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Fault Detection and Identification with Application to Advanced Vehicle Control Systems

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Abstract

A preliminary design of a health monitoring system for automated vehicles is developed and results of tests in a high-fidelity nonlinear simulation are very encouraging. The approach is to fuse data from dissimilar instruments using modeled dynamic relationships and fault detection and identification filters. The filters are constructed so that the residual process has directional characteristics associated with the presence of a fault, that is, static patterns. Sensor noise, process disturbances, system parameter variations, unmodeled dynamics and nonlinearities all contribute to the blurring of these static patterns. A neural network residual processor is trained to form a threshold detection mechanism that announces a fault when one is present by recognizing fault patterns embedded in the residual. A health monitoring system based on this concept has been constructed for the longitudinal mode and monitors seven sensors and two actuators. Work also continues in refining a detailed nonlinear vehicle simulation which is used as a testbed for evaluating the performance of the health monitoring system.

Keywords. Automated Highway Systems, Automatic Vehicle Monitoring, Fault Detection and Fault Tolerant Control, Neural Networks, Reliability, Sensors, Vehicle Monitoring.

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Executive Summary

A preliminary design of a health monitoring system for automated vehicles is developed and results of tests in a high-fidelity nonlinear simulation are very encouraging. The approach is to fuse data from dissimilar instruments using modeled dynamic relationships and fault detection and identification filters. The filters are constructed so that the residual process has directional characteristics associated with the presence of a fault, that is, static patterns. Sensor noise, process disturbances, system parameter variations, unmodeled dynamics and nonlinearities all contribute to the blurring of these static patterns. A neural network residual processor is trained to form a threshold detection mechanism that announces a fault when one is present by recognizing fault patterns embedded in the residual. A health monitoring system based on this concept has been constructed for the longitudinal mode and monitors seven sensors and two actuators. Work also continues in refining a detailed nonlinear vehicle simulation which is used as a testbed for evaluating the performance of the health monitoring system.

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$\underline{e}_x$, $\underline{e}_y$, $\underline{e}_z$	Unit vectors directed along earth-fixed coordinates	6
$\underline{a}_x$, $\underline{a}_y$, $\underline{a}_z$	Unit vectors $\underline{e}_x$, $\underline{e}_y$, $\underline{e}_z$ rotated about $\underline{e}_z$ through angle ϵ	6
$\underline{b}_x$, $\underline{b}_y$, $\underline{b}_z$	Unit vectors $\underline{a}_x$, $\underline{a}_y$, $\underline{a}_z$ rotated about $\underline{a}_y$ through angle ϕ	6
$\underline{c}_x$, $\underline{c}_y$, $\underline{c}_z$	Unit vectors directed along vehicle-fixed coordinates	6
X, Y, Z	Vehicle position in earth-fixed coordinates	6
x, y, z	Vehicle position in vehicle-fixed coordinates	6
v_x, v_y, v_z	Vehicle mass center velocity in the x, y, z direction	8
ϕ	Vehicle roll rotation about x-axis	6
θ	Vehicle pitch rotation about y-axis	6
ϵ	Vehicle yaw rotation about z-axis	6
ω_x, ω_y, ω_z	Vehicle angular velocity about the x, y, z axes	7
m_a	Intake manifold air mass	10

Symbol	Description	Page
ω_e	Engine speed	10
ω_{fl}, ω_{fr}, ω_{rl}, ω_{rr}	Speed of the front left, front right, rear left and rear right tires	10
$\bar{\omega}_f$	Sum of the front wheel speeds: $\omega_{fl} + \omega_{fr}$	12
$\bar{\omega}_r$	Sum of the rear wheel speeds: $\omega_{rl} + \omega_{rr}$	12
$\tilde{\omega}_f$	Difference of the front wheel speeds: $\omega_{fl} - \omega_{fr}$	12
$\tilde{\omega}_r$	Difference of the rear wheel speeds: $\omega_{rl} - \omega_{rr}$	12
yr, $\dot{y}r$	Lateral deviation and lateral deviation rate from road center	10
ϵ_{des}	Desired vehicle yaw angle	10
α	Throttle angle (rad)	10
τ_b	Brake torque (Nm)	10
β	Tire steering angle	10
α_c	Commanded throttle angle	10
τ_{bc}	Commanded brake torque	10
β_c	Commanded tire steering angle	10
m	Vehicle mass	9
I_x, I_y, I_z	Moments of inertia of the sprung mass about the x, y, z axes	7
m_x, m_y, m_z	Moments tending to rotate the vehicle about the x, y, z axes	7
F_x, F_y, F_z	Force acting on the vehicle along the x, y, z axes	9
y_m	Measured engine manifold airmass (kg)	15
y_ω	Measured engine speed (rad/sec)	15
$y_{\ddot{x}}$	Measured longitudinal acceleration (m/sec^2)	15
$y_{\ddot{z}}$	Measured lateral acceleration (m/sec^2)	15
y_q	Measured pitch rate (rad/sec)	15
$y_{\omega fs}$	Measured front symmetric wheel speed (rad/sec)	15
$y_{\omega rs}$	Measured rear symmetric wheel speed (rad/sec)	15
A, B, C, D	Linearized dynamics system matrices	10
x, u, y	System state, control and output	9
$\tilde{x}$, $\tilde{u}$, $\tilde{y}$	Small perturbation of the system state, control and output	10
n, p, m	Dimensions of system state, control and output	63

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CHAPTER 1
Introduction

A PROPOSED TRANSPORTATION SYSTEM with vehicles traveling at high speed, in close formation and under automatic control demands a high degree of system reliability. This requires a health monitoring and maintenance system capable of detecting a fault as it occurs, identifying the faulty component and determining a course of action that restores safe operation of the system. This report is concerned with vehicle fault detection and identification and describes a vehicle health monitoring system approach based on analytic redundancy.

