PREDICTIVE CODING USING NONCAUSAL MODELS *

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Abstract

In this paper, we introduce a new technique for compression of images. The method models the images as noncausal random fields. Our approach couples a noncausal predictive technique with vector quantization to provide image compression of quality largely superior to that provided by DCT based techniques or by vector quantization (VQ) alone.

1. Introduction

In this paper, we present a new technique for compression of images which is based on predictive coding with vector quantization. The distinctive feature of our method lies in the paradigm used to describe the images. The images are modeled as noncausal random fields, i.e., fields where the intensity at each pixel depends on the intensities at sites located in all directions around the given pixel. This is in contrast to conventional predictive coding methods (see, for example, [1]) where causal image models are used with 2D causality being defined arbitrarily by designating a portion of the image plane (usually the upper left quarter plane) as the "past". It has been shown, e.g., [2], that noncausal field models are a superior paradigm to causal models for image processing. The latter have often been favored because of the attractive computational characteristics of the recursive processing methods they lead to. The compression technique presented here is able to retain the use of recursive processing while modeling images as noncausal random fields. This is made possible by the use of the equivalent recursive representations that all finite lattice noncausal Gauss Markov random fields (GMRF's) have, see [3].

Experimental results from our technique are contrasted with two alternative methods, a DCT based method that is the baseline JPEG procedure with some optimizations in the parameters, obtained from [4], and vector quantization (VQ) alone. To reduce computational complexity, we used first order GMRF's as the image model in our algorithm. Our results demonstrate that the technique presented here produces good quality results at low bit rates without introducing any significant artifacts such as the blocking that is characteristic of DCT based methods.

The paper is organized as follows. In section 2, the salient features of the noncausal image model are outlined briefly. The compression/decompression algorithm is described in section 3. Experimental results are presented in section 4, and conclusions in section 5.

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2. Noncausal image model

After subtracting the global sample mean, the images are modeled as zero mean Gauss Markov random fields (GMRF). For simplicity, a square ($N \times N$) image lattice is assumed. For a first order GMRF, the MMSE representation [5] is:

$$\phi_{i,j}-\beta_v(\phi_{i,j-1}+\phi_{i,j+1})-\beta_h(\phi_{i-1,j}+\phi_{i+1,j})=\epsilon_{i,j} \quad (1)$$

Collecting in a matrix format:

$$A\phi=\epsilon \quad (2)$$

where,

$$\phi = [\phi^1 \cdots \phi^N] \quad (3)$$
$$\phi^i = [\phi_{i,1} \cdots \phi_{i,N}] \quad (4)$$

and A is referred to as the *potential matrix*, see [3], [6]. It is well known that

$$\mathrm{Cov}(\epsilon) = \Sigma_\epsilon = \sigma_\epsilon^2 A \quad (5)$$
$$\mathrm{Cov}(\phi) = \Sigma_\phi = \sigma_\epsilon^2 A^{-1} \quad (6)$$
$$\Sigma_{\phi\epsilon} = E\phi\epsilon^T = \sigma_\epsilon^2 I \quad (7)$$

In [3], we presented two recursive representations which are equivalent to the noncausal model (2). For example, the "backward" representation is

$$U_i\phi^i + O_i\phi^{i+1} = \xi_i \quad (8)$$

where ξ_i is white noise with variance σ_ϵ^2. The U_i's and O_i's are obtained from A by a Riccati type iteration

$$S_{i+1} = B - C^T S_i^{-1} C, \tag{9}$$

where B and C are submatrices in A, and

$$S_i = U_i^T U_i, \tag{10}$$
$$U_i^T O_i = C. \tag{11}$$

This Riccati equation converges quite rapidly. See [3], [6], for details of these matrices and theorems specifying the geometric rate of convergence of the Riccati iteration.

