Signal Processing for Bit-Patterned Media (BPM) Channels

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DSSC Channels Research Strategy

- **Low SNR will be one of the main challenges for future high-density storage systems**
  - LDPC codes
  - Iterative soft decoders
- **Channel models and signal processing approaches must be adapted to the new recording paradigms**
  - Bit-patterned media (BPM)
  - Two-dimensional magnetic recording (TDMR)
  - Heat-assisted magnetic recording (HAMR)
  - Multi-level cell flash memory & MRAM
- **Optimization of channels algorithms for hardware power/area/performance tradeoffs**
LDPC Codes: Contributions

- FPGA platform for LDPC decoder achieved $10^{-12}$ BER
- Design, evaluation and encoder/decoder architecture of hardware-friendly progressive edge growth (PEG) quasi-cyclic LDPC codes
- Evaluation of LDPC codes under realistic magnetic recording channel models (not just AWGN)
- Improved reverse concatenation of LDPC codes and RLL codes using bit-flipping & interleaving
- Semi-analytical method for error floor estimation by identifying trapping sets using FPGA platform; estimated error floors down to $10^{-13}$ frame error rates
- Investigation of the concatenation of LDPC codes and outer RS codes
Reed-Solomon (RS) codes are still of much use and interest.

Soft-decoding methods (e.g., Koetter-Vardy algorithm) proposed for error correction beyond half the minimum distance, but their complexity too high.

Soowoong Lee proposed a new Error-pattern flipping Chase2-type decoder, with parameter $p$:

- Flip dominant error-patterns before RLL decoder instead of bit-flipping or symbol-flipping.
- Repeat error-only decoding with $2^p$ modified received sequences while flipping $p$ most probable error-patterns.
**TDMR Concept (INSIC)**

**A.** 10 user bits

**B.** 40 channel bits from encoder

**C.** Writing 20 grains

**D.** Reading hi-res 2D read process (scanning)

**E.** Establish timing & position of bit-cells

**F.** Use 2D image analysis methods to decode user bits from recovered channel bits & soft info., and grain statistics

**Toy example:**

- Corner writer
- (shingled-) write process
- 40 channel bits get written on grains
- Not all channel bits get written on grains
- Can almost see grains but not grain-boundaries
- 10 bits in 1 \( \mu \text{in}^2 \) = 10 Terabits/in\(^2\)

**Soft 2D decode process**

**10 user bits**

**Courtesy: Roger Wood**
LDPC Error Floor Estimation via FPGA Emulation (Yu Cai & Prof. Ken Mai)

- Two PowerPC per FPGA
- Five FPGAs per board
- High speed interconnect
- 80GB on board DRAM

Future Plans:
- Two parallel SOVA_LDPC per FPGA (2X speedup)
- Utilize all 5 FPGAs (5X speedup)
- 50X speedup over previous FPGA implementation
- 1000X speedup over workstation implementation

Resource Usage:

<table>
<thead>
<tr>
<th>Resource</th>
<th>SOVA_LDPC</th>
<th>Total Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slices Used</td>
<td>10638 (32%)</td>
<td>17000 (51%)</td>
</tr>
<tr>
<td>Flip Flops</td>
<td>13859 (20%)</td>
<td>21380 (32%)</td>
</tr>
<tr>
<td>4-input LUTs</td>
<td>14040 (21%)</td>
<td>22286 (33%)</td>
</tr>
<tr>
<td>BRAM</td>
<td>47 (14%)</td>
<td>94 (28%)</td>
</tr>
<tr>
<td>Multiplier 18X18</td>
<td>58 (17%)</td>
<td>58 (17%)</td>
</tr>
<tr>
<td>Critical Path</td>
<td>9.979ns</td>
<td>9.979ns</td>
</tr>
</tbody>
</table>

- Clock rate improved from 20Mhz to 100Mhz (5X)
- Two parallel SOVA_LDPC fit in one FPGA (2X)
BPM Signal Processing: Outline

- Motivation for BPM
- BPM 2D Pulse response
- BER analysis of BPM
- Mitigating the effects of inter-track interference
- Modeling the media noise
Why bit-patterned media?

