Signal-Dependent Correlation-Sensitive Branch Metrics for Viterbi-like Sequence Detectors

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ABSTRACT—By applying the Euclidian branch metric in a Viterbi-like detector when the noise is signal-dependent and correlated, the receiver falls short of the maximum likelihood sequence detector (MLSD). We introduce new signal-dependent correlation-sensitive branch metrics for Viterbi-like implementations of the MLSD. We also provide an analytic analysis that calculates the probability of error of such a detector and compares it to the performance of the Euclidian detector. The new metric is well suited for magnetic recording applications where, especially at high recording densities, the noise is both correlated and signal-dependent.

1. Introduction

The Viterbi implementation of the maximum likelihood sequence detector (MLSD) was derived for a finite state machine source (or equivalently a communications channel with a finite intersymbol interference length) and memoryless (usually white) noise [1], [2]. The branch metrics for this Viterbi-like implementation of the MLSD in a communications channel are then derived to be Euclidian distances between the received and the expected values in the received signal space. When the intersymbol interference (ISI) length is theoretically infinite, but negligible after some finite integer length K, it is common practice to consider the ISI to be practically finite and proceed with Viterbi detection using the Euclidian metric. Notice that for this approach to give optimal results, the noise still needs to be white. If the noise is colored (stationary and signal-independent), one may whiten the noise first and then approximate the ISI to a finite length to arrive to an asymptotically optimal detector, although the Viterbi trellis may have an impractically large number of states.

In magnetic recording applications, the solution strategy is typically to fix the number of states in the trellis (or a tree) to 2^K and find a filter [3], [4] that, under some optimality criterion, maximally whitens the noise, while making the ISI negligibly small after the length K. However, the noise filtered in this fashion is not white. Furthermore, at high recording densities the noise (consisting in large of media noise) is also signal dependent [5], [6], which means that it cannot be whitened nor can it be made signal independent by a linear time-invariant filter. In such environments, there is no justification (except maybe the simplicity of implementation) for using the the Euclidian metric as the branch metric of the Viterbi-like trellis/tree detector.

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In this paper, we address the question of choosing the optimal branch metric for the Viterbi-like implementation of the MLSD when the noise is both signal-dependent and correlated. In Section 2, we show that the Euclidian metric, as well as other previously derived signal-dependent metrics [7], [8], are only special cases of the general optimal signal-dependent correlation-sensitive branch metric. In the latter part of the section, we consider strategies for implementing the newly derived metric in Viterbi-like architectures. In Section 3, we perform a probability of error analysis that shows how much we gain by using the correlation-sensitive metric over the Euclidian metric. At the end of the paper, we show error rate performance results of applying different metrics in a magnetic recording channel where the noise is both signal-dependent and correlated.

2. Branch Metrics for MLSD

Let a_i, $1 \leq i \leq N$, be symbols (bits) transmitted through a communications channel (or written on a magnetic recording medium). Let r_i, $1 \leq i \leq N$, be sampled observations of the noisy waveform at the receiving end of the channel. We assume that the noise is additive and that the noise samples can be, in the most general case, both signal-dependent and correlated. The maximum likelihood sequence detector (MLSD) chooses that sequence $\hat{a}_1, \ldots, \hat{a}_N$ over all possible sequences $a_1, \ldots a_N$ for which the joint conditional probability density function (pdf)

$$f(r_1, r_2, \ldots, r_N | a_1, a_2, \ldots, a_N) \tag{1}$$

is maximized for the observed channel output $r_1, \ldots, r_N$.

In a channel where not only the signal, but also the noise, is dependent on the transmitted sequence $a_1, \ldots, a_N$, the functional form of (1) may be different for different transmitted symbol sequences. This distinction can be made by introducing indices to the function in (1), but we do not do it here to keep the notation as simple as possible.