From (8), it is straightforward to generate a state space field representation

$$\phi^i = F_i \phi^{i+1} + G_i \xi^i, \tag{12}$$

where

$$F_i = U_i^{-1} O_i, \tag{13}$$
$$G_i = U_i^{-1}. \tag{14}$$

It is important to stress that (8) or (12) are statistically equivalent to the original noncausal autoregressive representation (2). Results for general order GMRF's, proofs, and details are provided in the references mentioned above.

To be able to use these models (2) – (14) with real images, we need to fit the models to the images. For the first order GMRF, this means determining the three parameters $(\beta_v, \beta_h, \sigma_\epsilon^2)$. In [7], we have described the parameter space for general first order noncausal GMRF for a variety of boundary conditions (Dirichlet, Neumann symmetric, and Neumann asymmetric). For general fields, results are in [8], [6]. The model parameters may be estimated using a conjugate gradient based algorithm which is described in [8], [6].

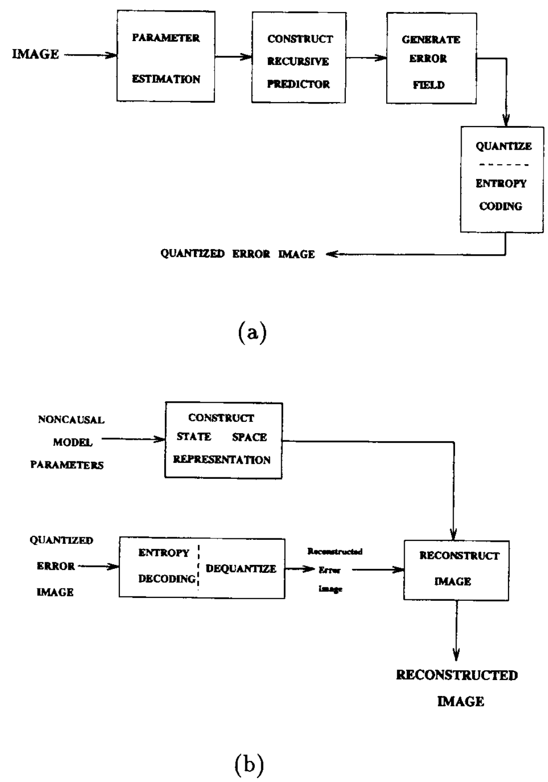

Figure 1: Noncausal predictive codec: (a) Compression, (b) Reconstruction.

3. Compression/decompression procedure

The compression procedure in the noncausal codec involves the following steps (see Figure 1 (a)):

1. *Parameter estimation:* A (first order) noncausal GMRF model is fitted to the image after subtracting out its sample mean. The parameters are estimated using the conjugate gradient estimation method outlined in [8].
2. *Construction of recursive predictor:* An equivalent recursive representation, e.g., the backward one given in (8, is constructed using the Riccati iteration (9). See [3] for details.
3. *Generation of error field:* The recursive predictor is used to generate a residual error field from the image (minus its sample mean).
4. *Quantization of error field:* The residual error field is quantized using vector quantization (VQ). This may be followed by entropy coding or some other form of lossless coding for additional compression. We have not added this at present.

The quantized error image, the global sample mean, and the noncausal model parameters are the output of the compression procedure.

The decompression procedure reconstructs the image through the following steps (see Figure 1(b)):

1. *Reconstruction of error field:* The error field is

decoded, if entropy coding was used, and then reconstructed using the VQ code book.

2. *Construction of state space regressor:* The state space equivalent of the recursive predictor used in the compression, (12), is constructed using the noncausal model parameters in the Riccati iteration (9), and the relationships in (13) and (14).

3. *Reconstruction of image:* The state space regressor is used to reconstruct the zero mean image from the reconstructed error image and the global mean is added back in.