- **SNR is proportional to the number of grains per bit**
- **Higher density can be achieved by scaling**
  - Size of bits should be reduced
  - The number of grains should remain the same

- For nano-scale magnetic grain, thermal energy is comparable to the energy of bit (superparamagnetic limit)
Bit-patterned media (BPM)

- Bits are stored in single domain magnetic islands
- The regions between the islands are nonmagnetic material
- Achieving densities over 1 Tb/in²

- Advantages
  - Reducing transition and track edge noise
  - Reducing or eliminating non-linear bit shift
  - Simplifying tracking
BPM challenges (signal processing)

- **Inter-track interference**
  - Bits or magnetic islands are very close
  - Besides ISI, there is ITI

- **Media noise**
  - Media noise is caused by the fluctuations in the islands
    - Location
    - Size
    - Thickness
    - Magnetization
    - Shape

- **Write synchronization**

- For more realistic channel model (modeling ITI and media noise) 2D pulse response needs to be used
2D pulse response

- **Readback voltage of the MR head**

\[ V_{MR}(\bar{x}, \bar{z}) = C \phi(\bar{x}, \bar{z}) \]

- \( \phi(\bar{x}, \bar{z}) = \phi(\bar{x}, \bar{y} = 0, \bar{z}) \) : signal flux injected into the MR element at the ABS

- **3D Reciprocity principle**

\[
\phi(\bar{x}, \bar{z}) = \frac{\mu_0}{i} \int_{-\infty}^{\infty}dx' \int_{d}^{d+\delta}dy' \int_{-\infty}^{\infty}dz' \Psi(x', y', z') \left[ \frac{\partial M_y(x' - \bar{x}, y', z' - \bar{z})}{\partial y'} \right]
\]

- \( M_y \): media magnetization
- \( i \): imaginary write current
- \( \Psi \): magnetic head potential

\[
\Psi(\bar{x}, \bar{y}, \bar{z}) = \frac{\bar{y}}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Psi_s(x', z')dx'dz' \left[ \frac{1}{(\bar{x} - x')^2 + \bar{y}^2 + (\bar{z} - z')^2} \right]^{3/2}
\]
**Example MR head surface magnetic potential**

- **Applying the head surface magnetic potential approximation from Wiesen et al.**

\[
\Psi_s(x, z) = 1 - \left( \frac{1}{\pi} \right) \arctan \left( \frac{\sqrt{2}}{1 - 2 \exp \left( \frac{2\pi z}{g} \right) \cos \left( \frac{2\pi x}{g} \right) + \exp \left( \frac{4\pi z}{g} \right)} \right) - 1 + \exp \left( \frac{2\pi z}{g} \right) \cos \left( \frac{2\pi x}{g} \right)
\]

MR head (\( W = 20 \text{ nm}, \ t = 4 \text{ nm}, \ g = 10 \text{ nm} \))

Example 2D Pulse Response

- **Numerical integration to obtain the magnetic potential and pulse response**
  - **MR head**
    - $t = 4 \text{ nm}, W = 20 \text{ nm}, \text{ gap} = 10 \text{ nm}$
  - **Magnetic island**
    - $a = 11 \text{ nm}, \delta = 10 \text{ nm}, d = 10 \text{ nm}$

Along-track $PW50 = 22.3 \text{ nm}$, Cross-track $PW50 = 30.1 \text{ nm}$
Readback signal

- **Using 2D linear superposition to obtain readback signal**
- **Bits can be represented by their locations**
- **Length of ITI assumed to be 3**
  - Main track and two adjacent tracks
- **Rectangular grid**

\[
s(\bar{x}) = \sum_{m=-1}^{1} \sum_{n=-N}^{N} x(m,n)P(-mT_z,\bar{x}-nT_x)
\]
Discrete-time BPM read channel