Application of a Viterbi-like algorithm that implements MLSD is conditioned on factoring (1). The simplest case when this is possible is when the noise samples in the channel are independent random variables [1], [2]. In [9], we show that a similar factorization is possible under two more general conditions. First, the observation r_j needs to be statistically independent of all observations r_{i+j} if j satisfies $j > L$, where L is a nonnegative integer. We call L the correlation length of noise. This formulation is anticausal. One can make an equivalent causal formulation, see [9] for details. The second condition that needs to be satisfied is that the intersymbol interference (ISI) length is finite. In general, let the leading (causal) ISI length be K_l and the trailing (anticausal) ISI length be K_t. We define $K = K_t + K_l + 1$ as the total ISI length, which we require to be finite. Under these conditions, the pdf in (1) factors to [9]

$$f(r_1, \ldots, r_N | a_1, \ldots, a_N) = \\ = \prod_{i=1}^{N} \frac{f(r_i, r_{i+1}, \ldots, r_{i+L} | a_{i-K_l}, \ldots, a_{i+L+K_t})}{f(r_{i+1}, \ldots, r_{i+L} | a_{i-K_l}, \ldots, a_{i+L+K_t})}. \quad (2)$$

The form given in (2) is suitable for applying a Viterbi-like detection algorithm that is now signal-dependent and correlation-sensitive. The branch metrics of the branches connecting the nodes (states) in the trellis/tree are the negative logarithms of the factors on the right-hand side of Equation (2). Notice that since each one of these factors is based on a joint pdf of observations $r_i, \ldots, r_{i+L}$, the metric is correlation-sensitive. Also, notice that since each of the factors in (2) is conditioned on the transmitted sequence $a_{i-K_l}, \ldots, a_{i+L+-K_t}$, the metric is signal-dependent.

A. Gaussian Branch Metrics

We next consider the noise to be Gaussian and, after different assumptions on its correlation and signal-dependent statistics, derive different branch metrics.

Euclidian metric. If the noise is white and signal-independent, then due to the factorization of uncorrelated (independent) Gaussian pdfs, the reciprocal of each factor in (2) is

$$\sqrt{2\pi\sigma^2} \exp\left[\frac{(r_i - m_i)^2}{2\sigma^2}\right], \quad (3)$$

where σ^2 is the white noise variance, and m_i is the mean of the sample observation r_i. Taking the logarithm of (3), and canceling common constant (signal-independent) additive and multiplicative terms, we obtain the Euclidian distance metric

$$\mathcal{M}_i = N_i^2 = (r_i - m_i)^2 \quad (4)$$

as the branch metric for the Viterbi-like implementation of the MLSD.

Variance dependent metric. If the noise is now Gaussian, whose samples are uncorrelated (independent), but whose variances σ_i^2 are possibly dependent on the transmitted signal, due to the factorization of Gaussian pdfs, the reciprocal of each factor in (2) is

$$\sqrt{2\pi\sigma_i^2} \exp\left[\frac{(r_i - m_i)^2}{2\sigma_i^2}\right]. \quad (5)$$

Taking the logarithm of (5) and canceling common constant (signal-independent) additive and multiplicative terms, we obtain the signal-dependent correlation-insensitive branch metric

$$\mathcal{M}_i = \log \sigma_i^2 + \frac{N_i^2}{\sigma_i^2} = \log \sigma_i^2 + \frac{(r_i - m_i)^2}{\sigma_i^2}, \quad (6)$$

Notice that this same metric was derived in [7], [8] and used on real data in [10].

Correlation-sensitive metric. In the general case, the Gaussian noise is both correlated and signal-dependent. This is especially the case in high density magnetic recording. In that case, the reciprocal of every factor in (2) takes the general multivariate Gaussian form

$$\sqrt{\frac{(2\pi)^{L+1} \det \mathbf{C}_i}{(2\pi)^L \det \mathbf{c}_i}} \frac{\exp\left[\underline{N}_i^{\mathrm{T}} \mathbf{C}_i^{-1} \underline{N}_i\right]}{\exp\left[\underline{n}_i^{\mathrm{T}} \mathbf{c}_i^{-1} \underline{n}_i\right]}. \quad (7)$$

