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OPTIMAL ESTIMATION OF TIME-VARYING DELAY

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ABSTRACT

The paper reports on time-varying delay estimation in a multisource single direct acoustic path environment. It follows up the work already presented at ICASSP 85 [1] for the single source case. The signals are stochastic nonstationary processes. The time delays are deterministic time-varying functions described by a finite dimensional vector θ of unknown parameters. The observation noise is spatially correlated. The observation time interval is arbitrary.

The estimation structure, based on maximum likelihood (ML) techniques, performs the joint estimation of the signals along with the identification of the parameter vector θ.

Under stationary, long observation time interval (SLOT), and time-invariant delay assumptions, two special problem categories are discussed. The first assumes signals with no overlapping frequency spectra. The second considers the mixing of strong and weak signals. For both class of problems, non-optimal simplified estimation structures are suggested. Monte Carlo simulation results illustrate how the optimal and non-optimal processors mean square error performances compare to the Cramér-Rao bound.

1. PROBLEM FORMULATION

The signal observed at sensor s of the receiver is

$$z(t,s) = \sum_{\ell=1}^{L} \alpha_\ell^s y_\ell(t - D_\ell^s(t)) + v(t,s) \ , \qquad (1)$$

where $t \in T$ ($T = [0,T]$) is the time parameter, s (sensor) and ℓ (source) are space variables that take values on the sets of coordinates S ($S = \{1,2,\cdots,S\}$) and L ($L = \{1,2,\cdots,L\}$), respectively; $D_\ell^s(t)$ and α_ℓ^s represent, respectively, the delay function and the attenuation coefficient associated to each path (s,ℓ): the time-varying delay function $D_\ell^s(t)$, $\forall t \in [0,T]$, satisfies the following conditions:

(i) it is a continuous function of t;

(ii) it has first derivative with respect to t;

(iii) it is described by a deterministic time-varying function of a finite dimensional vector θ_ℓ of unknown parameters, i.e.,

$$D_\ell^s(t) = \tau_\ell(t,\theta_\ell) \ . \qquad (2)$$

The observation noise $v(t,s)$ is a Gaussian white noise process with cross-covariance matrix $R_{sm}(t)\delta(t-\sigma)$ $(s,m \in S)$. The nonstationary signal $y_\ell(t)$ $(\ell \in L)$ is

$$\frac{d}{dt}x_\ell(t) = A_\ell(t)x_\ell(t) + B_\ell(t)u_\ell(t) \qquad (3)$$

$$y_\ell(t) = C_\ell(t)x_\ell(t) \ , \ t \geq t_0 \ . \qquad (4)$$

The state $x_\ell(t)$ initial condition $x_\ell(t_0)$ is a Gaussian random vector with mean $\bar{x}_\ell(t_0)$ and cross-covariance matrix $\Sigma_{\ell k}(t_0,t_0)$ $(\ell,k \in L)$. The dynamics disturbance $u_\ell(t)$ $(\ell \in L)$ is a Gaussian white noise process, independent of the observation noise $v(t,s)$ and of the random vector $x_k(t_0)$ $(k \in L)$, with cross-covariance matrix $Q_{\ell k}(t)\delta(t-\sigma)$.

On occasion, when no ambiguity arises, function arguments may be omitted.

2. ESTIMATOR STRUCTURE

Let us define the time-varying delay matrix

$$D(t) = [D_\ell^s(t)] \ . \qquad (5)$$

The delay estimate $\hat{D}(t)$ is constructed based on the ML parameter vector estimate (see expression (2)) [2], [3]

$$\hat{\theta}(t) = \arg\max_{\theta\in\Theta} J(t; D^t(\theta)) \ , \qquad (6)$$

where

$$D^t = \{D(\sigma) = \tau(\sigma,\theta) \ , \ 0 \leq \sigma \leq t\} \qquad (7)$$

represents the time-varying delay realization. The log-likelihood function is [2], [3]

$$J(t;D^t) = \int_0^t [Z^T R^{-1}\hat{Y} - \frac{1}{2}\hat{Y}^T R^{-1}\hat{Y} - \frac{1}{2}\mathrm{tr}(R^{-1}P_Y)]d\sigma \quad (8)$$

where

$$Z(t) = [z^T(t,1) \vdots z^T(t,2) \vdots \cdots \vdots z^T(t,S)]^T \qquad (9)$$

is the observation vector, and $R(t)\delta(t-\sigma)$ with

$$R(t) = [R_{sm}(t)] \qquad (10)$$

is the observation noise covariance matrix.