- **Readback signal is sampled at the nominal bit period** $T_x$

\[
H(m,n) = P(mT_z, nT_x)
\]

- **Discrete time readback signal**

\[
r(k) = \sum_{m=-1}^{1} \sum_{n=-N}^{N} x(m,n)H(-m, k-n) + v(k)
\]

- **Equalizer output**
  - Assuming symmetric channels

\[
z(k) = f(k) * r(k)
= f(k) * h_0(k) * x(0,k) + f(k) * h_1(k) * [x(-1,k) + x(1,k)] + f(k) * v(k)
\]
Viterbi detector input signal

\[ z(k) = y(k) + x(0, k) \cdot c_{ISI}(k) + b(k) \cdot c_{ITI}(k) + v(k) \cdot f(k), \]

where

\[ y(k) = x(0, k) \cdot g(k), \quad \text{where} \quad g(k) (g^T = [g_{-K}, \ldots, g_0, \ldots g_K]) \text{ is the PR target} \]

\[ c_{ISI}(k) = h_0(k) \cdot f(k) - g(k), \quad \text{where} \quad f(k) (f^T = [f_{-L}, \ldots, f_0, \ldots f_L]) \text{ is the equalizer} \]

\[ b(k) = x(-1, k) + x(1, k) \]

\[ c_{ITI}(k) = h_1(k) \cdot f(k), \quad \text{assuming} \quad h_1(k) = h_{-1}(k) \]

Using vector representation

\[ z(k) = y(k) + c^T_{ISI} x_{0,k} + c^T_{ITI} b_k + f^T v_k \]

where

\[ c^T_{ISI} = [c_{ISI}(-N-L) \ldots c_{ISI}(N+L)] \]

\[ c^T_{ITI} = [c_{ITI}(-N-L) \ldots c_{ITI}(N+L)] \]

\[ n(k) : \text{overall noise} \]
For performance analysis of a partial response channel we need to first determine the error events.

For the Viterbi detector, an error event $\varepsilon$ is an erroneous estimate state sequence.
Probability of an error event, $\varepsilon$

- **Probability of a particular error event $\varepsilon$ for the Viterbi detector**

  \[
  \Pr(\varepsilon) \leq \Pr(m_e < m_c) \left( \frac{1}{2} \right)^{w_H(\varepsilon)}
  \]

  where
  - $m_c$ is the path metric for the correct path
    \[
    m_c = \|z_k - y_k\|^2 = \|n_k\|^2
    \]
  - $m_e$ the path metric for the erroneous path of a particular error event $\varepsilon$
    \[
    m_e = \|z_k - \hat{y}_k\|^2 = \|y_k + n_k - \hat{y}_k\|^2 = \|\varepsilon + n_k\|^2
    \]
  - $w_H(\varepsilon)$: The number of input bit errors or the Hamming weight

- **For a specific error event $\varepsilon$**

  \[
  \Pr(m_e < m_c) = \Pr(\|n_k + \varepsilon_y\|^2 < \|n_k\|^2) = \Pr(\varepsilon_y^T n_k < -\frac{1}{2} \|\varepsilon_y\|^2)
  \]
The PDF of the overall noise and the output error sequence inner product

- **pdf of** $\tilde{\epsilon}_y^T \mathbf{n}_k$

\[
\begin{align*}
    f(x) & \quad \text{pdf of } f(b) \\
    \frac{1}{2} & \quad \frac{1}{2} \\
    -1 & \quad 1 \\
    x & \quad b
\end{align*}
\]

\[
f(\tilde{\epsilon}_y^T \mathbf{n}_k) = f(\mathbf{w}_{ISI}^T x_{0,k} + \mathbf{w}_{ITI}^T \mathbf{b}_k + \mathbf{w}_f^T \mathbf{v}_k)
\]

\[
= f(\mathbf{w}_{ISI}^T x_{0,k} + \mathbf{w}_{ITI}^T \mathbf{b}_k) * f(\mathbf{w}_f^T \mathbf{v}_k)
\]