Analytic redundancy methods for fault detection and identification use a modeled dynamic relationship between system inputs and measured system outputs to form a residual process. Nominally, the residual process is nonzero only when a fault has occurred and is zero at other times. For an observable system, this simple definition is met by the innovations process of any stable linear observer. A detection filter is a linear observer with the gain constructed so that when a fault occurs, the residual responds in a known and fixed direction. Thus, when a nonzero residual is detected, a fault can be announced and identified.

In applications it is unrealistic to expect that a residual process would be nonzero only when a fault has occurred. Sensor noise, process disturbances, system parameter variations, unmodeled dynamics and nonlinearities all contribute to the magnitude of a residual. There are many methods to reduce the impact of these effects on the residual but none reduce their effect to zero. This means that some threshold detection mechanism must be built.

A simple threshold detection mechanism announces a fault when the size of a residual exceeds some prescribed value. This prescribed value could be determined from empirical studies which balance a rate of false alarm against a rate of miss alarm. A more complicated residual processor might take into account the thresholds of all other residuals as well. Reasoning that if the probability of simultaneous failures is very small, no fault is announced when more than one residual exceeds a threshold. It is more likely that the nonzero residuals are caused by noise or nonlinearities or some cause other than multiple faults. A neural network residual processor is described in this report.

A complication arises when there are many possible faults because a fault detection filter can only be designed to detect a limited number of faults. This is related to the order of the vehicle dynamics. When more faults need to be identified, several fault detection filters have to be used with each filter designed to detect and identify some but not all possible faults. The vehicle fault detection system described in this report has four fault detection filters. This raises two difficult design issues. First, some and probably all faults will not be included in the design of one or more fault detection filters. When such a fault occurs, the residual of all filters will respond, even the residuals of the filters that do not have the fault included in their design. If a fault is not included in a fault detection filter design, the directional characteristics of the residual will be undefined and the fault cannot be properly identified. The challenge is to build a mechanism that recognizes when a fault detection filter is responding to a fault for which it has not been designed and then to exclude the residual of all such filters from the fault identification process. If it can be assumed that only one fault occurs at a time, then the residual processor can exclude the residual of any fault detection filters that point to two or more faults.

A second design issue is how the faults should be grouped and identification delegated among the fault detection filters. In a fault detection system that consists of a bank of fault detection filters and a residual processor such as a neural network, fault isolation is done through the combined effort of both system elements. The fault detection filter is a carefully tuned device that uses known dynamic relationships to isolate a fault. The neural network residual processor combines the residuals from several filters and resolves any ambiguity. It is suggested that identifying a fault among a group of dynamically similar faults requires the precision of and is best delegated to the fault detection filters. Furthermore, it is suggested that the reliability of the neural network training would be improved if the fault groups associated with each of the fault detection filters are dynamically dissimilar.

This paper is organized as follows. Section 2 describes the car models. Low-dimensional linear models are used for fault detection filter design. A high fidelity nonlinear model is used for evaluation and to obtain the linear models used for design. Section 3 describes the faults to be identified by the fault detection system. Section 4 describes the design of the fault detection filters. This includes how the faults are grouped for each fault detection filter design, how the fault detection filter eigenstructure placement is done and how reduced-order fault detection filters are formed. Section 5 presents an evaluation of the performance of the fault detection filters in a nonlinear simulation. Section 6 describes a fault detection filter residual processing system. Here a neural network is used to process residuals from all fault detection filters to detect and identify which if any fault has occurred. Finally, appendix A provides a very quick theoretical review of the Beard-Jones detection filter problem.

CHAPTER 2

Vehicle Model and Simulation Development

IN THIS SECTION, vehicle models are developed for the design and evaluation of fault detection filters. Three models are considered: (1) a six degree of freedom (DOF) nonlinear vehicle model, (2) a computer model obtained from the Berkeley PATH research team and derived in (Peng 1992), and (3) a linearized model used for detection filter design. The derivation of equations for the six DOF nonlinear model is independent of that used for the Berkeley simulation. The independent derivation was performed to be sure that we understood all the assumptions, definitions and issues which underlie the Berkeley simulation model. This exercise proved worthwhile in that we did uncover some differences between our model and the Berkeley model, and we have contacted them to clarify these differences. Resolution of these issues is pending.

All models can be used to describe a four-wheel-steering, four-wheel-drive vehicle. This report, however, only considers rear-wheel-drive vehicles. The road gradient and superelevation are assumed to be zero.

2.1 Nonlinear Model

Equations that describe the six degree of freedom motion of a vehicle are developed here. First, the coordinate systems are described. Then, the rotational equations of motion are developed followed by the translational equations of motion.