The $(L+1) \times (L+1)$ matrix $\mathbf{C}_i$ is the covariance matrix of the data samples $r_i, r_{i+1}, \ldots, r_{i+L}$, when a sequence of symbols $a_{i-K_l}, \ldots, a_{i+L+K_t}$ is written (transmitted). The matrix $\mathbf{c}_i$ in the denominator of (7) is the $L \times L$ lower principal submatrix of $\mathbf{C}_i = \begin{bmatrix} \cdot & \cdot \\ \cdot & \mathbf{c}_i \end{bmatrix}$. The $(L+1)$-dimensional vector $\underline{N}_i$ is the vector of differences between the observed samples r_i and their expected values m_i when the sequence of symbols $a_{i-K_l}, \ldots, a_{i+L+K_t}$ is written (transmitted), i.e.,

$$\underline{N}_i = \left[(r_i - m_i) \quad (r_{i+1} - m_{i+1}) \cdots (r_{i+L} - m_{i+L})\right]^{\mathrm{T}}. \quad (8)$$

The vector $\underline{n}_i$ collects the last L elements of $\underline{N}_i$, $\underline{n}_i = \left[\ (r_{i+1} - m_{i+1}) \quad \cdots \quad (r_{i+L} - m_{i+L})\ \right]^{\mathrm{T}}$. With this notation, by taking the logarithm of (7), we get the general correlation-sensitive metric

$$\mathcal{M}_i = \log \frac{\det \mathbf{C}_i}{\det \mathbf{c}_i} + \underline{N}_i^{\mathrm{T}} \mathbf{C}_i^{-1} \underline{N}_i - \underline{n}_i^{\mathrm{T}} \mathbf{c}_i^{-1} \underline{n}_i. \quad (9)$$

Notice that this metric is now both signal-dependent and correlation-sensitive.

The metric in (9) involves matrices $\mathbf{C}_i$ of size $(L+1) \times (L+1)$. If the correlation length is, say, $L = 2$, then the size of the matrices $\mathbf{C}_i$ is 3×3, and we shall refer to this metric as the the C3 metric. The metric in (6) is then, with this notation, the C1 metric. We can then refer to the Euclidian metric as the C0 metric to be consistent with this short-hand notation.

B. Implementation Issues

The metrics in (4), (6) and (9) are all Gaussian branch metrics. Their use is not limited to the Viterbi algorithm [1], [2], but can be used in any Viterbi-like suboptimal detector that operates on a tree/trellis structure. Such algorithms are PRML detection [11], [12], or hybrids between

MLSD and decision feedback equalization (DFE), such as FDTS/DF [13], MDFE [14] or RAM-RSE [15].

When implementing the Euclidian metric, the noise statistics (variances and covariance matrices) are assumed to be signal independent, and therefore do not need to be known in the metric computations. To implement the metrics in (6) and (9), on the other hand, we need to know these statistics. Since they are not known a priori, they need to be estimated from the noisy signal observations at the receiver end. We think that adaptive estimation techniques are best suited for magnetic recording applications. This is because the signal is time-varying since its characteristics (transition separations, bit cell time, normalized density, etc.) vary from track to track. Since the noise is signal dependent, its statistics are time-varying as well. The adaptive estimation of these noise statistics is beyond the scope of this paper. For an example of a recursive statistics estimation method based on past detector decisions, we refer the reader to [9]

3. Performance Analysis

Define H_1 and H_0 as the hypotheses that the written sequences are $a_1^0, \dots, a_N^0$ and $a_1^1, \dots, a_N^1$, respectively. Allow $a_j^0 \neq a_j^1$ to happen only for $j \in [i, i+M-1]$, i.e., the two sequences of symbols differ on a segment of $M \geq 1$ symbols. We consider the hypothesis H_0 to be the correct one and find the probability of choosing H_1 for both the correlation-sensitive and the Euclidian metrics. If the distance between these two paths in the trellis/tree corresponds to the minimum distance error event then the probability of bit errors is bounded by bounds that are proportional to the probability of this error event [1], [2].

