j|x_i)
\]

\[
\leq \sum_{x_j \in \mathcal{X}^n} P_{X}(x_j|x_i) \frac{P_{Y}(y|x_j)^{s_{ij}}}{P_{Y}(y|x_i)^{s_{ij}}}
\]

Using (3) and (4) in (2),

\[
P_{e_{i}} \leq \sum_{x_i \in \mathcal{X}^n} \sum_{y \in \mathcal{Y}^n} P_{X}(x_i|x_i) \frac{P_{Y}(y|x_i)^{s_{ij}}}{P_{Y}(y|x_i)^{s_{ij}}}
\]

\[
\sum_{j \notin D} \sum_{x_j \in \mathcal{X}^n} P_{X}(x_j|x_i) \frac{P_{Y}(y|x_j)^{s_{ij}}}{P_{Y}(y|x_i)^{s_{ij}}}
\]

The bound (5) has an exponential number of terms. However, it was argued earlier that in our sensor network, \( P_{X}(x_i) = P_{Y}(x_j) \) depends only on the type \( \gamma \) of the \( i \)th target vector, while \( P_{X}(x_j|x_i) = \lambda_{ij}(x_j|x_i) \) depends on the joint type of the \( i \)th and \( j \)th target vectors . Thus, we can rewrite (5) by grouping terms according to their joint type \( \lambda \).

\[
\sum_{j \notin D} \sum_{x_j \in \mathcal{X}^n} P_{X}(x_j|x_i) \frac{P_{Y}(y|x_j)^{s_{ij}}}{P_{Y}(y|x_i)^{s_{ij}}}
\]

\[
\sum_{\lambda \in S(D)} \beta(i,\lambda;k) \sum_{x_j \in \mathcal{X}^n} P_{\lambda|\gamma}(x_j|x_i) \frac{P_{Y}(y|x_j)^{s_{ij}}}{P_{Y}(y|x_i)^{s_{ij}}}
\]

(6)
where

\[ S(D) = \{ \lambda: \lambda \in P_k(\{0,1\}^2), \lambda_0 + \lambda_1 > D, \lambda_0 + \lambda_1 = \gamma_0, \lambda_1 + \lambda_1 = \gamma_1 \} \quad (7) \]

and where we choose \( s_{ij} = s_\lambda \) for all \( \{i, j\} \) of joint type \( \lambda \). Here \( \beta(i, \lambda; k) \) is the number of vectors \( v_j \) which have a joint type \( \lambda \) with respect to \( v_i \). This is bounded as,

\[ \beta(i, \lambda; k) \leq \left( \frac{k_{\lambda_0} k_{\lambda_1}}{k_{\gamma_0} k_{\gamma_1}} \right)^k \leq 2^{k(H(\lambda) - H(\gamma))} \quad (8) \]

Combining equations (5), (6) and (8),

\[ P_{e, i} \leq \sum_{x_i \in X^n} \sum_{y_i \in Y^n} P^\gamma_{n}(x_i) P_{Y|X}(y|x_i) \sum_{x_j \in X^n} 2^{k(H(\lambda) - H(\gamma))} \cdot \sum_{a_i, a_j \in X} P^\lambda_{n}(x_j|x_i) \frac{P_{Y|X}(y|x_j|x_i) \rho^\lambda}{P_{Y|X}(y|x_i) \rho^\lambda} \quad (9) \]

Using the independence of the sensor outputs, the joint p.m.f.s can be simplified as below.

\[ P_{e, i} \leq \sum_{\lambda \in S(D)} 2^{\rho \lambda (H(\lambda) - H(\gamma))} \left[ \left( \sum_{a_i, a_j \in X} P^\gamma_{n}(a_i) P_{Y|X}(b|a_i) \right)^n \right] \left( \sum_{a_j \in X} P^\lambda_{n}(a_j|a_i) P_{Y|X}(b|a_j) \right) \quad (10) \]

We define the following quantity.

\[ E(\rho, \lambda) = -\log \left( \sum_{a_i, a_j \in X} P^\gamma_{n}(a_i) P_{Y|X}(b|a_i) \right)^n \left( \sum_{a_j \in X} P^\lambda_{n}(a_j|a_i) P_{Y|X}(b|a_j) \right) \rho^\lambda \quad (11) \]

Since the number of types of \( \lambda \) is upper bounded by \((k + 1)^4\), and \( k = \lfloor n R \rfloor \), implying \( k < n R + 1 \), (10) is bounded as,

\[ P_{e, i} \leq \max_{\lambda \in S(D)} \min_{0 \leq \rho \lambda \leq 1} 2^{-n \left( \frac{4 \log_n (n + 1)}{2} \right)} \cdot 2^{-n \left( H(\lambda) - H(\gamma) + E(\rho, \lambda) \right)} \]

We seek to bound \( \max_i P_{e, i} \). However, \( P_{e, i} \) only depends on the type \( \gamma \) of \( v_i \). Thus, we have the bound,

\[ P_{e, \max} \leq 2^{-n \left( -\alpha_1(n) + E_{\max}(R, D) \right)} \]