2.1.1 Coordinate Systems

The motion of the vehicle will be referred to an Earth-fixed reference frame E which is described by a right handed orthogonal axis system (X, Y, Z) fixed on the Earth. The unit vectors along the X, Y, Z-axes are $\underline{e}_x$, $\underline{e}_y$ and $\underline{e}_z$, respectively. A second reference frame C fixed in the sprung mass of the vehicle is described by a right handed orthogonal axis system (x, y, z) fixed along the central principal axes of the vehicle. The origin is at the vehicle mass center where x points in the forward direction, y points to the left, and z points upward. We assume that x and y are horizontal when the vehicle is at rest. Unit vectors $\underline{c}_x$, $\underline{c}_y$, and $\underline{c}_z$ are directed along x, y, and z, respectively. The orientation of C with respect to E is given by a sequence of three angular rotations. First, there is a yaw rotation ϵ about the aligned Z and z-axes. Let $\underline{a}_x$, $\underline{a}_y$ and $\underline{a}_z$ be unit vectors along the displaced x, y, z-axes. Then there is a roll rotation ϕ about the displaced y-axis $\underline{a}_y$. Let $\underline{b}_x$, $\underline{b}_y$ and $\underline{b}_z$ describe the directions of x, y, z-axes after this roll rotation. Last, there is a pitch rotation θ about the displaced x-axis $\underline{b}_x$. The unit vectors $\underline{c}_x$, $\underline{c}_y$ and $\underline{c}_z$ describe the final orientation of C. The relationships among the various unit vectors are

$$\begin{bmatrix} \underline{a}_x \\ \underline{a}_y \\ \underline{a}_z \end{bmatrix} = \begin{bmatrix} \cos\epsilon & \sin\epsilon & 0 \\ -\sin\epsilon & \cos\epsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \underline{e}_x \\ \underline{e}_y \\ \underline{e}_z \end{bmatrix} \tag{2.1a}$$

$$\begin{bmatrix} \underline{b}_x \\ \underline{b}_y \\ \underline{b}_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \underline{a}_x \\ \underline{a}_y \\ \underline{a}_z \end{bmatrix} \tag{2.1b}$$

$$\begin{bmatrix} \underline{c}_x \\ \underline{c}_y \\ \underline{c}_z \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \underline{b}_x \\ \underline{b}_y \\ \underline{b}_z \end{bmatrix} \tag{2.1c}$$

2.1.2 Rotational Equations of Motion

The angular velocity of the car relative to the Earth is

$$\underline{\omega} = \dot{\epsilon}\underline{e}_z + \dot{\phi}\underline{a}_x + \dot{\theta}\underline{b}_y$$

Using the coordinate system transformations (2.1) the angular velocity is also given by

$$\begin{aligned}\underline{\omega} &= (\dot{\phi}\cos\theta - \dot{\epsilon}\cos\phi\sin\theta)\underline{c}_x + (\dot{\theta} + \dot{\epsilon}\sin\phi)\underline{c}_y + (\dot{\phi}\sin\theta + \dot{\epsilon}\cos\phi\cos\theta)\underline{c}_z \\ &= \omega_x\underline{c}_x + \omega_y\underline{c}_y + \omega_z\underline{c}_z\end{aligned}$$

Thus, the angular velocities of the car expressed in vehicle fixed axes, which are measured numbers, become

$$\begin{aligned}\begin{bmatrix}\omega_x \\ \omega_y \\ \omega_z\end{bmatrix} &= \begin{bmatrix}0 \\ \dot{\theta} \\ 0\end{bmatrix} + \begin{bmatrix}\cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta\end{bmatrix}\begin{bmatrix}\dot{\phi} \\ 0 \\ 0\end{bmatrix} + \begin{bmatrix}\cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta\end{bmatrix}\begin{bmatrix}1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}0 \\ 0 \\ \dot{\epsilon}\end{bmatrix} \\ &= \begin{bmatrix}\cos\theta & 0 & -\cos\phi\sin\theta \\ 0 & 1 & \sin\phi \\ \sin\theta & 0 & \cos\phi\cos\theta\end{bmatrix}\begin{bmatrix}\dot{\phi} \\ \dot{\theta} \\ \dot{\epsilon}\end{bmatrix}\end{aligned}$$

If this expression is solved for the angular rates $\dot{\phi}$, $\dot{\theta}$ and $\dot{\epsilon}$, one obtains the rotational kinematic equations of motion:

$$\begin{bmatrix}\dot{\phi} \\ \dot{\theta} \\ \dot{\epsilon}\end{bmatrix} = \begin{bmatrix}\cos\theta & 0 & \sin\theta \\ \sin\theta\tan\phi & 1 & -\cos\theta\tan\phi \\ -\sin\theta\cos^{-1}\phi & 0 & \cos\theta\cos^{-1}\phi\end{bmatrix}\begin{bmatrix}\omega_x \\ \omega_y \\ \omega_z\end{bmatrix} \tag{2.2}$$

The rotational dynamic equations governing the angular motions of the vehicle are obtained from the Euler equations:

$$\dot{\omega}_x = \frac{m_x}{I_x} + \omega_y\omega_z\frac{I_y - I_z}{I_x} \tag{2.3a}$$

$$\dot{\omega}_y = \frac{m_y}{I_y} + \omega_z\omega_x\frac{I_z - I_x}{I_y} \tag{2.3b}$$

$$\dot{\omega}_z = \frac{m_z}{I_z} + \omega_x\omega_y\frac{I_x - I_y}{I_z} \tag{2.3c}$$

The applied moments m_x, m_y and m_z come from aerodynamic forces and interaction forces between the tires and pavement. Expressions for these moments are discussed in a later section.