(not Gaussian) → Gaussian

- *: convolution
- **pdf of** $\mathbf{w}_f^T \mathbf{v}_k$
  - $\mathbf{v}_k$: AWGN electronics noise

\[
\mathbf{v}_k \rightarrow N(0, \sigma_v^2) \quad \Rightarrow \quad \mathbf{w}_f^T \mathbf{v} \rightarrow N(0, \sigma_v^2 \left\| \mathbf{w}_f \right\|^2)
\]
Distribution of ITI and un-equalized ISI

- pdf of  \( w_{\text{ISI}}^T x_{0,k} + w_{\text{ITI}}^T b_k \)

- Assuming uniform binary input data \( f(w_{\text{ISI}}^T x_{0,k} + w_{\text{ITI}}^T b_k) \) is a train of impulses

Monic constraint target \( (g^T = [g_{-L}, 1, g_{L}]) \),

Density of 2 Tbit/in², The minimum distance error event
Most probable error events

- **Using computer search to obtain most probable error events**
  - **Monic constraint target** \((g^T = [g_{-1}, 1, g_1])\)
  - **Density of 2 Tbit/in²**
  - \(\sigma_v^2 = 0.01\),

  The minimum distance error event, \((d_{\text{min}})^2 = 4.03\)

<table>
<thead>
<tr>
<th>(\varepsilon_x)</th>
<th>(|\varepsilon_y|^2)</th>
<th>(\Pr(\varepsilon))</th>
</tr>
</thead>
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<tr>
<td>+</td>
<td>4.03</td>
<td>1.66e-03</td>
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<tr>
<td>++</td>
<td>8.98</td>
<td>1.48e-06</td>
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<tr>
<td>+0--</td>
<td>8.03</td>
<td>4.77e-06</td>
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<tr>
<td>+0+</td>
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<td>6.33e-06</td>
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<tr>
<td>+++</td>
<td>10.25</td>
<td>2.89e-07</td>
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<td>12.08</td>
<td>1.13e-08</td>
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<td>+0-0--</td>
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<tr>
<td>++0-0-0-0+</td>
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<td>7.03e-08</td>
</tr>
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<td>7.03e-08</td>
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</table>
Conventional PR4 and EPR4 targets do not perform well

- SNR_e = 20 \log_{10}(V_p / \sigma_v)
  - \( V_p = 1 \)
  - \( \sigma_v \): noise standard deviation
Analytical and simulation BER vs SNR

- **Monic constraint target**
  - \( g^T = [g_{-1}, 1, g_1] \)

- **Density of 2 Tbit/in\(^2\)**
  - \( T_x \) and \( T_z \) 18 nm
  - MR head
    - \( t = 4 \text{ nm}, W = 20 \text{ nm}, g = 10 \text{ nm} \)
  - Magnetic island
    - \( a = 11 \text{ nm}, \delta = 10 \text{ nm}, d = 10 \text{ nm} \)
  - \( H = \begin{bmatrix} 0.021 & 0.213 & 0.021 \\ 0.101 & 1 & 0.101 \\ 0.021 & 0.213 & 0.021 \end{bmatrix} \)
**Effect of Inter-track Interference (ITI)**

- **GPR equalizer**
- **Target length : 3**
  \[ g = [g_{-1}, 1, g_1] \]
- **Density of 2 Tbit/in²**
  - \( T_x \) and \( T_z \) 18 nm
  - **MR head**
    - \( t = 4 \text{ nm}, W = 18 \text{ nm}, g = 6 \text{ nm} \)
  - **Magnetic island**
    - \( a=11 \text{ nm}, \delta = 10 \text{ nm}, d=10 \text{ nm} \)
- \[ H = \begin{bmatrix} 0.038 & 0.294 & 0.038 \\ 0.113 & 1 & 0.113 \\ 0.038 & 0.294 & 0.038 \end{bmatrix} \]

