U2.6

Cramer-Rao Bounds for Passive Range and Depth in a Vertically Inhomogeneous Medium

M. João D. Rendas

CAPS, Dep. Eng. Electr. e Comp.
Instituto Superior Técnico
1096 Lisboa Codex, Portugal

José M. F. Moura

LASIP, Depart. Elect. and Comp. Eng.
Carnegie Mellon University
Pittsburgh, PA, 15213

Abstract

We establish Cramer-Rao Bounds for passive location in a multipath environment with an array of multiple sensors. The source signature is wideband stationary and the background noise is white. We derive general expressions for the bounds and then compare the performance gain contributed by the inter-path delays over location techniques based on wavefront curvature only (i.e., spatial processing across the array of sensors).

1. Introduction

There is an increasing interest in incorporating detailed medium descriptions in passive location mechanisms. This improves the performance and eliminates ambiguities arising in simpified propagation models, such as isovelocity models.

Here, we study the performance (Cramer-Rao Bound (**CRB**)) of passive systems, under the following simplifying assumptions:

1. The ocean is a vertically inhomogeneous medium;
2. The source signal is a stationary wideband Gauss signal of known spectral density function;
3. The observations correspond to a multisensor array of known geometry and location;
4. Source and receiver have no relative horizontal motion.

Assumption 1 implies the existence of multiple paths for the energy propagating from a point source to a point receiver, leading to the following general expression for the signal received at a sensor:

$$x(t) = \sum_{p=1}^{P} a_p s(t - \tau_p) \qquad (1)$$

where $s(t)$ is the signal emitted by the source.

The information about the source location is encoded in the **propagation parameters** $P, \{a_p\}$, and $\{\tau_p\}$. Here, we focus on the τ_p's. In the **CRB** study,

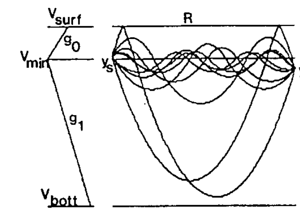

Figure 1: Configuration

we consider only situations where the number of propagation paths P is constant in a neighborhood of the source, and we asssume that the variation of the attenuation along different sensors with the source position is negligible.

The paper is organized as follows: first, the basic equations of the propagation model are described, in section 2. In section 3, the expressions for the **CRB** of the **propagation parameters** are presented, assuming perfect knowledge of the **environmental parameters**. Finally, in section 4. we plot the **CRB** for configurations of pratical interest.

2. Propagation Model

We assume that the ocean is a horizontally homogeneous medium, with flat boundaries, and that the sound velocity profile (**SVP**) is a bilinear function of depth, as in Fig. 1. With this **SVP**, the rays bent towards the depth corresponding to the minimum of the **SVP**, (duct axis). The rays are classified according to their interference with the medium's boundaries, into purely refracted rays (SOFAR), surface (SR) or bottom (BR) reflected rays, and surface/bottom reflected rays (SRBR).

We give in this section the basic equations of the propagation model, assuming that both source and receiver are above the duct axis. For the case of source and receiver at the same depth, simpler equations are given in [4].

The equations are parametrized according to the number K of complete bounces around the duct axis.

CH2847-2/90/0000-2779 $1.00 © 1990 IEEE

We present the expressions for the delays and their derivatives with respect to the **location parameters** (range and depth)

The delay from a point source located at (R, y_s) to a point receiver located at depth y_{rec} along a path with launching angle θ_s is,

$$\begin{aligned}\tau(\theta_s) = {} & \tfrac{\gamma}{g_0}\mathcal{F}(c_s) - \tfrac{\delta}{g_0}\mathcal{F}(c_r) + A_1\mathcal{F}(c_{duct}) \\ & + \tfrac{A_2}{g_0}\mathcal{F}(c_{surf}) - \tfrac{A_3}{g_1}\mathcal{F}(c_{bott})\end{aligned} \tag{2}$$