2.1.3 Translational Equations of Motion

The position vector from an Earth-fixed point to the center of mass of the car may be described in terms of the earth-fixed unit vectors $\underline{e}_x$, $\underline{e}_y$ and $\underline{e}_z$ or the vehicle-fixed unit vectors $\underline{c}_x$, $\underline{c}_y$ and $\underline{c}_z$. Thus,

$$\begin{aligned} \underline{p} &= X\underline{e}_x + Y\underline{e}_y + Z\underline{e}_z \\ &= x\underline{c}_x + y\underline{c}_y + z\underline{c}_z \end{aligned}$$

The velocity of the center of mass of the car then becomes,

$$\begin{aligned} \underline{v} &= \dot{X}\underline{e}_x + \dot{Y}\underline{e}_y + \dot{Z}\underline{e}_z \\ &= (\dot{x}\underline{c}_x + \dot{y}\underline{c}_y + \dot{z}\underline{c}_z) + (\omega_x\underline{c}_x + \omega_y\underline{c}_y + \omega_z\underline{c}_z) \times (x\underline{c}_x + y\underline{c}_y + z\underline{c}_z) \\ &= (\dot{x} - y\omega_z + z\omega_y)\underline{c}_x + (\dot{y} - zw_x + xw_z)\underline{c}_y + (\dot{z} - xw_y + yw_x)\underline{c}_z \\ &= v_x\underline{c}_x + v_y\underline{c}_y + v_z\underline{c}_z \end{aligned}$$

Solving for $\dot{x}$, $\dot{y}$, and $\dot{z}$ in terms of x, y, z and ω_x, ω_y, ω_z one obtains:

$$\dot{x} = v_x + y\omega_z - z\omega_y \tag{2.4a}$$

$$\dot{y} = v_y + z\omega_x - x\omega_z \tag{2.4b}$$

$$\dot{z} = v_z + x\omega_y - y\omega_x \tag{2.4c}$$

The acceleration of the center of mass of the car in both earth-fixed and vehicle-fixed axes is

$$\begin{aligned} \underline{a} &= \ddot{X}\underline{e}_x + \ddot{Y}\underline{e}_y + \ddot{Z}\underline{e}_z \\ &= (\dot{v}_x\underline{c}_x + \dot{v}_y\underline{c}_y + \dot{v}_z\underline{c}_z) + (\omega_x\underline{c}_x + \omega_y\underline{c}_y + \omega_z\underline{c}_z) \times (v_x\underline{c}_x + v_y\underline{c}_y + v_z\underline{c}_z) \\ &= (\dot{v}_x + \omega_y v_z - \omega_z v_y)\underline{c}_x + (\dot{v}_y + \omega_z v_x - \omega_x v_z)\underline{c}_y + (\dot{v}_z + \omega_x v_y - \omega_y v_x)\underline{c}_z \end{aligned}$$

Expressing the forces acting on the vehicle $\underline{F}$ as

$$\underline{F} = F_x\underline{c}_x + F_y\underline{c}_y + F_z\underline{c}_z$$

and using Newtons 2nd Law, $\underline{F} = m\underline{a}$, leads to the following dynamic translational equations of motion.

$$\dot{v}_x = \omega_z v_y - \omega_y v_z + \frac{F_x}{m} \quad (2.5a)$$

$$\dot{v}_y = \omega_x v_z - \omega_z v_x + \frac{F_y}{m} \quad (2.5b)$$

$$\dot{v}_z = \omega_y v_x - \omega_x v_y + \frac{F_z}{m} \quad (2.5c)$$

As before, the applied forces F_x, F_y and F_z come from gravity, aerodynamic forces and interaction forces between the tires and pavement. Equations (2.2), (2.3), (2.4) and (2.5) describe the motion of the car provided the applied forces and moments are known. These expressions would be required if our objective were to construct a complete analytical model or a computer simulation. At the present time, we have not taken this next step, and have instead used the Berkeley simulation model for subsequent work. More work on force and moment models may be attempted at a later date.

2.2 Linear Model

The nonlinear model in the previous section was generated primarily to better understand and verify the Berkeley model. In this section, we generate a linearized model directly from the Berkeley model. This will be done numerically rather than analytically. The procedure is as follows.

First, a computer run is made in which the car goes straight at a constant speed of 25 m/s ($\simeq$ 56 mph) to obtain steady state values for each state. The nonlinear model is then linearized about this nominal operating point using the central difference method. The use of an analytical approach, that is taking partial derivatives, is impractical because this model is too complicated.