According to (2), the hypothesis H_1 will be chosen over the hypothesis H_0 if

$$\prod_{j=i-K_t-L}^{i+K_l+M-1} \frac{f(r_j, r_{j+1}, \dots, r_{j+L} | a_{j-K_l}^p, \dots, a_{j+L+K_t}^p)}{f(r_{j+1}, \dots, r_{j+L} | a_{j-K_l}^p, \dots, a_{j+L+K_t}^p)} \tag{10}$$

is greater for $p=1$ than for $p=0$. Since $a_j^0 \neq a_j^1$ can happen only for $j \in [i, i+M-1]$, the product in (10) contains only $M+L+K_l+K_t$ terms. If we multiply (10) by $f(r_{i+M+K_l}, \dots, r_{i+M+K_l+L} | a_{i+M}^p, \dots, a_{i+M+K_l+K_t+L}^p)$, which is the same for $p=0$ and $p=1$ since $a_j^0 = a_j^1$ for $j \geq i+M$, we get a joint pdf $f_{\underline{R}|H_p}(\underline{r}|H_p)$, where $\underline{r}$ is a vector collecting samples $r_{i-K_t-L}, \dots, r_{i+M+K_l+L}$ and p denotes the hypothesis 1 or 0. The hypothesis test is then

$$f_{\underline{R}|H_1}(\underline{r}|H_1) \mathop{\gtrless}_{H_0}^{H_1} f_{\underline{R}|H_0}(\underline{r}|H_0). \tag{11}$$

Under the Gaussian assumption, when H_p holds, $\underline{r}$ is normally distributed with mean $\underline{m}_p$ and covariance $\mathbf{\Sigma}_p$, $p \in \{0,1\}$. In general ($\mathbf{\Sigma}_0 \neq \mathbf{\Sigma}_1$ and $\underline{m}_0 \neq \underline{m}_1$) the likelihood test in (11) is the general Gaussian test that has been exhaustively studied, see e.g. [16]. Here we do not study the general case, but simplify the problem to get quick analytic results to show how much we gain by using the correlation-sensitive metric over the Euclidian metric.

Let $\mathbf{\Sigma}_0 = \mathbf{\Sigma}_1 = \mathbf{\Sigma}$. Then the test in (11) is

$$(\underline{m}_1 - \underline{m}_0)^{\mathrm{T}} \mathbf{\Sigma}^{-1} \underline{r} \mathop{\gtrless}_{H_0}^{H_1} \frac{1}{2}\left[\underline{m}_1^{\mathrm{T}} \mathbf{\Sigma}^{-1} \underline{m}_1 - \underline{m}_0^{\mathrm{T}} \mathbf{\Sigma}^{-1} \underline{m}_0\right]. \tag{12}$$

The probability of error (probability that H_1 is chosen over H_0) when using the correlation-sensitive metric is

$$P_c(\varepsilon) = \frac{1}{2}\mathrm{erfc}\left[\frac{\frac{1}{2}(\underline{m}_1 - \underline{m}_0)^{\mathrm{T}} \mathbf{\Sigma}^{-1} (\underline{m}_1 - \underline{m}_0)}{\sqrt{2(\underline{m}_1 - \underline{m}_0)^{\mathrm{T}} \mathbf{\Sigma}^{-1} (\underline{m}_1 - \underline{m}_0)}}\right], \tag{13}$$

where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-t^2} dt$. On the other hand, if we use the Euclidian metric, the test is

$$(\underline{m}_1 - \underline{m}_0)^{\mathrm{T}} \underline{r} \mathop{\gtrless}_{H_0}^{H_1} \frac{1}{2}\left[\underline{m}_1^{\mathrm{T}} \underline{m}_1 - \underline{m}_0^{\mathrm{T}} \underline{m}_0\right]. \tag{14}$$

and the probability of error when applying the Euclidian metric to correlated noise is

$$P_e(\varepsilon) = \frac{1}{2}\mathrm{erfc}\left[\frac{\frac{1}{2}(\underline{m}_1 - \underline{m}_0)^{\mathrm{T}} (\underline{m}_1 - \underline{m}_0)}{\sqrt{2(\underline{m}_1 - \underline{m}_0)^{\mathrm{T}} \mathbf{\Sigma} (\underline{m}_1 - \underline{m}_0)}}\right], \tag{15}$$

To study the improvement of (13) over (15), we construct a simple PR4 example. In PR4, the ISI length is $K=2$ ($K_l = 0$, $K_t = 1$). Let the noise correlation length be $L = 1$. For a PR4 system, the minimum number of consecutive places where two sequences can differ is $M=2$. Let the 2×2 conditional covariance matrix be $\mathbf{C}_i = \begin{bmatrix} \sigma^2 & \rho \\ \rho & \sigma^2 \end{bmatrix}$ for all branches and for all i, where $|\rho| < \sigma^2$. The joint covariance matrix is then $\mathbf{\Sigma} = \sigma^2 \cdot \mathbf{T}(q)$, where $\mathbf{T}(q)$ is a symmetric Toeplitz matrix whose first row is $[1, q, q^2, q^3, q^4, q^5]$ and $q = \rho/\sigma^2$ is the correlation coefficient.