where[1], $\mathcal{F}(c) = \ln\left|(1+\sqrt{1-c^2})/c\right|$, and,

$$\gamma = \mathrm{sign}(\theta_s) \quad \delta = \mathrm{sign}(\theta_r) \tag{3}$$

and,

$$\begin{aligned}
A_1(K) &\triangleq \begin{cases} 2(K+1)\left(\frac{1}{g_1} - \frac{1}{g_0}\right), & \text{SOFAR. SR. SRBR} \\ 0, & \text{DIRECT RAYS} \end{cases} \\
A_2(K) &\triangleq \begin{cases} \begin{cases} 2K & \gamma = 1, \delta = -1 \\ 2(K+1) & \gamma\delta = 1 \\ 2(K+2) & \gamma = -1, \delta = 1 \end{cases}, & \text{SR. SRBR} \\ 0 & \text{SOFAR} \\ 2. & \text{DIRECT SR} \end{cases} \\
A_3(K) &\triangleq \begin{cases} 0, & \text{SOFAR. SR} \\ 2(K+1), & \text{SRBR} \end{cases}
\end{aligned} \tag{4}$$

In the previous equations, $\theta_s, \theta_{duct}, \theta_{surf}, \theta_{bott}$ and θ_r are the ray angles at the source (launching angle), the duct, the surface, the bottom, and the receiver, respectively. We denote the cosine of these angles by c_s, c_{duct}, etc. These angles are related through Snell's law. The launching angle is determined from the following equation:

$$\begin{aligned}R = {} & \tfrac{v_s}{g_0}\tan\theta_s - \tfrac{v_{rec}}{g_0}\tan\theta_r + A_1 v_{min}\tan\theta_{duct} \\ & + A_2\tfrac{v_{surf}}{g_0}\tan\theta_{surf} - A_3\tfrac{v_{bott}}{g_1}\tan\theta_{bott}\end{aligned} \tag{5}$$

The derivatives of the delays with respect to the **location parameters**, are:

$$\begin{aligned}\frac{\partial\tau}{\partial R} &= \left(\left.\frac{\partial\tau}{\partial\theta_s}\right|_{y_{rec}}\right) / \left(\frac{\partial R}{\partial\theta_s}\right) \\ \frac{\partial\tau}{\partial y_{rec}} &= \left.\frac{\partial\tau}{\partial\theta_s}\right|_{y_{rec}}\left(\left.\frac{\partial R}{\partial y_{rec}}\right|_{\theta_s}\right) / \left(\frac{\partial R}{\partial\theta_s}\right) + \left.\frac{\partial\tau}{\partial y_{rec}}\right|_{\theta_s}\end{aligned} \tag{6}$$

where,

$$\begin{aligned}\frac{\partial\tau}{\partial\theta_s} = -\Big(& \tfrac{\gamma}{g_0}\dot{\mathcal{F}}(c_s) - \tfrac{\delta}{g_0}\dot{\mathcal{F}}(c_r)\tfrac{v_{rec}}{v_s} \\ & - A_1\dot{\mathcal{F}}(c_{duct})\tfrac{v_{duct}}{v_s} + \tfrac{A_2}{g_0}\dot{\mathcal{F}}(c_{surf})\tfrac{v_{surf}}{v_s} \\ & - \tfrac{A_3}{g_1}\dot{\mathcal{F}}(c_{bott})\tfrac{v_{bott}}{v_s}\Big)\sin\theta_s\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\partial R}{\partial\theta_s} = \tfrac{v_s}{g_0 c_s^2}\Big(1 & - \tfrac{\sin\theta_s}{\sin\theta_r} + g_0 A_1\tfrac{\sin\theta_s}{\sin\theta_{duct}} + A_2\tfrac{\sin\theta_s}{\sin\theta_{duct}} \\ & - \tfrac{g_0}{g_1}A_3\tfrac{\sin\theta_s}{\sin\theta_{bott}}\Big)\end{aligned} \tag{8}$$