The nonlinear model has the form :

$$\dot{x} = f(x, u) \quad (2.6a)$$

$$y = Cx + D\dot{x} \quad (2.6b)$$

Suppose the nominal operating point is $(x_0,\ u_0)$ where $f(x_0, u_0) = 0$. Take perturbations $\tilde{x}$, $\tilde{u}$ about the nominal point, that is, let

$$\begin{aligned} x &= x_0 + \tilde{x} \\ u &= u_0 + \tilde{u} \end{aligned}$$

Also approximate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial u}$ as

$$\begin{aligned} \frac{\partial f}{\partial x} &\approx \frac{\Delta f}{\Delta x} = \left.\frac{f(x+\tilde{x}, u) - f(x-\tilde{x}, u)}{2\tilde{x}}\right|_{x=x_0, u=u_0} \\ \frac{\partial f}{\partial u} &\approx \frac{\Delta f}{\Delta u} = \left.\frac{f(x, u+\tilde{u}) - f(x, u-\tilde{u})}{2\tilde{u}}\right|_{x=x_0, u=u_0} \end{aligned}$$

Equation (2.6a) may now be approximated as

$$\dot{x}_0 + \dot{\tilde{x}} = f(x_0, u_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0, u=u_0} \tilde{x} + \left.\frac{\partial f}{\partial u}\right|_{x=x_0, u=u_0} \tilde{u} + \cdots$$

Dropping out higher order terms and using the approximations given above for the partial derivatives, one obtains

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + B\tilde{u} \\ \tilde{y} &= C\tilde{x} + D\dot{\tilde{x}} \\ &= (C + DA)\tilde{x} + DB\tilde{u} \end{aligned}$$

where

$$\begin{aligned} \tilde{x} &= [m_a \quad w_e \quad v_x \quad x \quad v_y \quad y \quad v_z \quad z \quad \phi \quad \dot{\phi} \quad \theta \quad \dot{\theta} \quad \epsilon \quad \dot{\epsilon} \\ &\qquad w_{fl} \quad w_{fr} \quad w_{rl} \quad w_{rr} \quad X \quad Y \quad yr \quad \dot{y}r \quad \epsilon_{des} \quad \alpha \quad \tau_b \quad \beta]^T \\ \tilde{y} &= [m_a \quad w_e \quad \dot{v_x} \quad \dot{v_y} \quad \dot{v_z} \quad \dot{\phi} \quad \dot{\theta} \quad \dot{\epsilon} \quad w_{fl} \quad w_{fr} \quad w_{rl} \quad w_{rr}]^T \\ \tilde{u} &= [\alpha_c \quad \tau_{bc} \quad \beta_c]^T \\ A &= \left.\left[\frac{\Delta f}{\Delta x}\right]\right|_{x=x_0, u=u_0} \\ B &= \left.\left[\frac{\Delta f}{\Delta u}\right]\right|_{x=x_0, u=u_0} \end{aligned}$$

and where A is a 26×26 real matrix and B is a 26×3 real matrix. Symbols in $\tilde{x}$, $\tilde{y}$ and $\tilde{u}$ are defined in the list of symbols.

Several sizes of perturbations must be taken to find one that gives the best approximation of the partial derivatives. If the perturbation is too small, there is a truncation error in computing the difference $f(x+\tilde{x},u)-f(x-\tilde{x},u)$. If the perturbation is too large, a roundoff error occurs in computing $f(x+\tilde{x},u)$ and $f(x-\tilde{x},u)$; also nonlinearities become important. According to our experience, $\frac{\tilde{x}}{x}$ and $\frac{\tilde{u}}{u} \approx 10^{-4}$ is a good rule for selecting the size of the perturbation when using the central differences method. The resulting linear model can then be tested in a simulation to see how well it describes the nonlinear model over the speed range of 23 m/s to 27 m/s. When this was done, we found that the errors were under 10%.

The linear model generated as described above was intended for use in designing the fault detection filters. This model has 26 states. Before using the model for filter design, we decided to try to simplify the model to the extent possible without significant loss of accuracy. The model simplification was accomplished in two steps, the first of which resulted in no loss of accuracy.

By inspection of the equations, it was found possible to rearrange the sequence of states such that the linearized equations assume the following partitioned form:

$$\begin{aligned}
\dot{\tilde{x}} &= \begin{bmatrix} \dot{\tilde{x}}_3 \\ \dot{\tilde{x}}_4 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ A_2 & A_3 \end{bmatrix} \begin{bmatrix} \tilde{x}_3 \\ \tilde{x}_4 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tilde{u} \\
\tilde{y} &= \begin{bmatrix} C_1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_3 \\ \tilde{x}_4 \end{bmatrix}
\end{aligned}$$

where

$$\begin{aligned}
\tilde{x}_3 &= [m_a \quad w_e \quad v_x \quad v_z \quad z \quad \theta \quad \dot{\theta} \quad w_{fl} \quad w_{fr} \quad w_{rl} \quad w_{rr} \quad \alpha \quad \tau_b \quad v_y \quad \phi \quad \dot{\phi} \quad \dot{\epsilon} \quad \beta]^T \\
\tilde{x}_4 &= [x \quad X \quad y \quad \epsilon \quad Y \quad yr \quad \dot{yr} \quad \epsilon_{des}]^T
\end{aligned}$$

In this form, we see that both $\tilde{x}_3$ and $\tilde{y}$ are independent of $\tilde{x}_4$. Thus $\tilde{x}_4$ can be deleted from the model without affecting the transfer function from $\tilde{u}$ to $\tilde{y}$. Based on this observation,

$\tilde{x}_4$ is removed from the model, which then becomes

$$\dot{\tilde{x}}_3 = A_1\tilde{x}_3 + B_1\tilde{u}$$
$$\tilde{y} = C_1\tilde{x}_3$$

where A_1 is an 18×18 matrix, B_1 is an 18×3 matrix and C_1 is a 12×18 matrix.