In (8), the received signal minimum mean square error (MMSE) vector estimate is

$$\hat{Y}(t) = [\hat{Y}^s(t)] \qquad (11)$$

where

$$\hat{Y}^s(t) = \sum_{\ell=1}^{L} \alpha_\ell^s \hat{y}_\ell(t, D_\ell^s(t)) \ , \qquad (12)$$

and the error estimate covariance matrix is

$$P_Y(t) = [P_Y^{sm}(t)] \qquad (13)$$

where

$$P_Y^{sm}(t) = \sum_{\ell,k=1}^{L} \alpha_\ell^s \alpha_k^m P_{\ell k}(t, D_\ell^s, D_k^m) \ . \qquad (14)$$

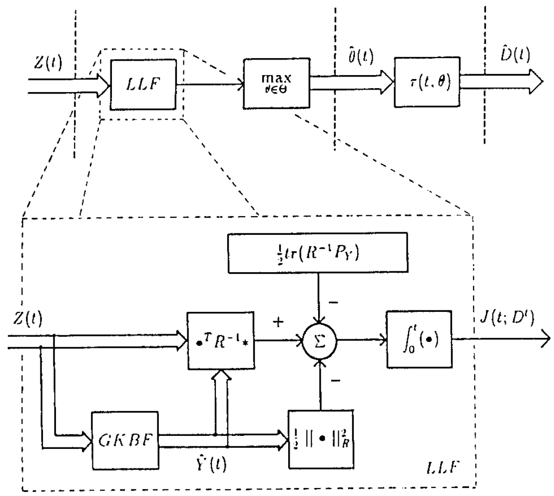

Figure 1: Estimator structure

In (12) and (14). $\hat{y}_\ell(t, D_\ell^s)$ is the signal $y_\ell(t - D_\ell^s)$ MMSE estimate and $P_{\ell k}(t, D_\ell^s, D_k^m)$ the corresponding cross-covariance matrices obtained with the generalized Kalman-Bucy filter (GKBF) [2]. [3]. The GKBF generalizes to systems with multiple time-varying delays in the observation process, the filter developed by Kwakernaak [4] for linear systems with multiple time-invariant delays in both the dynamics and/or the observations.

Figure 1 shows the time-varying delay estimator structure. The delay estimate $\hat{D}(t)$ is computed on-line based on the ML parameter vector estimate $\hat{\theta}(t)$. It represents a generalized cross-correlator.

3. SPECIAL PROBLEM CATEGORIES

Introduction

Under stationary, long observation time interval (SLOT) and time-invariant delay assumptions along with the hypothesis of observability and controllability of the emitted signals model, the GKBF is asymptotically time-invariant [2]. In this context, we are going to consider two special problem categories:

(i) separable signal processes;

(ii) uncorrelated strong and weak signals.

For both class of problems, non-optimal simplified structures are suggested.

Separable Signal Processes

Denoting Ω_ℓ the signal $y_\ell(t)$ equivalent rectangular bandwidth [5], let us assume the emitted signals $y_\ell(t)$, $\forall \ell \in L$,

(i) are almost band-limited, i.e., its power spectral density $S_{\ell\ell}(\omega)$ satisfies

$$\forall \omega \in \Re \setminus \Omega_\ell \ , \ S_{\ell\ell}(\omega) \simeq 0 \ ; \tag{15}$$

(ii) have no overlapping frequency spectra, i.e.,

$$\forall k \in L : \ \ell \neq k, \ \Omega_\ell \cap \Omega_k = \emptyset \tag{16}$$

where $\emptyset$ represents the empty set.