[1]sign(x) denotes the signal function: $\mathrm{sign}(x) = 1, x > 0$, $\mathrm{sign}(x) = -1, x < 0$.

and,

$$\begin{aligned}\left.\frac{\partial\tau}{\partial y_{rec}}\right|_{\theta_s} = {} & \delta\dot{\mathcal{F}}(c_r)c_r/v_s - A_1\dot{\mathcal{F}}(c_{duct})g_0 c_{duct}/v_s \\ & - A_2\dot{\mathcal{F}}(c_{surf})c_{surf}/v_s + A_3 g_0/g_1\dot{\mathcal{F}}(c_{bott})\end{aligned} \tag{9}$$

$$\begin{aligned}\left.\frac{\partial R}{\partial y_{rec}}\right|_{\theta_s} = {} & \tan(\theta_s) - 1./c_s\sin\theta_s + A_1 g_0/\sin\theta_{duct}c_s \\ & + A_2/\sin\theta_{surf}c_s - A_3 g_0/g_1\sin\theta_{bott}c_s\end{aligned} \tag{10}$$

In the above equations, $\dot{\mathcal{F}}(c) = -\left[c\sqrt{1-c^2}\right]^{-1}$

3. Cramer Rao Bound

In this section, we present expressions for the **CRB** of the vector of location parameters: $\alpha^T = [R.\ y]$ The **CRB** is related to $J(\alpha)$, the Fisher Information Matrix (FIM) of the vector α, by [7]

$$\mathbf{CRB}(\alpha) = J(\alpha)^{-1} \tag{11}$$

In the Gaussian stationary case under consideration, the generic element of this matrix is given asymptotically (in the large sample limit) by [8,1]

$$\begin{aligned}[J(\alpha)]_{ij} = \tfrac{N}{4\pi}\int \mathrm{tr}\Big\{ & \tfrac{\partial\mathbf{S}_y(\omega)}{\partial\alpha_i}\mathbf{S}_y(\omega)^{-1} \\ & \tfrac{\partial\mathbf{S}_y(\omega)}{\partial\alpha_j}\mathbf{S}_y(\omega)^{-1}\Big\}\,d\omega\end{aligned} \tag{12}$$

where $\mathbf{S}_y(\omega)$ is the spectral density matrix (sdm) of the vector of observations,

$$y_k(t) = x_k(t) + w(t), \qquad 1,\ldots,K \tag{13}$$

which for the coherent case is

$$\mathbf{S}_y(\omega) = S_s(\omega)\mathbf{h}(\omega)\mathbf{h}(\omega)^H + \Sigma(\omega). \tag{14}$$

For simplicity, we consider here only the case of spatially incoherent noise, i.e., $\Sigma(\omega) = \sigma^2(\omega)I_K$.

In (14), the complex vector $\mathbf{h}(\omega)$ has generic element $\mathbf{h}(\omega)_k = \sum_{p=1}^{P} a_p e^{j\omega\tau_{kp}}$, where a_p is the attenuation along path p, and τ_{kp} is the travel time from source to sensor k along path p. To be able to analyze the impact of both the temporal (multipath) and spatial (geometric) structures of the incoming wavefield, we decompose each individual travel time:

$$\tau_{kp} = \tau_{11} + \theta_{mp_P} + \theta_{s_{kp}}. \tag{15}$$

With this decomposition, $\mathbf{h}(\omega) = \mathbf{A}(\omega)\mathbf{b}(\omega)$ where

$$\begin{aligned}\mathbf{A}(\omega) &= [\mathbf{a}_1(\omega)|\cdots|\mathbf{a}_P(\omega)] \\ \mathbf{b}(\omega)^T &= [a_1 e^{j\omega\theta_{MP_1}}\cdots a_P e^{j\omega\theta_{MP_P}}]\end{aligned} \tag{16}$$

i.e., $\mathbf{A}(\omega)$ is the usual steering matrix for the P incoming wavefronts, dependent only on the intersensor delays, and $\mathbf{b}(\omega)$ gives the phase relations between the P coherent replicas, and depends only on the inter-path delays. Note that $\mathbf{S}_y(\omega)$ is independent of τ_{11}, and that the delays θ_{mp} are relative to the reference path 1.