If the four wheel speed state variables w_{fl}, w_{fr}, w_{rl}, w_{rr} are replaced by four new state variables $\bar{w}_f$, $\bar{w}_r$, $\tilde{w}_f$, $\tilde{w}_r$ defined as:

$$\begin{aligned}
\bar{w}_f &= w_{fl} + w_{fr} \\
\bar{w}_r &= w_{rl} + w_{rr} \\
\tilde{w}_f &= w_{fl} - w_{fr} \\
\tilde{w}_r &= w_{fl} - w_{rr}
\end{aligned}$$

then the model exactly decouples into two subsystems. These are the longitudinal and lateral dynamics, that is,

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix}$$

where

$$\begin{aligned}
\tilde{x}_1 &= [m_a \quad w_e \quad \dot{x} \quad \dot{z} \quad z \quad \theta \quad \dot{\theta} \quad \bar{w}_f \quad \bar{w}_r \quad \alpha \quad \tau_b]^T \\
\tilde{x}_2 &= [\tilde{w}_f \quad \tilde{w}_r \quad \dot{y} \quad \phi \quad \dot{\phi} \quad \dot{\epsilon} \quad \beta]^T \\
\tilde{u}_1 &= [\alpha_c \quad \tau_{bc}]^T \\
\tilde{u}_2 &= \beta_c
\end{aligned}$$

Therefore, the longitudinal model becomes:

$$\dot{\tilde{x}}_1 = A_1\tilde{x}_1 + B_1\tilde{u}_1$$

and the lateral model becomes:

$$\dot{\tilde{x}}_2 = A_2\tilde{x}_2 + B_2\tilde{u}_2$$

2.3 Reduced-Order Model

Previous manipulation has not involved any approximation. For further model simplification, some approximation must occur. First, the actuator dynamics are neglected because they are relatively fast and also they are in series with the other dynamics. At this point, we are more concerned about simplifying the highly coupled dynamics and will return to consider the actuator dynamics later. Hence, the actuator dynamic states are deleted from the model. So the states for the longitudinal model are $\tilde{x}_1$ and for the lateral model are $\tilde{x}_2$.

$$\begin{aligned}
\tilde{x}_1 &= [m_a \quad w_e \quad \dot{x} \quad \dot{z} \quad z \quad \theta \quad \dot{\theta} \quad \bar{w}_f \quad \bar{w}_r]^T \\
\tilde{x}_2 &= [\tilde{w}_f \quad \tilde{w}_r \quad \dot{y} \quad \phi \quad \dot{\phi} \quad \dot{\epsilon}]^T
\end{aligned}$$

After the linear models are derived, the first thing one should do is check the eigenvalues. Then, three approaches are presented to get reduced-order models. The first approach one may consider to reduce the model is to set the derivatives of certain fast states to zero. Using this philosophy, states with large negative eigenvalues can be dropped. However, a correction should be made using the deleted states to remove the steady state error. Consider a linear system modeled as :

$$\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{aligned}$$

Suppose this model is written in a partitioned form.

$$\begin{aligned}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \\
y &= \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Du
\end{aligned}$$

where x_2 contains the 'fast states'. Set the derivative of x_2 to zero and solve the resulting equations for x_2 as a function of x_1 and u. This leads to

$$x_2 = -A_{22}^{-1} A_{21} x_1 - A_{22}^{-1} B_2 u$$

Substitute this result into the expressions for $\dot{x}_1$ and y to obtain the reduced order model:

$$\begin{aligned} \dot{x}_1 &= \left[A_{11} - A_{12}A_{22}^{-1}A_{21}\right]x_1 + \left[B_1 - A_{12}A_{22}^{-1}B_2\right]u \\ y &= \left[C_1 - C_2A_{22}^{-1}A_{21}\right]x_1 + \left[D - C_2A_{22}^{-1}B_2\right]u \end{aligned}$$

this model preserves the static input-output relationships.

A second approach is to use balanced realization before implementing the method just described. Balancing refers to an algorithm which finds a realization that has equal and diagonal controllability and observability grammians. The diagonal of the joint grammian $g(i)$ can be used to reduce the order of the model. Since $g(i)$ reflects the combined controllability and observability of individual states, it is reasonable to remove those states from the model that have a small $g(i)$. Elimination of these states retains the most important input-output characteristics of the original system. After balanced realization has been done, the first method is used to obtain a reduced order model.

A third approach is a little different from the second one. After balanced realization has been done, a truncation is used instead of the first method. For example, if the full-order model is

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}u \\ y &= \begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Du \end{aligned}$$

then, the reduced-order model is

$$\dot{x}_1 = A_{11}x_1 + B_1u$$

This is the approach originally proposed by Moore (Moore 1981). Using this approach it is possible to calculate a bound on the error introduced by deleting states.

2.3.1 Longitudinal Model

At the end of the previous section, section 2.2 which deals with the linear model, a decoupled longitudinal model is developed. Its eigenvalues are -212.11, -166.04, -31.46, -26.27, -0.04, $-2.3 \pm 6.65i$ and $-1.53 \pm 5.69i$. Observe that two of these eigenvalues are significantly larger than the rest. From this we conclude that at least two state variables can be dropped. In method one, by looking at the eigenvectors corresponding to the large eigenvalues, we find that the two fast mode states are the sum of the front wheel speeds $\bar{w}_f$ and the sum of the rear wheel speeds $\bar{w}_r$. So, these two states are dropped to get a seventh-order model. In methods 2 and 3, two states with smallest grammians are dropped. These methods combine the states in such a way that they lose their physical significance, so we can not explicitly identify the states that are being deleted. Here are some results using the three methods for model reduction described earlier.

