Resolving Narrowband Coherent Paths With Non-Uniform Arrays

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Abstract

In this paper, we present a new algorithm to estimate the directions of arrival (DOA's) of multiple narrow band (NB) sources (possibly completely correlated) with observations by an array of arbitrary geometry. As a preprocessor, the new algorithm extends the application of NB High Resolution (HR) direction finding schemes beyond the linear uniform array configuration, which these methods usually assume when handling coherent sources. The novelty of the approach consists in the fact that both on geometrical considerations and results from optimal estimation theory are used.

1. Introduction

It is well known that conventional HR algorithms for multiple source direction finding (as proposed in [1,7]) fail when the sources are perfectly correlated. Several authors have proposed extensions to these algorithms that allow for completely correlated (or coherent) sources. These consist in the addition of a preprocessor to the conventional HR algorithm, that does some kind of "smoothing" of the received waveform. We distinguish broadly between Narrow Band (NB) methods (that use a single frequency component of the observations) and Wide Band (WB) methods (that process several frequency components).

One of the most successful NB methods appears to be the Spatial Smoothing (SS) technique, initially proposed in [3]. Designed for uniform linear arrays (ULA's), it generates a "smoothed" covariance matrix with rank equal to the number of directive components in the observations, irrespective of the correlation among them. Alternative algorithms have appeared, all of which are also fundamentally dependent on the ULA assumption.

The Coherent Signal Subspace Method (CSSM), presented in [8], can be applied to an array of arbitrary configuration, being based on the nonsingularity of the integrated source spectral density matrix. Alternative wideband methods have been proposed, e.g. [2], that assume a particular shape of the source spectrum. We concentrate on the Coherent Signal Subspace Method (here shortly denoted by CSSM) since it is independent of any such assumption.

Non-uniform arrays are frequently encountered in practice, and it is consequently important to have methods that can detect and estimate the directions of arrival independently of the array configuration. Herein, we present an algorithm to resolve **coherent narrowband** replicas, with observations by an array of **arbitrary geometry**.

The paper is organized as follows: first, we reinterpret the CSSM method in the framework of estimation theory. Motivated by this discussion, we propose, in section 3, a new algorithm, that extends the basic idea of the CSSM method to the narrow band case. Finally, we present simulation studies that assess the performance of the proposed technique.

2. The CSSM Method

To motivate our algorithm, we give a brief description of the CSSM technique [8]. Consider the following model of the (WB) signal received at an array of K sensors:

$$r_k(t) = \sum_{p=1}^{P} a_k(\theta_p) f_p(t - \tau_k(\theta_p)) + w_k(t), \quad \begin{matrix} t \in T \\ k \in \{1, \ldots, K\} \end{matrix} \tag{1}$$

where $r(t) = [r_1(t) \cdots r_K(t)]$ is the K dimensional observation vector; P is the total number of directive incoming replicas; $a_k(\theta_p), \tau_k(\theta_p)$ are the attenuation and delay from source p to sensor k; $f_p(t)$ is the signal emitted by the p-th source; $w_k(t)$ is the observation noise of known covariance matrix.

For a sufficiently large observation interval T, the DFT components of the observation vector $r(t)$ at

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each frequency are:

$$r(\omega_n) = A(\Theta, \omega_n)s(\omega_n) + w(\omega_n), n = 1, \ldots, N \quad (2)$$

where $r(\omega_n)$ is the $K-$dimensional complex vector of DFT's at the frequency ω_n; $A(\Theta; \omega_n) = [a(\theta_1; \omega_n) | \cdots | a(\theta_P; \omega_n)]$ is a $(K \times P)$ full column rank matrix whose columns are the steering vectors for each one of the P incoming directions $\{\theta_p\}_{p=1}^{P}$ at the frequency ω_n; $s(\omega_n)$ is the $P-$dimensional vector of the Fourier components of the source signals at the frequency ω_n, as seen by the first sensor (note that the interpath delays are included here); $w(\omega_n)$ is the DFT of the random vector $w(t)$, with known covariance matrix $\sigma^2\Sigma(\omega_n)$.