Let σ_c^2 be the noise power needed to produce an error rate $P(\varepsilon)$ when using the correlation-sensitive metric. The corresponding noise covariance matrix in (13) is $\mathbf{\Sigma}_c = \sigma_c^2 \cdot \mathbf{T}(q)$. Similarly, let σ_e^2 be the noise power needed to produce the same error rate $P(\varepsilon)$, but using the Euclidian metric. The corresponding covariance matrix in (15) is then $\mathbf{\Sigma}_e = \sigma_e^2 \cdot \mathbf{T}(q)$. Since these two noise powers (σ_c^2 and σ_e^2) produce the same probability of error $P(\varepsilon)$ for their respective tests, we can equate (13) to (15) (note that the covariance matrices are now $\mathbf{\Sigma}_c \neq \mathbf{\Sigma}_e$), and solve to get

$$\frac{\sigma_c(q)^2}{\sigma_e(q)^2} = \frac{\Delta\underline{m}^{\mathrm{T}} \mathbf{T}(q)^{-1} \Delta\underline{m} \Delta\underline{m}^{\mathrm{T}} \mathbf{T}(q) \Delta\underline{m}}{(\Delta\underline{m}^{\mathrm{T}} \Delta\underline{m})^2}, \tag{16}$$

where $\Delta\underline{m} = \underline{m}_1 - \underline{m}_0$. Notice that the ratio in (16) is not dependent on $P(\varepsilon)$. We define the gain in dB as

$$G(q) = 10 \log_{10}\left[\frac{\sigma_c(q)^2}{\sigma_e(q)^2}\right]. \tag{17}$$

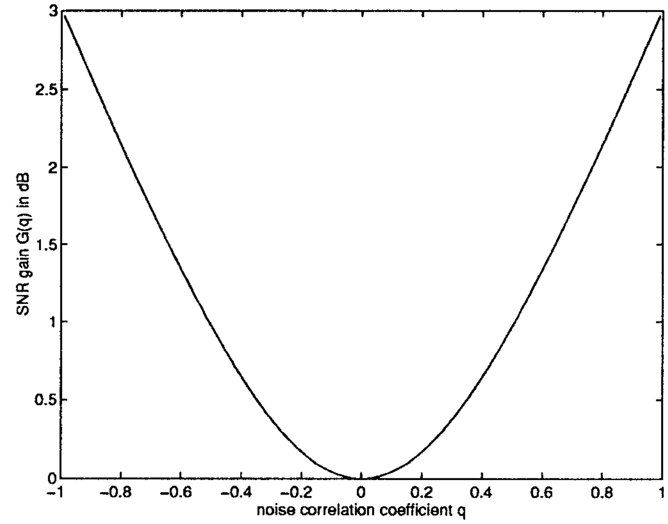

Fig. 1. SNR gains achieved by using the correlation-sensitive over the Euclidian metric as a function of the correlation coefficient q.

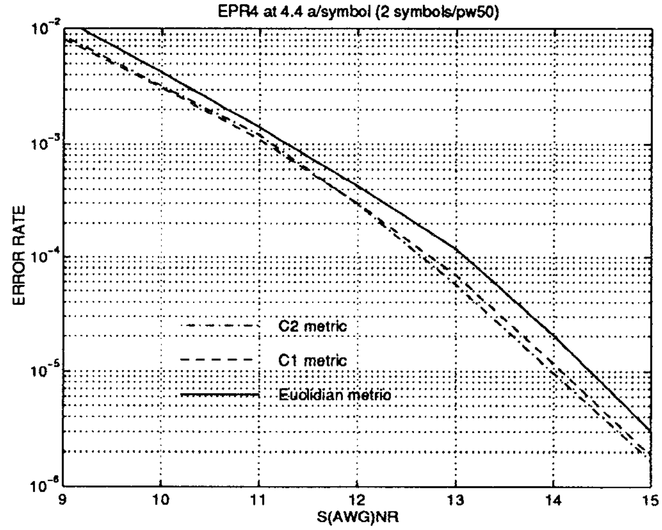

Fig. 2. EPR4 detection results at $4.4a$/symbol.