The location parameters α do not appear explicitly in (14). Actually, α affects $\mathbf{S}_y(\omega)$ through the set of inter-delays θ_{mp} and θ_s. We group all the inter-delays into a $KP-1$ vector:

$$\theta^T = \left[\theta_{mp}^T | \mathrm{vec}(\Theta_S)^T\right] \tag{17}$$

where $\Theta_S = [\underline{\theta}_1 | \cdots | \underline{\theta}_P]$ and $\underline{\theta}_p$ groups all the inter-sensor delays corresponding to the p-th path.

The FIM's for α and θ are related by

$$J(\alpha) = \frac{\partial \theta^T}{\partial \alpha} J(\theta) \frac{\partial \theta}{\partial \alpha} \tag{18}$$

where $\partial\theta/\partial\alpha$ is the matrix of first order derivatives, and $J(\theta)$ is the FIM for the inter-delays θ.

Using the chain rule once more, to express the derivatives with respect to the inter-sensor delays in terms of the elements of the complex vector $\mathbf{h}(\omega)$, and after algebraic manipulations, the following expression for $J(\theta)$ is obtained:

$$\begin{aligned} J(\theta) = \tfrac{N}{4\pi} \int \Big\{ & K_1 \mathrm{Re}\left\{ \tfrac{\partial \mathbf{h}^H}{\partial \theta} \mathbf{P}_\perp \tfrac{\partial \mathbf{h}}{\partial \theta} \right\} \\ & + K_2 \mathrm{Re}\left\{ \tfrac{\partial \mathbf{h}^H}{\partial \theta} \mathbf{h} \right\} \mathrm{Re}\left\{ \mathbf{h}^T \tfrac{\partial \mathbf{h}^*}{\partial \theta} \right\} \Big\} d\omega \end{aligned} \tag{19}$$

and $\mathbf{P}_\perp$ is the projection matrix in the orthogonal complement of the vector $\mathbf{h}(\omega)$:

$$\mathbf{P}_\perp = I_K - \frac{1}{||\mathbf{h}(\omega)||^2} \mathbf{h}(\omega)\mathbf{h}(\omega)^H \tag{20}$$

and K_1 and K_2 are

$$K_1 = 2\frac{S_s(\omega)^2}{\sigma^2(\omega)\mathbf{E}} ||\mathbf{h}(\omega)||^2 , \quad K_2 = 4\frac{S_s(\omega)^2}{\mathbf{E}^2} \tag{21}$$

where $\mathbf{E} = \sigma^2 + S_s(\omega)||\mathbf{h}(\omega)||^2$. The derivation of (19) is lengthy and is given in [3].

We note that the above expression yields as a particular case the expression for the **CRB** for a single sensor given in [5], and presents close relations with the expression for the narrowband uncoherent case given in [6].

When there is only a single path, $\mathbf{h}(\omega) = e^{j\omega D(\theta)}\mathbf{1}$ where $\mathbf{1}$ is a one-form, and $D(\theta) = \mathrm{diag}(\theta_{s_{k1}})$. It is easy to show that $\partial\mathbf{h}(\omega)/\partial\theta = j\omega e^{j\omega D(\theta)}$, which implies, $\mathrm{Re}\{\partial\mathbf{h}(\omega)^H/\partial\theta\mathbf{h}(\omega)\} = 0$, leaving just the first term in (19). Simple algebraic manipulations give,

$$\frac{\partial \mathbf{h}(\omega)^H}{\partial \theta} \mathbf{P}_\perp \frac{\partial \mathbf{h}(\omega)}{\partial \theta} = \omega^2 (I_K - \frac{1}{M}\mathbf{1}^H\mathbf{1}) \tag{22}$$

which is the expression in [5].

