Reconstruction of 3-D Dense Cardiac Motion from Tagged MRI Sequences

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Outline

• Introduction

• Methodology: Prior knowledge + MRI data
  – Myocardial Fiber Based Structure
  – Continuum Mechanics
  – Constrained Energy Minimization

• Results and Conclusions
2-D Cardiac MRI Images

- $N$ slices
- $M$ frames per slice

Y. Sun, Y.L. Wu, K. Sato, C. Ho, and J.M.F. Moura,
Proc. Annual Meeting ISMRM 2003
3-D Reconstruction: myocardial fiber model

Use a fiber based model to find the correspondence between transversal slices.
3-D Reconstruction: fiber deformation model

Use continuum mechanics to describe the motion of fibers.

Fit the model to MRI data by constrained energy minimization.
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Prior Knowledge: myocardial anatomy

Multiple-layer view:

-60° Epicardium

+60° Endocardium

Mid-wall

Streeter, in *Handbook of Physiology Volume 1: the Cardiovascular System*, American Physiological Society, 1979
Prior Knowledge: fiber dynamics

Motion of a small segment

Displacement:
\[ u(t) = a(t) - a(0) \]

\[ a(t) + da(t) \]
\[ da(t) \]
\[ da(0) \]
\[ a(0) \]
\[ a(0) + da(0) \]

Notations are column vectors, ex:
\[ a(t) = \begin{bmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \end{bmatrix} \]

\[ da(t) = \frac{\partial a(t)}{\partial a(0)} da(0) \]
Deformation Gradient Matrix

\[ F(t) = \frac{\partial a(t)}{\partial a(0)} = \begin{bmatrix}
\frac{\partial a_1(t)}{\partial a_1(0)} & \frac{\partial a_1(t)}{\partial a_2(0)} & \frac{\partial a_1(t)}{\partial a_3(0)} \\
\frac{\partial a_2(t)}{\partial a_1(0)} & \frac{\partial a_2(t)}{\partial a_2(0)} & \frac{\partial a_2(t)}{\partial a_3(0)} \\
\frac{\partial a_3(t)}{\partial a_1(0)} & \frac{\partial a_3(t)}{\partial a_2(0)} & \frac{\partial a_3(t)}{\partial a_3(0)}
\end{bmatrix} \]

\[ = I + dF(t) = I + \frac{\partial u(t)}{\partial a(0)} = I + \begin{bmatrix}
\frac{\partial u_1(t)}{\partial a_1(0)} & \frac{\partial u_1(t)}{\partial a_2(0)} & \frac{\partial u_1(t)}{\partial a_3(0)} \\
\frac{\partial u_2(t)}{\partial a_1(0)} & \frac{\partial u_2(t)}{\partial a_2(0)} & \frac{\partial u_2(t)}{\partial a_3(0)} \\
\frac{\partial u_3(t)}{\partial a_1(0)} & \frac{\partial u_3(t)}{\partial a_2(0)} & \frac{\partial u_3(t)}{\partial a_3(0)}
\end{bmatrix} \]

**Deformation gradient** \( F(t) \) **is a function of displacement** \( u(t) \).
Strain

- Strain is the displacement per unit length, and is written mathematically as

\[
S = \frac{1}{2} (F^T F - I)
\]

\[
S = \frac{1}{2} [(I + dF)^T (I + dF) - I] = \frac{1}{2} [dF^T + dF + dF^T dF]
\]


- When strain is small, it is approximated as

\[
S \approx \frac{1}{2} [dF^T + I + dF + I] - I = \frac{1}{2} (F^T + F) - I
\]

(\textbf{Note:} S is symmetric)
Linear Strain Energy Model

- $S$ is symmetric, so we vectorize the entries at upper triangle.

$$S = \begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{12} & S_{22} & S_{23} \\
S_{13} & S_{23} & S_{33}
\end{bmatrix} \quad \Rightarrow \quad s = [S_{11}, S_{22}, S_{33}, S_{12}, S_{13}, S_{23}]^T$$

- Let $C$ describe the material properties. It can be shown the linear strain energy is $e = s^T Cs = e(u)$

- The entire energy of the heart:

$$E(U) = \sum_{\forall \text{fibers}} \sum_{\forall \text{segments}} e(u) = \sum_{\forall \text{fibers}} \sum_{\forall \text{segments}} s^T Cs$$
Constrained Energy Minimization

\[ E(U, \lambda) = \gamma_1 E_{\text{int}}(U) + \gamma_2 E_{\text{ext}}(U) + \lambda E_{\text{con}}(U) \]

- Internal energy: continuum mechanics governs the fibers to move as smooth as possible.

\[
E_{\text{int}}(U) = \sum \sum s^T Cs
\]

- External energy: pixel intensities of fibers should be kept similar across time.

\[
E_{\text{ext}}(U) = \| I(t) - I(t+1) \|^2
\]
2-D Displacement Constraints

\[
E(U, \lambda) = \gamma_1 E_{int}(U) + \gamma_2 E_{ext}(U) + \lambda E_{con}(U)
\]

**D**: 2-D displacements of the taglines

**ΩU**: picks the entries of \( U \) corresponding to \( D \)

2-D displacement constraints: \( \Omega U = D \)

\( \lambda \): Lagrange multiplier
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Data Set

- Transplanted rats with heterotopic working hearts.
- MRI scans performed on a Bruker AVANCE DRX 4.7-T system

4 slices

256×256 pixels per image

10 frames per slice

Fiber Based Model

Whole left ventricle

endocardium

mid-wall

epicardium
3-D Reconstruction of the Epicardium
Conclusions

- Take into account the *myocardial fiber based structure*.
- Adopt the *continuum mechanics* framework.
- Implement *constrained energy minimization* algorithms.