It shows the loss in SNR that can be afforded when using the correlation-sensitive metric over the Euclidian metric. Figure 1 shows this gain for a PR4 system, where we assume that the minimum distance error event occurs when $\underline{m}_0 = [0\ 0\ 0\ 0\ 0\ 0]^{\mathrm{T}}$ and $\underline{m}_1 = [0\ 1\ 0\ -1\ 0\ 0]^{\mathrm{T}}$, i.e., when a dibit is decided instead of an all-zero pattern. In this example, depending on the value of the correlation coefficient q, gains ranging from 0 to 3 dB can be made by using the correlation-sensitive metric instead of the Euclidian metric.

4. Simulation Results

We study the gains obtained with the correlation-sensitive metric over the Euclidian metric. Due to limited space, we confine the comparisons to the EPR4 channel. To create realistic waveforms, corrupted by media noise, we used the triangle zig-zag model [17], [18]. The waveforms were then corrupted by additive white Gaussian noise (AWGN), low-pass filtered, sampled, equalized to the EPR4 target, and passed through a Viterbi detector. A Lindholm head [19] was used for both writing and reading. The recording parameters were: remanence M_r = 450kA/m, coercivity H_c = 160kA/m, media thickness δ = 20nm, media cross-track correlation width s = 20nm, head-media separation d = 15nm, head field gradient factor $Q = .8$, head gap length $g = .135\mu$m, track width $TW = 2\mu$m, transition width parameter $a = 19nm$, percolation threshold $L_P = 1.4a = 26.6$nm, 50%-pulse width $PW50 = .167\mu$m.

We tested three branch metrics for the EPR4 algorithm, the Euclidian metric, the variance dependent metric for which L=0 and the 2×2 correlation-sensitive metric for which L=1. We refer to these metrics as the Euclidian, the C1 and the C2 metric, respectively. We studied the performance at three normalized densities: 2, 2.5 and 3 bits/PW50, corresponding to symbol separations of $4.4a$, $3.5a$ and $2.9a$, respectively. The error rates in Figures 2 to 4 are given as a function of the signal to AWGN ratio

$$\mathrm{S(AWG)NR} = 10\log_{10}\frac{A_{iso}^2}{\sigma_n^2}, \tag{18}$$

where A_{iso} is the isolated pulse amplitude and σ_n^2 is the variance of AWGN. This noise is added to a signal that is already media noise corrupted, hence we make a distinction between S(AWG)NR and the total SNR.

Figures 2 to 4 show that the performance margin between the Euclidian metric and the C2 metric increase with density. Figure 5 gives the S(AWG)NRs needed to achieve an error rate of 10^{-5} when using the EPR4 detector with the three different metrics for a range of normalized densities between 2 and 2.5 bits/PW50. We see form Figure 5 that the C2 metric outperforms the Euclidian metric by 0.4dB at 2 bits/PW50 and by 0.8dB at 2.5 bits/PW50. This margin is much greater at 3 bits/PW50, but Figure 4 shows that the error rates are too large for this case to be of a practical importance. We can also read from Figure 5 that with an S(AWG)NR of 15dB we can operate the Euclidian detector at 2.2 bits/PW50, while the C2 detector operates at 2.4 bits/PW50, achieving a 10% gain in linear density.