Combining equations (5), (6) and (8),

\[ E_{\max}(R, D) = \min_{\gamma} \alpha_1(n) \max_{\lambda \in S(D)} \min_{0 \leq \rho \lambda \leq 1} (E(\rho, \lambda) - \rho \lambda (H(\lambda) - H(\gamma))) \]

\[ \alpha_1(n) = \frac{4 \log(n + 2)}{n} + 1 \rho \lambda (H(\lambda) - H(\gamma)) \quad \text{(12)} \]

where \( \gamma = (\gamma_0, \gamma_1) \) is over all p.m.f.s, and \( S(D) \) is as in (7), with \( \gamma \). Note that \( \alpha_1(n) \to 0 \) as \( n \to \infty \), so we have not included it in the error exponent \( E_{\max}(R, D) \). Observing that \( E(0, \lambda) = 0 \ \forall \ \lambda \), we let \( \rho_\lambda \) go to zero, rather than optimizing it, thus resulting in a lower bound on \( E_{\max}(R, D) \). In the above expression, this implies that in order for \( R \) to be achievable \( E_{\max}(R, D) \to 0 \), it must be positive for all \( \lambda \), even as \( \rho_\lambda \to 0 \). But this implies that the derivative of \( E(\rho, \lambda) \) with respect to \( \rho_\lambda \) at \( \rho_\lambda = 0 \) must be greater than \( R(\lambda) - H(\gamma) \).

Write this derivative below.

\[ \frac{\partial E(\rho, \lambda)}{\partial \rho_\lambda} \bigg|_{\rho_\lambda=0} = \sum_{a_i, a_j \in X} \sum_{b \in Y} P^\gamma_{n}(a_i) P_{Y|X}(b|a_i) \cdot \sum_{a_j \in X} P^\lambda_{n}(a_j|a_i) P_{Y|X}(b|a_j) = \sum_{a_i, a_j \in X} \sum_{b \in Y} P^\gamma_{n}(a_i, b) P^\lambda_{n}(a_j|a_i) \frac{Q^\lambda_{n}(a_i, b)}{Q^\lambda_{n}(a_j|a_i)} = \sum_{a_i, a_j \in X} \sum_{b \in Y} \frac{Q^\lambda_{n}(a_i, b)}{Q^\lambda_{n}(a_j|a_i)} \quad (13) \]

IV. CAPACITY BOUND EXAMPLES

We compute the capacity bound \( C_{LB}(D) \) in (1) for various distortions, noise levels, and sensor resolutions. The sensor noise model assumed is that the probability of counting error decays exponentially with the error magnitude. In the figures, ‘Noise = p’ indicates that for a sensor, \( P(Y \neq X) = p \), with \( Y = X \) assumed. Also, ‘sensor resolution’ in bits is simply \( \log_2(e + 1) \). In Figure 2, we demonstrate \( C_{LB}(D) \) for various sensor noise levels and sensor resolutions. In all cases, \( C_{LB}(D = 0) = 0 \), since each sensor only has a fixed number of connections. Other obvious conclusions about the effect of noise and resolution can also be drawn.

Figure 3 shows \( C_{LB}(D) \) at \( D = 0.1 \), as a function of sensor noise level. This figure demonstrates that the random sensor network is more efficient than a strategy of simple sensor replica-
tion, which is a popular practical method to minimize error probability. For example, for 2-bit sensors, a rate of 0.38 is achievable at noise level 0.2. If instead, each sensor is replicated thrice (thus, requiring three times as many sensors, while also reducing the noise level to $3 \times (0.2)^2 \times 0.8 + (0.2)^3 = 0.1$ due to majority-decoding), then the resulting rate falls to $C_{LB}(0.1)/3 = 0.19$. Thus, the bound indicates that cooperative sensor strategies are significantly more efficient than sensor replication.

V. EXTENSIONS OF THE SENSOR NETWORK MODEL

We consider two straightforward extensions to our sensor network model. The first extension considers non-binary target vectors. Binary target vectors indicate the presence or absence of targets at the spatial positions. A target vector over a general finite alphabet may indicate, in addition to the presence of targets, the class of targets in the various positions. Assuming a target vector over alphabet $\mathcal{A}$, and a sensor model in which a sensor can indicate the number of occurrences of each letter of $\mathcal{A}$ that it senses, we obtain the capacity bound below.

$$C(D) \geq C_{LB}(D) = \min_{\gamma} \min_{\lambda} \sum_{\lambda_{ab} > D} D\left(\frac{P_{X,Y}^{\lambda}}{Q_{X,Y}^{\lambda}}\right)$$

where $\gamma = (\gamma_a, a \in \mathcal{A})$ and $\lambda = (\lambda_{ab}, (a, b) \in \mathcal{A}^2)$ are two arbitrary probability mass functions.

The second extension considers the case of heterogeneous sensors, where each class of sensor has a different output alphabet $\mathcal{Y}$ and noise model $P_{Y|X}$. Let the sensor of class $l$ be used with a given relative frequency $\alpha_l$. For such a model,

$$C(D) \geq C_{LB}(D) = \min_{\gamma} \min_{\lambda} \sum_{\lambda_{al1} > \lambda_{l1} > \lambda_{l0} > \lambda_{01}} \frac{\alpha_l D\left(\frac{P_{X,Y}^{\lambda}}{Q_{X,Y}^{\lambda}}\right)}{H(\lambda) - H(\gamma)}$$

where $\gamma = (\gamma_0, \gamma_1)$ and $\lambda = (\lambda_{00}, \lambda_{01}, \lambda_{10}, \lambda_{11})$ are two arbitrary probability mass functions.

VI. CONCLUSIONS

We introduced a notion of sensing capacity for discrete target detection. We proved a lower bound to this ‘sensing capacity’ (as opposed to ‘channel capacity’) and computed the bound for an illustrative example at various sensor resolutions, noise levels, and tolerable distortions. By examining this bound, we concluded that under some situations, simple sensor replication is inefficient compared to sensor cooperation. Future work will concentrate on generalizing the sensor network model.

REFERENCES