*ITI significantly degrades the performance of the channel*
2D-GPR equalizer

- **Using 2D approach to mitigate the effect of ITI**
- **Assuming the parallel readback of the data in three adjacent tracks**

![Diagram showing the 2D-GPR equalizer process]

- **Using minimum mean squared error (MMSE) approach to optimize the 2D equalizer and target**
- **Viterbi algorithm does not exist for the true 2D motion**
Vector representation for 2D GPR

- \( j=0 \) is representing the main track

\[
\begin{align*}
\mathbf{f} &= \begin{bmatrix} f_{-M,-N} & \ldots & f_{0,0} & \ldots & f_{M,-N} \end{bmatrix}^T \\
\mathbf{g} &= \begin{bmatrix} g_{-L,-L} & \ldots & g_{0,0} & \ldots & g_{L,-L} \end{bmatrix}^T \\
\mathbf{y}_k &= \begin{bmatrix} y_{M,k+N} & \ldots & y_{0,k} & \ldots & y_{-M,k-N} \end{bmatrix}^T \\
\mathbf{x}_k &= \begin{bmatrix} x_{L,k+L} & \ldots & x_{0,k} & \ldots & x_{-L,k-L} \end{bmatrix}^T
\end{align*}
\]

\[
e(k) = \mathbf{f}^T \mathbf{y}_k - \mathbf{g}^T \mathbf{x}_k
\]

- **Mean square error**

\[
E\{e(k)^2\} = \mathbf{f}^T \mathbf{R} \mathbf{f} - 2 \mathbf{f}^T \mathbf{T} \mathbf{g} + \mathbf{g}^T \mathbf{A} \mathbf{g}
\]

where

\[
\mathbf{R} = E\{\mathbf{y}_k \mathbf{y}_k^T\}, \quad \mathbf{T} = E\{\mathbf{y}_k \mathbf{x}_k^T\}, \quad \mathbf{A} = E\{\mathbf{x}_k \mathbf{x}_k^T\}\]
Minimizing the $E\{e(k)^2\}$ with considering the constraint

$$G = \begin{bmatrix} 0 & 0 & 0 \\ g_{0,-1} & 1 & g_{0,1} \\ 0 & 0 & 0 \end{bmatrix} \quad \text{or} \quad g = \begin{bmatrix} 0 & 0 & 0 & g_{0,-1} & 1 & g_{0,1} & 0 & 0 & 0 \end{bmatrix}^T$$

Lagrange functional to be minimized

$$J = f^T R f - 2f^T T g + g^T A g - 2\lambda^T (E^T g - c)$$

where

$$E^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Taking derivative respect to $f$, $g$ and $\lambda$

$$\lambda = (E^T (A - T^T R^{-1} T)^{-1} E)^{-1} c$$

$$g = (A - T^T R^{-1} T)^{-1} E \lambda$$

$$f = R^{-1} T g$$
Results of 2D GPR ZIFE

- **2D GPR ZIFE**
  - 3×3 target
  - Obtained for $\sigma_v^2=0.01$
  - $G = \begin{bmatrix} 0 & 0 & 0 \\ 0.132 & 1 & 0.130 \\ 0 & 0 & 0 \end{bmatrix}$

- **1D GPR (multi-track)**
  - Obtained for $\sigma_v^2=0.01$
  - $g = [0.076 \; 1 \; 0.077]^T$

2D GPR ZIFE improves the BER
2D-GPR equalizer

- **Pseudo 2D Viterbi algorithm**

- **For a 3×3 target, tracing the history of two consecutive data in three tracks**
  - 64 states
  - 8 branches

\[
G = \begin{bmatrix}
g_{-1,-1} & g_{-1,0} & g_{-1,1} \\
g_{0,-1} & 1 & g_{0,1} \\
g_{1,-1} & g_{1,0} & g_{1,1}
\end{bmatrix}
\]
2D GPR equalizers