If we further assume the signal $y_\ell(t - D_\ell^s)$ estimation error is small, the signal estimate $\hat{y}_\ell(t, D_\ell^s)$ power spectral density $\hat{S}_{\ell\ell}^{ss}(\omega)$ is

$$\hat{S}_{\ell\ell}^{ss}(\omega) \simeq S_{\ell\ell}(\omega) \ , \ \forall \omega \in \Re, \ \forall \ell \in L, \ s \in S \ . \tag{17}$$

Factorization of the steady-state generalized Riccati equation [2], along with expressions (15-17), lead to the condition on the signal estimates $\hat{y}_\ell(t, D_\ell^s)$ and $\hat{y}_k(t, D_k^m)$ $(\ell \neq k)$ power cross-spectral density [2]

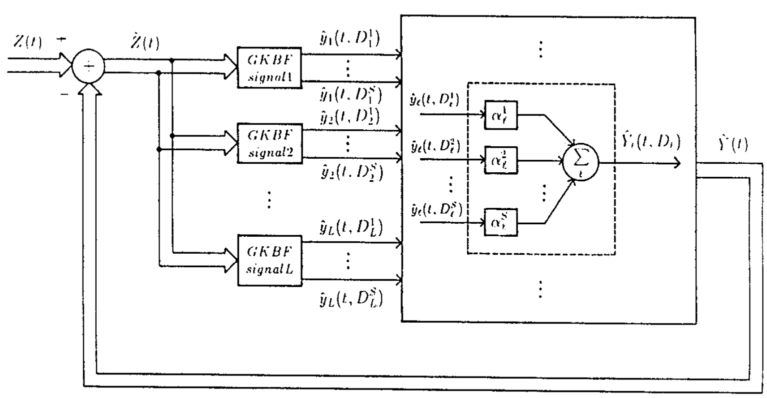

Figure 2: Generalized Kalman-Bucy filter: L frequency separated signals

$$\dot{S}_{\ell k}^{sm} \simeq 0 \ , \ \forall \omega \in \Re, \ \forall s, m \in S, \ \ell, k \in L : \ \ell \neq k \ , \quad (18)$$

which gives rise to

$$P_{\ell k}^{sm}(r,1) C_k^T \simeq 0 \ , \ \forall r \in [0,1], \ \forall s, m \in S, \ \ell, k \in L : \ \ell \neq k \ , \quad (19)$$

where $P_{\ell k}^{sm}(r_1, r_2)$ is the steady-state state estimation error cross-covariance matrix. In this case, the covariance matrix $P_{\ell\ell}^{sm}(r_1, r_2)$ description, as well as the GKBF gain matrix, is not based on the cross-covariance matrices $P_{\ell k}^{sm}(r_1, r_2)$, $\forall \ell, k \in L : \ \ell \neq k$. Therefore the multisource GKBF structure decouples into L parallel one source GKBF, being the effect of the reminder $L-1$ sources introduced through the innovations process (figure 2).

Uncorrelated Strong and Weak Signals

Consider an L source configuration. At the array of hydrophones, the average power of arrival of L_1 ($L_1 < L$ - strong signals) of the total number L of signals is much greater than that of the remainder L_2 signals (weak signals). For instance, this may model the presence of L_2 remote sources superimposed to L_1 close signal sources.

Assume zero mean mutually uncorrelated stationary signal processes, and denote $\mathcal{P}_\ell$ the signal $y_\ell(t)$ average power. The observed process is described by equation (1) where we consider the attenuation factor

$$\alpha_\ell^s = \begin{cases} 1 & \text{if} \quad \ell \in L_1 = \{1, 2, \cdots, L_1\} \\ \alpha \ll 1 & \text{if} \quad \ell \in L_2 = \{L_1 + 1, \cdots, L\} \end{cases} . \quad (20)$$

Therefore, if we assume the emitted signals average power is

$$\mathcal{P}_\ell = \mathcal{P}_k \ , \ \forall \ell, k \in L \ , \quad (21)$$

at the receiver's array we have

$$\mathcal{P}_\ell \gg \mathcal{P}_k^\alpha = \alpha^2 \mathcal{P}_k \ , \ \forall \ell \in L_1, \ k \in L_2 \ . \quad (22)$$

In (22) α^2 represents the weak to strong signals average power ratio.