Comparison of (19) with the expression given in [6], shows that in the case of mutiple incoherent sources, the second term is not present, while the remaining term has essentially the same meaning in both cases. We can, therefore, attribute the second term to the multipath (coherent) structure of the observations. This additional contribution to the **CRB** is affected by the factor K_2, which, contrary to K_1 does not increase indefinitely with the signal to noise ratio, i.e., the additional information in the temporal structure of the data given by the second term in (19) attains a limit.

According to (17) the FIM for the inter-delays is partitioned as:

$$J(\theta) = \begin{bmatrix} J(\theta_{mp}) & J(\theta_{mp} : \theta_s) \\ J(\theta_s : \theta_{mp}) & J(\theta_S) \end{bmatrix}. \tag{23}$$

Using the above equations,

$$\begin{aligned} \tfrac{\partial \mathbf{h}(\omega)}{\partial \theta_{mp}} &= j\omega \mathbf{A}(\omega) \mathrm{diag}(\mathbf{b}(\omega)) \\ \tfrac{\partial \mathbf{h}(\omega)}{\partial \theta_s} &= j\omega \left[\mathbf{b}_1 \mathrm{diag}(\mathbf{a}_1) | \cdots | \mathbf{b}_1 \mathrm{diag}(\mathbf{a}_P)\right] \end{aligned} \tag{24}$$

which, together with the general equation (19) yield for the multipath sub-block of $J(\theta)$,

$$\begin{aligned} J(\theta_{mp}) = \tfrac{N}{4\pi} \int \omega^2 \big\{ & K_1 \mathrm{Re}\left\{ \mathrm{diag}(\mathbf{b}^*) \mathbf{A}^H \mathbf{P}_\perp \mathbf{A} \mathrm{diag}(\mathbf{b}) \right\} \\ & + K_2 \mathrm{Im}\left\{ \mathrm{diag}(\mathbf{b}^*) \mathbf{A}^H \mathbf{A} \mathbf{b} \right\} \\ & \mathrm{Im}\left\{ \mathbf{b}^T \mathbf{A}^T \mathbf{A}^* \mathrm{diag}(\mathbf{b}^*) \right\} \big\} d\omega \end{aligned} \tag{25}$$

and for the inter-sensor sub-blocks, corresponding to the paths p and q,

$$\begin{aligned} J(\theta_s)_{pq} = \tfrac{N}{4\pi} \int \omega^2 \big\{ & K_1 \mathrm{Re}\left\{ \mathbf{b}_p^* \mathrm{diag}(\mathbf{a}_p^*) \mathbf{P}_\perp \mathrm{diag}(\mathbf{a}_q) \mathbf{b}_q \right\} \\ & + K_2 \mathrm{Im}\left\{ \mathbf{b}_p^* \mathrm{diag}(\mathbf{a}_p^*) \mathbf{A} \mathbf{b} \right\} \\ & \mathrm{Im}\left\{ \mathbf{b}^T \mathbf{A}^T \mathrm{diag}(\mathbf{a}_q^*) \mathbf{b}_q^* \right\} \big\} d\omega \end{aligned} \tag{26}$$

For the terms involving both types of parameters,

$$\begin{aligned} & J(\theta_{mp} : \theta_s)_q = \\ & \tfrac{N}{4\pi} \int \omega^2 \big\{ K_1 \mathrm{Re}\left\{ \mathrm{diag}(\mathbf{b}^*) \mathbf{A}^H \mathbf{P}_\perp \mathrm{diag}(\mathbf{a}_q) \mathbf{b}_q \right\} \\ & \quad + K_2 \mathrm{Im}\left\{ \mathrm{diag}(\mathbf{b}^*) \mathbf{A}^H \mathbf{A} \mathbf{b} \right\} \\ & \quad \mathrm{Im}\left\{ \mathbf{b}^T \mathbf{A}^T \mathrm{diag}(\mathbf{a}_q^*) \mathbf{b}_q^* \right\} \big\} d\omega \end{aligned} \tag{27}$$

We note that for $K = 1$, i.e., for single sensor observations, the first term in (25) is zero, and we are left with the single sensor **CRB** formula for the delay estimates given in [2].