5. Conclusion

In the presence of signal-dependent correlated noise, the Viterbi algorithm with Euclidian branch metrics does not perform maximum likelihood sequence detection (MLSD). We have shown that, in order to have MLSD performance, the branch metrics need to be modified such that they include the effects of signal dependence and correlation. We gave an explicit expression for the new branch metric for Gaussian noise statistics. We also provided an analytic tool for evaluating the probability of error of the MLSD detector that implements this metric. Through an example, we illustrated that the performance margin between

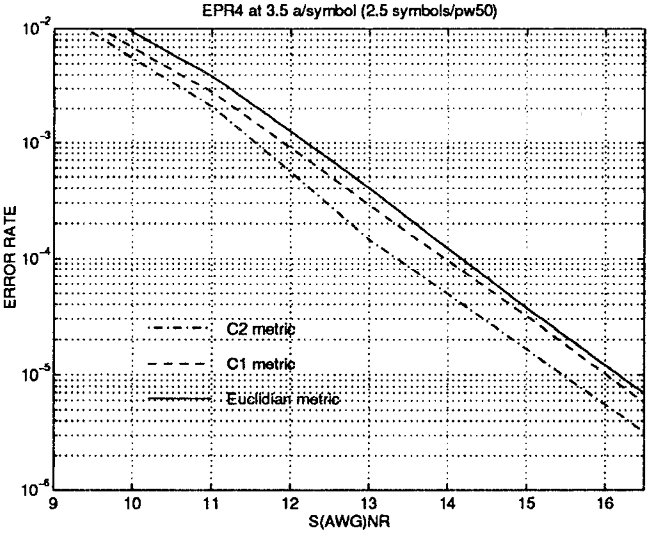

Fig. 3. EPR4 detection results at $3.5a$/symbol.

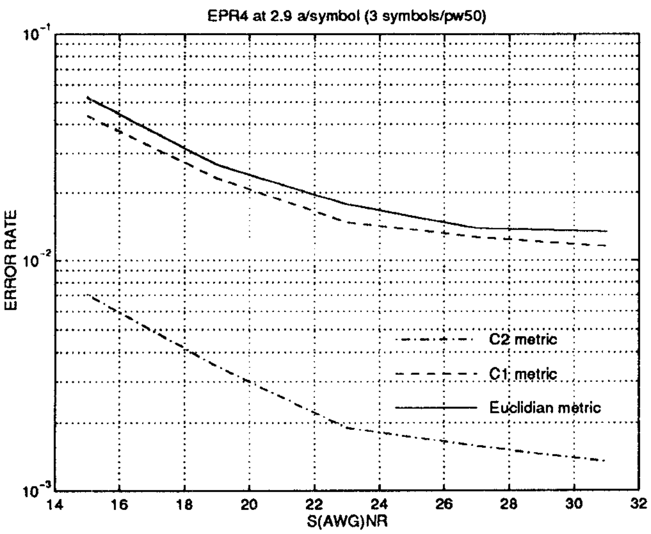

Fig. 4. EPR4 detection results at $2.9a$/symbol.

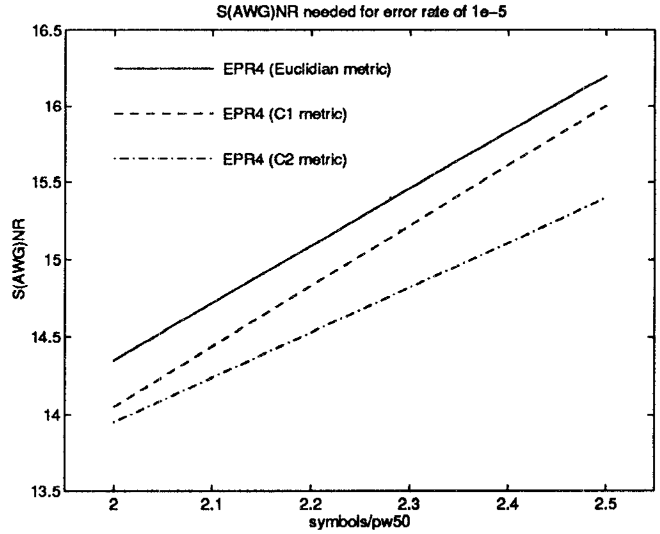

Fig. 5. S(AWG)NRs needed for an error rate of 10^{-5} with EPR4.

the correlation-sensitive metric and the Euclidian metric increases as the correlation between the noise samplesd grows. In a magnetic recording channel at high recording densities, the media noise and nonlinearities corrupt the readback waveform, causing signal-dependent correlation between the noise samples. We have shown that, in this environment, the new metric provides gains of up to 1dB over the Euclidian detector for the EPR4 channel.

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