The delay estimate $\hat{D}(t)$ is given by expressions (6-8) with

$$D = \tau(\theta) = \theta \ . \quad (23)$$

The received signal estimate $\hat{Y}(t)$ (expressions (11-12)) and the error covariance matrix $P_Y(t)$ (expressions (13-14)) are both functions of α. When α is small, a second order approximation to the log-likelihood function (LLF) can be considered [2], i.e.,

$$J(t; D) \simeq J^{(0)}(t; D) + \alpha J^{(1)}(t; D) + \frac{\alpha^2}{2} J^{(2)}(t; D) \ , \quad (24)$$

where

$$J^{(0)}(t; D) = J(t; D^{L_1}) \quad (25)$$

$$J^{(1)}(t; D) = 0 \ , \quad (26)$$

and

$$J^{(2)}(t : D) = \int_0^t [(Z - \hat{Y}_{opt}^{L_1})^T R^{-1} \hat{Y}^{(2)} - \frac{1}{2} \mathrm{tr}(R^{-1} P_Y^{(2)})] d\sigma \ . \quad (27)$$

In the above expressions, D^{L_1} represents the delay matrix related to the strong signals subset L_1, $J(t; D^{L_1})$ and $\hat{Y}_{opt}^{L_1}$ being, respectively, the LLF and the signal estimate established for the L_1 sources problem.

The LLF $J(t; D^{L_1})$ is not a function of the remainder L_2 signal delay matrix D^{L_2}. On the other hand, α being small, one should expect that $J(t; D^{L_1})$ dominates the second order term $\frac{\alpha^2}{2} J^{(2)}(t; D)$. These facts suggest the following two steps non-optimal delay estimate procedure [2]:

(i) compute $\hat{D}^{L_1}(t)$ by maximization of $J(t; D^{L_1})$;

(ii) taking $D^{L_1} = \hat{D}^{L_1}(t)$, maximize $J^{(2)}(t; \hat{D}^{L_1}(t), D^{L_2})$ to get $\hat{D}^{L_2}(t)$.

Figure 3 shows this non-optimal delay processor structure. Block PL_1 represents the optimal delay processor developed for the L_1 sources problem. Block LLF^2 generates the log-likelihood function second order term (expression (27)). If there is no need on estimating D^{L_2}, we just have to implement the reduced processor PL_1.

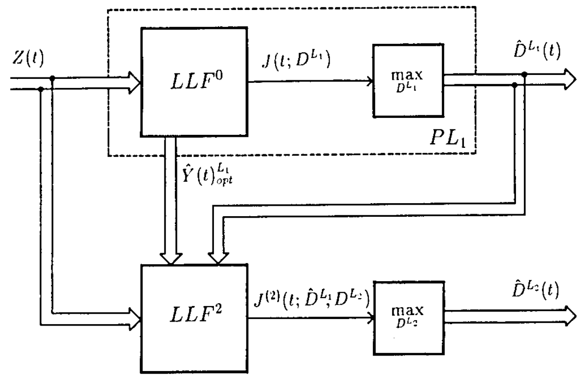

Figure 3: Non-optimal delay processor structure

4. SIMULATION RESULTS

In the presence of a perturbation source located broadside to the receiving array, consider the problem of estimating the time delay between the signals observed at two spatially separated sensors.