The contributions to $\mathbf{CRB}(\alpha)$ are:

$$\begin{aligned} \mathrm{CRB}^{-1}(\alpha)_{mp} = \tfrac{N}{4\pi} \int \omega^2 \big\{ & K_1 \mathrm{Re}\left\{ \psi_{mp}^H \mathbf{A}^H \mathbf{P}_\perp \mathbf{A} \psi_{mp} \right. \\ & + K_2 \mathrm{Im}\left\{ \psi_{mp}^H \mathbf{A}^H \mathbf{A} \mathbf{b} \right\} \mathrm{Im}\left\{ \mathbf{b}^H \mathbf{A}^H \mathbf{A} \psi_{mp}^H \right\} \big\} d\omega \end{aligned} \tag{28}$$

for the multipath block, where we defined $\psi_{mp} \triangleq \mathrm{diag}(\mathbf{b})\partial\theta_{mp}/\partial\alpha$ and for the inter-sensor block,

$$\begin{aligned} \mathrm{CRB}^{-1}(\alpha)_s = \tfrac{N}{4\pi} \int \omega^2 \big\{ & K_1 \mathrm{Re}\left\{ \psi_s^H \mathbf{P}_\perp \psi_s \right\} \\ & + K_2 \mathrm{Im}\left\{ \psi_s^H \mathbf{A} \mathbf{b} \right\} \mathrm{Im}\left\{ \mathbf{b}^H \mathbf{A} \psi_s^H \right\} \big\} d\omega \end{aligned} \tag{29}$$

where, $\psi_s \triangleq \sum_{p=1}^{P} \mathbf{b}_p \mathrm{diag}(\mathbf{a}_p) \partial \underline{\theta}_p / \partial \alpha$. Similar expressions are obtained for the cross-terms.

We see from the pevious expressions that the multipath contribution depends heavily on the Gram matrix for the steering vectors: $\Lambda = \mathbf{A}\mathbf{A}^H$ We analyze (25) in two extreme cases: $\Lambda \simeq KI$, which corresponds to orthogonal direction vectors, and the case $\Lambda \simeq K\mathbf{1}\mathbf{1}^T$, when the incoming wavefronts are undistinguishable for the array.

In the first case, $\Lambda \simeq KI$, and noting that the matrix product in the first term of $J(\theta_{mp})$,

$$\mathbf{A}^H \mathbf{P}_\perp \mathbf{A} = K\mathbf{P}_{b\perp} \tag{30}$$

it is easily checked that

$$\mathbf{CRB}_{mp}^{-1} = K\frac{N}{4\pi}\int \omega^2 \frac{\partial \theta_{mp}^T}{\partial \alpha}\left(I_P - \frac{\mathbf{a}\mathbf{a}^T}{||\mathbf{a}||^2}\right)\frac{\partial \theta_{mp}}{\partial \alpha} d\omega \tag{31}$$

that is equivalent to having K independent arrays of P sensors receiving a single wavefront with steering vector $\mathbf{b}(\omega)$.

In the other extreme case, $\Lambda \simeq K\mathbf{1}\mathbf{1}^T$, we are lead to:

$$\begin{aligned}\mathbf{CRB}_{mp}^{-1} = K\tfrac{N}{4\pi}\int \omega^2 \tfrac{\partial \theta_{mp}^T}{\partial \alpha} \mathrm{Im}\{\mathbf{b}^* \textstyle\sum_p \mathbf{b}_p\} \\ \mathrm{Im}\{\textstyle\sum_p \mathbf{b}_p \mathbf{b}^H\} \tfrac{\partial \theta_{mp}}{\partial \alpha} d\omega\end{aligned} \tag{32}$$

which corresponds to having K independent sensors observing the P incoming wavefronts. In these two extreme cases, we were lead to the existence of a "virtual" array, whose size depends on the number of spatially resolved sources, observing a fictitious wavefront with direction vector described by the vector $\mathbf{b}(\omega)$. The geometry of this "virtual" array is determined both by the multipath delays θ_{mp} and by their derivatives.