Assuming that the gains of each sensor are the same, $a_k(\theta_p) = a_p$, we incorporate the amplitude factors a_p in the source vector, resulting in the following definition of the steering vectors:

$$a(\theta_p; \omega_n) = [\ 1 \quad e^{j\omega_n \zeta_2(\theta_p)} \quad \ldots \quad e^{j\omega_n \zeta_K(\theta_p)}\ ]^T \quad (3)$$

where ζ_i is the delay relative to sensor 1.

The covariance matrix of the DFT components (2) is (asymptotically):

$$R(\omega_n) = A(\Theta; \omega_n)S(\omega_n)A(\Theta; \omega_n)^H + \sigma^2\Sigma(\omega_n) \quad (4)$$

The CSSM method computes the "smoothed" covariance matrix:

$$\underline{R} = \sum_{n=1}^{N} T_n R(\omega_n) T_n^H. \quad (5)$$

where T_n are $(K \times K)$ nonsingular matrices that map the signal subspace at each frequency ω_n into the signal subspace at a reference frequency ω_0, and satisfy the following set of equations:

$$T_n A(\Theta; \omega_n) = A(\Theta; \omega_0) \qquad n = 1, \ldots, N \quad (6)$$

Using (4) and (6) in (5) yields

$$\begin{array}{rcl} \underline{R} & = & A(\Theta; \omega_0)\overline{S}A(\Theta; \omega_0)^H + \sum_{n=1}^{N} T_n\Sigma(\omega_n)T_n^H \\ & = & A(\Theta; \omega_0)\underline{S}A(\Theta; \omega_0)^H \end{array} \quad (7)$$

with obvious definitions of $\overline{S}$ and $\underline{S}$.

In [8], it is proposed that the vector of estimates of the directions of arrival be made of dimension K, say, augmenting an initial set of estimates obtained with a given method, Θ_0, with another set of directions, Φ_0, and use the following transformation matrices:

$$T_n = A(\Theta_0|\Phi_0; \omega_0)A(\Theta_0|\Phi_0; \omega_n)^{-1} \quad (8)$$

Using in (7) equation (8) and the expression for the sample covariance matrix:

$$\hat{R}(\omega_n) = \frac{1}{L}\sum_{l=1}^{L} r^l(\omega_n)r^l(\omega_n)^H \quad (9)$$

where l denotes the snapshot index, the matrix used by the CSSM method can be written

$$\underline{R} = A(\Theta_0|\Phi_0; \omega_0)\sum_{n=1}^{N}\left(\frac{1}{L}\sum_{l=1}^{L}\hat{s}^l(\omega_n)\hat{s}^l(\omega_n)^H\right)A(\Theta_0|\Phi_0; \omega_0) \quad (10)$$

where we have defined

$$\hat{s}^l(\omega_n) \triangleq A(\Theta_0|\Phi_0; \omega_n)^{-1}r^l(\omega_n) \quad (11)$$

This algorithm has been justified on the basis of geometric/algebraic relations. Using (10) it can be given an alternative interpretation. Consider the model (2) together with the following assumptions:

- $s(\omega_n)$ is, for each n, a $P-$dimensional **unknown deterministic** vector ;
- $w(\omega_n)$ is a $K-$dimensional normal random vector, with zero mean and known covariance matrix, $\sigma^2\Sigma(\omega_n)$.

Under the above, the minimum variance unbiased estimate (MVUE) of $s(\omega_n)$ given $r(\omega_n)$ is the projection of $r(\omega_n)$ on the subspace spanned by the columns of the matrix $A(\Theta; \omega_n)$ (see , e.g. [6]):

$$\hat{s}(\omega_n)_{MVUE} = (A(\Theta; \omega_n)^H A(\Theta; \omega_n))^{-1}A(\Theta; \omega_n)^H r(\omega_n) \quad (12)$$

When $A(\Theta; \omega_n)$ has an inverse, (12) simplifies to

$$\hat{s}(\omega_n)_{MVUE} = A(\Theta; \omega_n)^{-1}r(\omega_n) \quad (13)$$

which has covariance matrix

$$R_{\hat{s}} = A(\Theta; \omega_n)^{-1}R(\omega_n)A(\Theta; \omega_n)^{-H}. \quad (14)$$

Comparing (14) with (10), we conclude that the "smoothed source covariance matrix" $\underline{S}$ is in fact a scaled version of the covariance matrix of the estimates of the unknown deterministic vectors $s(\omega_n)$ for the model described [1],

$$\begin{array}{rcl} \underline{S} & \longrightarrow & \sum_n E_l\left[\hat{s}^l(\omega_n)\hat{s}^l(\omega_n)^H\right] \\ & \propto & E_n E_l\left[\hat{s}^l(\omega_n)\hat{s}^l(\omega_n)^H\right] \end{array} \quad (15)$$

when the DOA's are those in the augmented set $(\Theta_0|\Phi_0)$.