4. Experimental results

A 128×128 portion of the Lenna image from the USC database was compressed to 0.375 bits per pixel (bpp) using our algorithm with the image being modeled as a first order GMRF with Dirichlet boundary conditions. In the first step of the compression procedure, the image sample mean was computed to be 110.4 and subtracted from the image prior to parameter estimation. The first order model parameters were estimated as

$$\beta_h = 0.1038, \quad (15)$$
$$\beta_v = 0.3943, \quad (16)$$
$$\sigma_\epsilon^2 = 130.4488. \quad (17)$$

The error field was quantized using a simple implementation of VQ based on the LBG algorithm [9] with a block size of 4×4 and 64 code vectors. The original and reconstructed images are shown in Figure 2 (a) and (b), respectively. A comparison shows that the reconstructed image reproduces most of the detail of the original image, such as the structure of the eye and the eyelashes, without introducing any significant artifacts. For comparison, the Lenna image was compressed to 0.375 bpp using the VQ algorithm alone, i.e., without the predictive coding steps. The reconstructed image, which is displayed in Figure 2 (c), shows significant blocking, as expected. A more meaningful comparison is with the JPEG procedure for still image compression. An optimized version of the baseline JPEG procedure using the 8×8 DCT was used to compress the Lenna image to a comparable bit rate, 0.3826 bpp. This bit rate includes lossless coding of the DCT coefficients using Huffman coding (while the bit rate given above for our algorithm does not.) The details of this procedure may be found in [4]. The reconstructed image is shown in Figure 2 (d). It shows considerable blocking as well as significant loss of detail, for example, around the eye.

5. Conclusions

A new image compression algorithm was presented in this paper. The algorithm combined the recursive processing characteristic of predictive coding with the use of a noncausal image model. The experimental results demonstrated the high quality of the reconstructed image at a low bit rate (0.375 bpp) and contrasted this with the significant loss of detail and blocking artifacts introduced by a JPEG type DCT method at the same bit rate. Work is in progress on improving the algorithm by introducing a more elaborate model for the deterministic component of the image to replace the global sample mean. This is expected to produce further improvements in image quality. A VLSI implementation of the algorithm for real–time image compression is also under development.

References

[1] M. Rabbani and P. W. Jones. *Digital Image Compression Techniques*. SPIE Optical Engineering Press, Bellingham, WA, 1991.

[2] N. Balram and J. M. F. Moura. Recursive enhancement of noncausal images. In *Proceedings ICASSP'91*, pages 2997–3000. IEEE, May 1991.

[3] J. M. F. Moura and N. Balram. Recursive structure of noncausal Gauss–Markov random fields. *IEEE Trans. Inform. Theory*, IT-38(2):334–354, March 1992.

[4] M. R. Souryal. The CCIT H.261 video coding algorithm: Evaluation of implementation options. Master's thesis, Information Networking Institute, Carnegie Mellon University, Pittsburgh, PA, October 1991.

[5] J. W. Woods. Two-dimensional discrete Markovian fields. *IEEE Trans. Inform. Theory*, IT-18(2):232–240, March 1972.

[6] N. Balram. *A Recursive Framework for Noncausal Gauss Markov Random Fields*. PhD thesis, Carnegie Mellon University, Pittsburgh, PA, January 1992.

[7] N. Balram and J. M. F. Moura. Parameter estimation in 2D fields. In *Proceedings ICASSP'92*. IEEE, Mar 1992.

[8] N. Balram and J. M. F. Moura. Noncausal Gauss Markov random fields: Parameter structure and estimation. Technical Report 91-03, LASIP, ECE, Carnegie Mellon University, April 1991. 43 pages. To appear in IEEE Trans. Inform. Theory.

[9] Y. Linde, A. Buzo, and R. M. Gray. An algorithm for vector quantizer design. *IEEE Trans. Comm.*, COM-28:84–95, January 1980.