The observations are described by equation (1) with

$$D_\ell^s = \begin{cases} 0 & \text{if} \quad s = 1 \quad \text{or} \quad \ell = 2 \\ D^a & \text{if} \quad s = 2 \quad \text{and} \quad \ell = 1 \end{cases} , \quad (28)$$

where $D^a = 0.002$ sec is the actual delay value. The observation noise is spatially and temporally white, with spectral height

$$R(s) = 0.1 \ , \ s = 1, 2 \ . \quad (29)$$

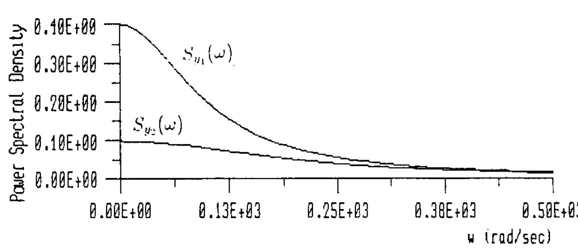

Figure 4: Power spectral density

Processes $y_\ell(t)$, $\ell = 1, 2$, are low-pass (figure 4) mutually uncorrelated stationary signals, modeled by equations (3-4). The signal $y_1(t)$, emitted by the source to be located, and the signal $y_2(t)$, radiated by the perturbation source, are given by

$$C_1 = C_2 = 1 \tag{30}$$

$$B_1 = B_2 = 10 \tag{31}$$

$$A_1 = -100 \tag{32}$$

$$A_2 = -200 \ . \tag{33}$$

The dynamic disturbance covariance is $40\delta_{\ell k}\delta(t - \sigma)$.

Assume we can restrain the delay domain to the time interval $[-0.01, 0.01]$ sec. and consider a discretization step of 0.001 sec. The optimal and the non-optimal (reduced processor PL_1 - figure 3) delay processors are then implemented through a bank of 21 blocks working in parallel.

For both the optimal and the non-optimal structures, a 200 sample Monte Carlo experiment was run. The delay estimate ensemble average and the mean square error (MSE) are plotted in figures 5 and 6, respectively. Although for each time instant t the bias achieved with the optimal estimator is smaller than the one obtained with the non-optimal structure, for both processors the delay estimate is asymptotically unbiased, and the MSE achieves the Cramér-Rao bound (CRB). The simulation results presented show the delay processor robustness when the emitted signals modeling is not accurately known, due to the presence of a weak interference.

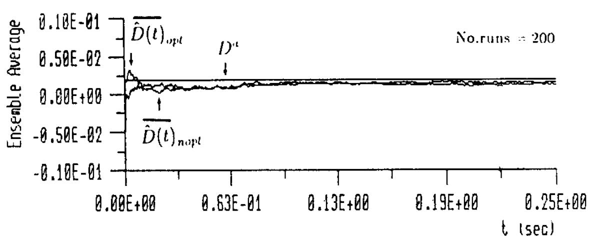

Figure 5: Delay estimate ensemble average

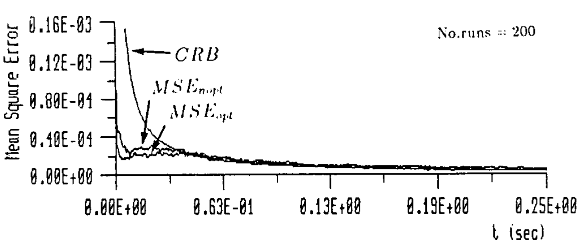

Figure 6: Delay estimate mean square error

5. CONCLUSION

The work presented in this paper reports on maximum likelihood (ML) time-varying delay estimation with mutually correlated stochastic nonstationary signals and mutually correlated observation noises. The general ML estimation structure presented can be interpreted as a generalized cross-correlator. In [3] we expand on the present processor, presenting its asymptotic study under stationary, long observation time interval (SLOT) and time-invariant delay assumptions. In this context, an approximate structure to this general ML processor can be achieved, that generalizes [2], [3]:

(i) Ng and Bar-Shalom [6] processor, developed for a multisource environment with spatially uncorrelated observation noise;

(ii) Kirlin and Dewey [7] estimator, established for a single source configuration with spatially correlated observation noise;

(iii) Knapp and Carter [8] generalized cross-correlator, obtained for a single source geometry with observation noise uncorrelated among sensors.

Said in another way, these classical ML solutions are SLOT approximations to the general ML estimator structure presented in this paper.

6. AKNOWLEDGMENTS

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