Consider the diagram in Fig. 1. The CSSM algorithm uses the fact that it has available the observations of the output of N different systems (one for each frequency) to excitation vectors $s(\omega_0)$ that span the whole P-dimensional complex space. It estimates these excitation vectors, and then "reconstructs" the output of one of the systems (the reference frequency ω_0), simulating in this manner N independent observations.

[1] E_l denotes average over snapshots, and E_n over frequency.

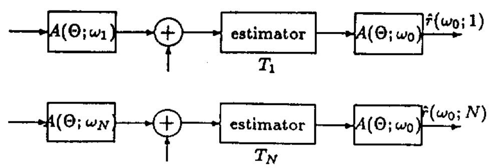

Figure 1: Estimation step of the CSSM.

It then applies the conventional NB HR solution to the "reconstructed" signal set $\{\hat{r}(\omega_0;i)\}_{i=1}^N$, with asymptotic covariance provided by (7).

Given the output, for each DFT component, of its associated observer $(A(\Theta;\omega_n))$ this method estimates the signal as it would be observed through a specific spatial observer $(A(\Theta;\omega_n))$. It is this "interpolation", or "reconstruction", idea that is central to our algorithm.

3. Description of the Algorithm

Consider the following (complex) model of the observations at an array of K sensors of arbitrary (but known) geometry. the observations are the superposition of P distant NB sources:

$$r(t) = A(\Theta)s(t) + w(t), \qquad t \in T \qquad (16)$$

where, for simplicity, we do not indicate frequency dependence.

In (16), $A(\Theta)$ is the matrix of steering vectors:

$$[A(\Theta)]_{kp} = a_{kp}(\Theta) = e^{j\omega\zeta_k(\theta_p)} \qquad (17)$$

and s(t) is the vector of source signals:

$$s(t)^T = [\; a_1 s_1(t)e^{j\omega\tau_{11}} \;\; \cdots \;\; a_P s_P(t)e^{j\omega\tau_{1P}} \;] \quad (18)$$

The asymptotic value of the sample covariance matrix has the same expression as (4):

$$\hat{R} \triangleq \frac{1}{N}\sum_{n=1}^{N} r(t_n)r(t_n)^H \longrightarrow A(\Theta)SA(\Theta)^H + \sigma^2\Sigma \quad (19)$$

We consider here the general case where the rank $r \triangleq \rho(S) \leq P$.

The algorithm is iterative, each iteration step being divided in the following functional blocks: First, we "reconstruct" the signal, as it would be observed by a uniform linear array; this "reconstructed" signal is then input to the spatial smoothing algorithm, yielding a covariance matrix with the properties required by the HR techniques. Finally, we compute a HR spectrum to estimate the DOA's.

Below, we describe briefly each step. Lack of space precludes going into the details of the algorithm.

3.1 Estimation/Reconstruction Step

The first step is functionally equivalent to the preprocessing of the CSSM method discussed in the previous section: it is in fact an estimation step. In contradistinction with the CSSM algorithm, where the estimation problem is set in a deterministic framework, we adopt here a stochastic model.

As the CSSM algorithm, our method starts with initial estimates of the DOA's that define the model used in the estimation of the source vector. We use as well an estimate of the source covariance matrix. Consider the observation equation (16), together with the following model:

- $s(t)$ is a Gaussian stationary vector random process, with zero mean and covariance matrix

$$S \triangleq E[s(t)s(t)^H] \qquad (20)$$

independent of the noise process $w(t)$.

For this model, the MVUE of $s(t)$ is given by:

$$\hat{s}(t) \triangleq Tr(t), \qquad (21)$$

where we have defined the "filter" matrix

$$T \triangleq S^{1/2}[S^{H/2}A(\Theta)^H\Sigma^{-1}A(\Theta)S^{1/2} + \sigma^2 I]S^{H/2}A(\Theta)^H\Sigma^{-1} \qquad (22)$$

where $S^{1/2}$ is a $(K \times r)$ square root of S.