- **2D GPR target**
  - Obtained for $\sigma_v^2 = 0.01$

$$G = \begin{bmatrix} 0.056 & 0.310 & 0.077 \\ 0.183 & 1 & 0.180 \\ 0.051 & 0.315 & 0.068 \end{bmatrix}$$

![Graph showing BER vs. SNR]
**Modified Viterbi algorithm (VA)**

- **The locations of bits (islands) are fixed**
- **One track is read**
- **Assuming symmetric channel**

\[
\begin{align*}
 & x(-1,k) \\
 & x(0,k) \\
 & x(1,k) \\
\end{align*}
\]

\[
\begin{array}{c}
\text{Channel} \\
H(m,n)
\end{array}
\quad
\begin{array}{c}
\nu(k) \\
r(k)
\end{array}
\quad
\begin{array}{c}
\text{Equalizer} \\
f(k)
\end{array}
\quad
\begin{array}{c}
z(k) \\
\hat{x}(0,k)
\end{array}
\]

\[
H = \begin{bmatrix}
- h_1(k) & \\
- h_0(k) & \\
- h_1(k) & \\
\end{bmatrix}
\]

\[
z(k) = f(k) * r(k)
\]

\[
= f(k) * h_0(k) * x(0,k) + f(k) * h_1(k) * (x(-1,k) + x(1,k)) + f(k) * \nu(k)
\]
Considering ITI in VA

- **ITI is mainly due to directly adjacent bits i.e.** \(x_{-1,k}, x_{1,k}\)
- **Assuming no track mis-registration**
- **Desired equalizer output**

\[
d_k \cong g(k) \ast x(0,k) + \alpha \left[ x(1,k) + x(-1,k) \right] + n(k)
\]

- **Where**

\[
x(j,k) \in \{1, -1\} \Rightarrow \left[ x(1,k) + x(-1,k) \right] \in \{-2, 0, 2\}
\]

\[
g(k) = h_0(k) \ast f(k)
\]

\[
\alpha = h_1(k) \ast f(k) \bigg|_{k=0}
\]

\[
n(k) = v(k) \ast f(k)
\]
- Modified VA trellis

\[
d_k = \begin{cases} 
    g(k) \ast x(0,k) + 2\alpha + n(k) \\
    g(k) \ast x(0,k) + n(k) \\
    g(k) \ast x(0,k) - 2\alpha + n(k)
\end{cases}
\]
Results of the modified VA

- Target: \( g = [0.1 \ 1 \ 0.1] \)
- \( \alpha = 0.2 \)

Modified VA improves the performance of the channel
Modeling the Media Noise

- Considering location fluctuations and size fluctuations

\[ r(k) = \sum_{m=-1}^{m=1} \sum_{n=-N}^{n=N} x(m, n) P(-mT_z + \Delta T_m, (k - n)T_x + \Delta T_n, a + \Delta a_{mn}) + v(k) \]
Obtaining an analytical 2D pulse response

- Along-track pulse response can be considered as subtraction of two transitions
- For \( a << PW50 \) derivative of transition pulses can be interpreted as pulse responses

2D Gaussian pulse
- Using \( \text{erf}(x) \) as a transition response
  \[
  \sigma_x = \frac{\text{along-track } PW50}{2.3548}
  \]
  \[
  \sigma_z = \frac{\text{cross-track } PW50}{2.3548}
  \]

2D Lorentzian pulse
- Using \( \text{arctan}(x) \) as a transition response
  \[
  P(x, z) = A \exp\left\{ -\frac{1}{2} \left( \frac{x^2}{\sigma_x^2} + \frac{z^2}{\sigma_z^2} \right) \right\}
  \]
  \[
  P(x, z) = \frac{A}{1 + \left( \frac{x}{W_x / 2} \right)^2 + \left( \frac{z}{W_z / 2} \right)^2}
  \]
  \( W_x \): along-track PW50
  \( W_z \): cross-track PW50
Comparison of the 2D Gaussian and numerical pulses

- **Channel density 2Tbit/inch²**
- **GPR equalizer**
  - Target: \([g_{-1} 1 g_1]\)