Order Reduction Method	Eigenvalues				
Method 1.	-33.05	-25.85	-0.0484	$-2.26 \pm 6.71i$	$-1.57 \pm 5.67i$
Method 2.	-31.58	-26.23	-0.0449	$-2.32 \pm 6.65i$	$-1.54 \pm 5.69i$
Method 3.	-32.08	-26.05	-0.0449	$-2.25 \pm 6.04i$	$-1.78 \pm 5.82i$

Table 2.1: Eigenvalues for the Longitudinal Dynamics Using Three Model Reduction Methods.

The eigenvalues of each reduced-order model are given in table 2.1. The second method is the best because the eigenvalues are closer to the true eigenvalues. This method uses the balanced realization and drops unimportant states by letting their derivatives be zero. One can also perform another test to see which method is best. That method is based on frequency response. Bode diagrams for each input to each output are plotted to see their responses to frequencies from 10^{-1} to 10^2 rad/s. The reason for choosing this frequency range is that it roughly corresponds to that of the control inputs to a car. The Bode diagrams also show that the frequency response of a model obtained with the second method is closest to that of the full-order model.

The seven-state model involves the longitudinal dynamics only. No lateral dynamics and no actuator dynamics are included. The states have no physical significance because they are derived from the balanced realization as stated in section 2.3. The measured outputs are

y_m Engine manifold air mass (kg).

y_ω Engine speed (rad/sec).

$y_{\ddot{x}}$ longitudinal acceleration (m/sec^2).

$y_{\ddot{z}}$ heave acceleration (m/sec^2).

y_q Pitch rate (rad/sec).

$y_{y_{\mathrm{fs}}}$ Forward symmetric wheel speed (rad/sec).

$y_{y_{\mathrm{rs}}}$ Rear symmetric wheel speed (rad/sec).

and the control inputs are

α Throttle angle (deg).

β Brake torque (Nm).

The system matrices are given by

$$A = \begin{bmatrix} -0.0514 & -0.2203 & 0.2670 & -0.0102 & 0.0145 & 0.0084 & -0.0074 \\ -0.2984 & -7.7825 & 18.5490 & -0.9359 & 0.1522 & 0.2418 & 0.0463 \\ -0.3247 & -19.1948 & -49.4179 & -3.2002 & -4.9689 & -2.3224 & -0.0652 \\ 0.0440 & 2.2616 & 14.8614 & -2.1396 & 6.4462 & -0.2283 & 0.0394 \\ 0.0216 & 1.0707 & 8.3103 & -7.1707 & -0.6642 & -0.2614 & 0.9221 \\ 0.0116 & 0.5739 & 3.6890 & -1.0911 & -0.6573 & -1.0090 & 5.9643 \\ 0.0150 & 0.7490 & 4.6068 & -1.4672 & -1.0353 & -6.5849 & -2.5807 \end{bmatrix} \quad (2.7a)$$

$$B = \begin{bmatrix} 0.9509 & -0.0341 \\ 2.8861 & -0.0107 \\ 2.9813 & 0.0082 \\ -0.4068 & 0.0116 \\ -0.2001 & 0.0185 \\ -0.1069 & 0.0040 \\ -0.1389 & 0.0109 \end{bmatrix} \quad (2.7b)$$

$$C = \begin{bmatrix} 0.0080 & 0.4605 & 0.3771 & 0.1010 & 0.0541 & 0.0340 & -0.0129 \\ 0.7411 & 2.8381 & -2.9156 & 0.1484 & -0.0552 & -0.0503 & -0.0048 \\ 0.0027 & 0.1650 & -0.2533 & 0.0732 & -0.0157 & 0.0094 & -0.0005 \\ 0.0000 & -0.0006 & -0.0007 & -0.0207 & -0.0500 & -0.0446 & 0.0694 \\ -0.0000 & -0.0024 & 0.0049 & 0.0108 & 0.0207 & -0.0026 & 0.0009 \\ 0.4222 & -0.1429 & 0.0360 & 0.2242 & -0.1731 & -0.0138 & 0.1051 \\ 0.4217 & 0.1241 & -0.4239 & -0.2778 & -0.0356 & -0.0740 & 0.0579 \end{bmatrix} \quad (2.7c)$$

$$D = \begin{bmatrix} 0.0000 & -0.0000 \\ -0.0005 & 0.0004 \\ 0.0010 & -0.0020 \\ -0.0000 & 0.0001 \\ 0.0000 & -0.0000 \\ -0.0001 & -0.0008 \\ 0.0013 & -0.0010 \end{bmatrix} \tag{2.7d}$$

2.3.2 Lateral Model

At the end of previous section, section 2.2 dealing with the Linear Model, we also have a decoupled lateral model. Its eigenvalues are -205.91 , -133.45 , $-3.29 \pm 5.96i$, $-8.39 \pm 1.43i$. This model also contains two high frequency modes, so we again conclude that two state variables can be dropped. By looking at the corresponding eigenvectors, we learn that the two fast mode states are the difference of the front wheel speeds $\tilde{w}_f$ and the difference of the rear wheel speeds $\tilde{w}_r$. So following the procedure of method 1, these two states are dropped to get a fourth-order model. In methods 2 and 3, two states with the smallest grammians are dropped. Here are some results by using these three methods for model reduction.