Finally, this result shows the importance of methods that exploit the double (temporal/spatial) structure of the observations, when the arrivals are correlated. Furthermore, the analogy now established with the classic array processing brings into this study the vast body of results of that area. This will be explored elsewhere.

5. Plots

Figure 2 shows plots of the normalized $\mathbf{CRB}_N\ (R) = 10\log\left(N\mathbf{CRB}(R)/R^2\right)$, for several values of the signal to noise ratio $10\log(S_s/\sigma^2)$ and signal bandwidth. The solid line corresponds to the total $\mathbf{CRB}$, given by (19). The dashed line is the $\mathbf{CRB}$ considering only the spatial structure, i.e., given by (29). We see that in this case, the contribution of the inter-path delays to the $\mathbf{CRB}$ is fundamental to attain a satisfactory performance. Similar plots are

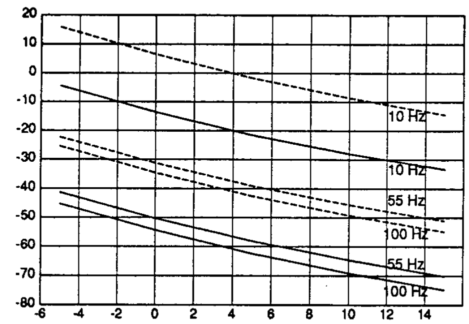

Figure 2: $\mathbf{CRB}_N(R)$.

obtained for $\mathbf{CRB}(y_s)$. Lack of space precludes full discussion of the behavior of the $\mathbf{CRB}$ here, and will be reported elsewhere.

In figure 2, the following channel parameters were used: $g_0 = -.035s^{-1}$, $g_1 = .013s^{-1}$, $v_{min} = 1480ms^{-1}$, duct depth $= 914m$, bottom depth $= 5000m$. The source is at depth $y_s = 10m$. The receiving array is a vertical linear uniform array of 4 sensors, with inter-sensor spacing of $10m$ and its first sensor is located at $(R = 3Km, y_1 = 100m)$.

References

[1] B. Friedlander. Accuracy of source location using multipath delays. *IEEE Trans. on Aerospace Electronic Eng.*, Vol. 24, No. 4:346–359, 1988.

[2] John P. Ianiello. Large and small error performance limits for multipath time delay estimation. *IEEE Trans. on Acoustic, Speech and Signal Processing*, Vol. ASSP-34, No.2:245–251, April 1986.

[3] M. João Rendas. *Dep. Eng. Electr. Comp., IST, Lisboa, Portugal*, forthcoming PhD thesis.

[4] M. João D. Rendas and José M. F. Moura. Location sensitivity in multipath environment. In *Proceedings of the IEEE OCEAN'89 Conference, Seattle, WA*, 1989.

[5] P. M. Schultheiss and K. Wagner. Active and passive location: similarities and differences. In Y.T.Chan, editor, *NATO ASI on Underwater Acoustic Data Processing, Kingston, C nada*, pages 215–230, 1988.

[6] P. Stoica and A. Nehorai. Music, Laximum Likelihood and Cramer-Rao-Bound. *IEEE Trans. on Acoustic, Speech and Signal Processing*, ASSP-37, No.5, pp. 720–741, May 1989.

[7] H.L. Van Trees. *Detection, Estimation, and Modulation Theory*. John Willey & Sons, Inc., 1971.

[8] P. Whittle. The analysis of multiple stationary time series. *J. Royal Statist. Soc.*, Vol.15:125–139, 1953.