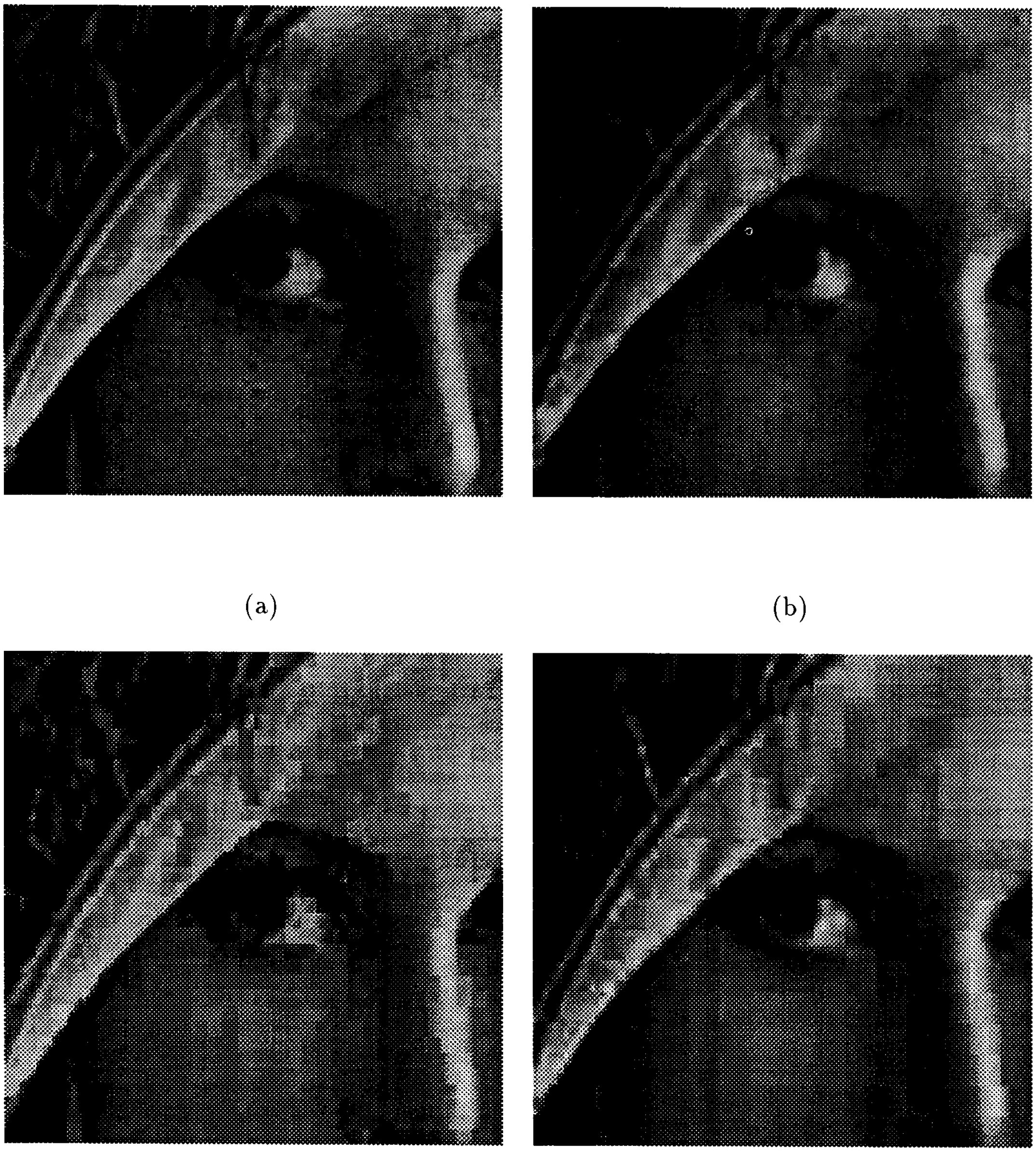

Figure 2: (a) Original 128 × 128 Lenna, (b)–(d) Comparison of reconstructed Lenna after compression: (b) to 0.375 bp using the compression procedure in Figure 1 with a first order GMRF as the noncausal image model and VQ (4 × 4 blocks, 64 codevectors) to quantize the error field, (c) to 0.375 bp using VQ (4 × 4 blocks, 64 codevectors) to quantize the image with the global mean removed, (d) to 0.3826 bp using the optimized baseline JPEG procedure (8 × 8 DCT).