Using the estimated source vector $\hat{s}(t)$, the signal at the output of an hypothetical ULA is now "reconstructed". Let $B(\Theta)$ be the steering matrix of this ULA, with steering vectors $b(\theta)$. The estimate of the signal at the output of this ULA is then:

$$\tilde{r}(t) \triangleq B(\Theta)\hat{s}(t) \qquad (23)$$

with covariance matrix,

$$\tilde{R} = B(\Theta)T\hat{R}T^H B(\Theta)^H \qquad (24)$$

3.2 Spatial Smoothing

The next step consists in the application of the Spatial Smoothing algorithm to the reconstructed signal. Here, the spatial degrees of freedom are used to "simulate" the missing degrees of freedom in the source vector, and make possible subsequent application of the HR algorithms.

Application of the SS algorithm yields the following "smoothed" matrix:

$$\underline{R}_{i,j} = \sum_{m=1}^{M} \tilde{R}_{i+m-1,j+m-1}. \qquad (25)$$

where M is the number of sub-arrays used.

The distribution of the eigenvalues of $\underline{R}$ will not show $K-P$ zero values, but two regions will be clearly defined, corresponding to "signal" and "noise" eigenvalues, yielding an estimate of the number of signals, and of the "signal" and "noise" subspaces.

3.3 Spatial Spectrum

Finally, we use the estimates of the number of sources, obtained from the distribution of the eigenvalues of $\underline{R}$, and the estimated noise subspace to build a spatial spectrum. Let

$$P(\theta) = b(\theta)^H U_N (\Lambda_N)^{-1} U_N^H b(\theta) \qquad (26)$$

where $b(\theta)$ is the steering vector for the tentative direction θ; U_N is a $(K \times (K-P))$ matrix formed by the noise eigenvectors; Λ_N is a $((K-P) \times (K-P))$ diagonal matrix, whose entries are the noise eigenvalues.

This spectrum, when compared to the usual MUSIC spectrum (which does not involve the inverse of the noise eigenvalues), is more robust to errors in the estimation of $\underline{R}$, and on the noise term characterization. In fact, since the model we are using in the estimation is not the true one (we don't know the DOA's, or S) we cannot expect to have a clear separation between the signal and noise subspaces. Weighting the eigenvectors accordingly to the inverse of the corresponding eigenvalues (which has been proposed previously in [5]) makes the spectrum more robust, giving less weight to those eigenvectors that are poorly estimated.

The directions of arrival are estimated as the $\hat{P}$ largest peaks of (26), with $\hat{P}$ determined from analysis of the eigenvalues of $\underline{R}$ generated by the Spatial Smoothing algorithm, in step 2 of the algorithm.

4. Simulation

Fig. 2 shows the spatial spectrum and the distribution of the eigenvalues after 5 iterations of the algorithm, using the asymptotic sample covariance matrix. There are 3 perfectly correlated sources, from directions 50^o, 70^o and 90^o. A 10-element non-uniform array is used, with the following consecutive sensor spacings (in half wavelengths): $[1, .5, 2.5, 1., 3., .5, .5, 2., 2., 2.]$. The SNR for the weaker signal (70^o) is 20dB and for the other two is 30dB.

For this example, the MUSIC spectrum does not peak at the true source directions. With the present algorithm, we see that the 3 sources were detected, and that the DOA's were correctly estimated. The small bias on the value of the estimates can be removed if the algorithm is run using in the estimation step a set of angles in the vicinity of the estimates found, as it is proposed for the CSSM method in [4].

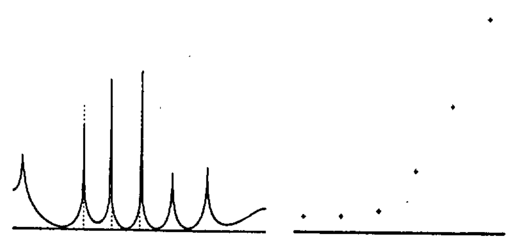

Figure 2: Spatial spectrum and eigenvalues after 5 iterations.

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