2D Gaussian pulse is a good candidate to model the BPM pulse response
Modeling media noise using analytical pulse response

- **Location fluctuations**
  - Randomness at the sampling points of the 2D Gaussian pulse,

- **Size fluctuations**
  - Amplitude, along-track PW50 and cross-track PW50 fluctuations

\[
P(x,z) = (A + \Delta_A) \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x + \Delta_x}{c(W_x + \Delta_{Wx})} \right)^2 + \left( \frac{z + \Delta_z}{c(W_z + \Delta_{Wz})} \right)^2 \right] \right\},
\]

- where \( c = 2.3548 \)

\[
\Delta_{Wx} \approx a_1 \Delta_a \\
\Delta_{Wz} \approx a_2 \Delta_a \\
\Delta_A \approx a_3 \Delta_a
\]
Media noise characteristics

- Extracting size and location statistics, using image processing techniques
- Using an image of BPM nano-mask

Courtesy of Chip Hogg and professor Sara Majetich
- **Location and size fluctuations can be modeled by Gaussian distributions**

Along-track location fluctuation

Along-track size fluctuations
Correlation coefficients

- **Location fluctuations are highly correlated**

  ![Along-track location fluctuations graph]

- **Size fluctuations are less correlated**

  ![Along-track Size fluctuations graph]
Modeling correlated noise

- Passing white noise through a digital filter
- Noise auto-correlation function modeled by a decaying exponential

\[ R_n(k) = \delta(k) \]

\[ R_v(k) = a^{|k|}, \ |a| < 1 \]

\[ v(k) = av(k-1) + (\sqrt{1-a^2})n(k) \]
Decaying exponential auto-correlation function is a good model for correlated media noise.

---

- **Along-track location fluctuations, \( a=0.9 \)**
  - Real fluctuation
  - Simulated corr. noise

- **Along-track Size fluctuations, \( a=-0.15 \)**
  - Real fluctuation
  - Simulated corr. noise
Read channel for BPM with media noise

- **Matched-pulse PR channels**
  - Minimizing noise amplification of the equalizers

- **GPR channels**
  - Noise whitening equalizers

- **NPML channels**
  - Noise predicting detectors
  - Predictor filter is embedded in the VA

\[
\begin{align*}
  v(k) & \rightarrow r(k) & + & y(k) & \rightarrow Viterbi detector \\
  x(-1,k) & \rightarrow Channel H(m,n) & + & r(k) & \rightarrow z(k) & \rightarrow NPML detector \\
  x(0,k) & \rightarrow & & & & \\
  x(1,k) & \rightarrow & & & & \\
  & \text{Media noise} & & & & \\
  & & & \hat{x}(0,k) & & \\
\end{align*}
\]
Error performance of the PBM channels with media noise

- **Channel density 2Tbit/inch²**
- **PR equalizer**
  - $g=[0.1 \ 1 \ 0.1]$  
- **GPR equalizer**
  - $g=[g_{-1} \ 1 \ g_{1}]$  
  - Obtained for $\sigma^2=0.05$ and 6% fluctuations
- **NPML detector**
  - $g=[0.1 \ 1 \ 0.1]$
- **Gaussian fluctuations**
- **Highly correlated normalized location and size fluctuations**
  - $\alpha=0.95$  
  - $\beta=\sqrt{1-\alpha^2}$

solid lines: no media noise, dashed lines: 8% size and location fluctuations and doted lines: 8% normalized correlated size and location fluctuations.
Summary

- 2D pulse response of BPM obtained using the 3D reciprocity principle
- The ITI has the dominant impact on the error performance of the BPM channels and taking ITI into account is important for BPM channels
- The 2D GPR equalizers and the modified VA improve the BER performance of the BPM channels
- Media noise is correlated and can be modeled by a Gaussian random process
- 2D Gaussian pulse can be used for the BPM 2D pulse response
- NPML channels improve the BER performance of the BPM channels in the presence of the correlated media noise