Order Reduction Method	Eigenvalues	
Method 1.	$-2.95 \pm 5.78i$	$-8.80 \pm 2.02i$
Method 2.	$-3.29 \pm 6.02i$	$-8.31 \pm 2.04i$
Method 3.	$-4.18 \pm 5.63i$	$-9.49 \pm 6.97i$

Table 2.2: Eigenvalues for the Lateral Dynamics Using Three Model Reduction Methods.

The eigenvalues of each lateral reduced-order model are given in table 2.2. Once again, the second method produces the best result. That is where we use balanced realization and drop states by letting their derivatives be zero. Bode diagrams for each input to each output also were plotted to see their responses to frequency from 10^{-1} to 10^2 rad/s. Looking at the Bode diagrams confirmed that the second method is best.

CHAPTER 3

Fault Selection

ANALYTIC REDUNDANCY is an approach to health monitoring that compares dissimilar instruments using a detailed model of the system dynamics. Therefore, to detect a fault in a given sensor, there must be a dynamic relationship between the sensor and other sensors or actuators. That is, the information provided by a monitored sensor must, in some form, also be provided by other sensors. Analytic redundancy also can be used to effectively monitor the health of system actuators and even the dynamic behavior of the system itself. But, as with sensors, if some part of the vehicle is to be monitored for proper operation, then that part has to produce some observable dynamic effect.

In automated vehicles, these requirements preclude monitoring, for example, nonredundant sensors such as obstacle detection sensors or lane position sensors. The information provided by a radar or infrared sensor designed to detect objects in the vehicle's path has no dynamic correlation with other sensors on the vehicle. A sensor that detects the vehicle's position in a lane is the only sensor that can provide this information. Similarly, the health of the actuator that controls the position of the driver's window is easily monitored by the driver. But, unless specialized sensors are installed, no other part of the car is affected by the operation of this actuator and there is no analytic redundancy.

Before describing how faults are modeled, it is necessary to describe how a fault detection filter works. Most of the detail is left to appendix A. For a thorough background, several references are available, a few of which are (Douglas 1993), (White and Speyer 1987) and (Massoumnia 1986). Consider a linear time-invariant system with q failure modes and no disturbances or sensor noise

$$\dot{x} = Ax + Bu + \sum_{i=1}^{q} F_i m_i \tag{3.1a}$$

$$y = Cx + Du \tag{3.1b}$$

The system variables x, u, y and the m_i belong to real vector spaces and the system maps A, B, C, D and the F_i are of compatible dimensions. Assume that the input u and the output y both are known. The F_i are the failure signatures. They are known and fixed and model the directional characteristics of the faults. The m_i are the failure modes and model the unknown time-varying amplitude of faults. The m_i do not have to be scalar values.

A fault detection filter is a linear observer that, like any other linear observer, forms a residual process sensitive to unknown inputs. Consider a full-order observer with dynamics and residual

$$\dot{\hat{x}} = (A + LC)\hat{x} + Bu - Ly \tag{3.2a}$$

$$r = C\hat{x} + Du - y \tag{3.2b}$$

Form the state estimation error $e = \hat{x} - x$ and the dynamics and residual are

$$\dot{e} = (A + LC)e - \sum_{i=1}^{q} F_i m_i$$

$$r = Ce$$

In steady-state, the residual is driven by the faults when they are present. If the system is (C, A) observable, and the observer dynamics are stable, then in steady-state and in the absence of disturbances and modeling errors, the residual r is nonzero only if a fault has occurred, that is, if some m_i is nonzero. Furthermore, when a fault does occur, the residual is nonzero except in certain theoretically relevant but physically unrealistic situations. This means that any stable observer can detect the presence of a fault. Simply monitor the residual and when it is nonzero a fault has occurred.

In addition to detecting a fault, a fault detection filter provides information to determine which fault has occurred. An observer such as (3.2) becomes a fault detection filter when the observer gain L is chosen so that the residual has certain directional properties that immediately identify the fault. The gain is chosen to partition the residual space where each partition is uniquely associated with one of the design fault directions F_i. A fault is identified by projecting the residual onto each of the residual subspaces and then determining which projections are nonzero.

Before the fault detection filter design (3.2) can begin, a system model with faults has to be found with the form (3.1). Seven sensors and two actuators are associated with the linearized longitudinal vehicle dynamics described in section 2.3.1. The sensors measure the engine manifold airflow and engine speed, the vehicle forward and heave accelerations, the pitch rate and the averaged speed of the forward wheels and the averaged speed of the rear wheels. The actuators control the engine throttle and the brake torque.

3.1 Sensor Fault Models

Sensor faults can be modeled as an additive term in the measurement equation

$$y = Cx + E_i \mu_i \tag{3.3}$$

where E_i is a column vector of zeros except for a one in the i^{th} position and where μ_i is an arbitrary time-varying real scalar. Now, for the fault detection filter design, faults are expressed as additive terms to the system dynamics as in (3.1). Sensor faults may be expressed in this way, as explained in (Douglas 1993), where the fault E_i in (3.3) is equivalent to a two-dimensional fault F_i

$$\dot{x} = Ax + F_i m_i \qquad \text{with } F_i = \left[F_i^1, F_i^2\right]$$

and where the directions F_i^1 and F_i^2 are given by

$$E_i = CF_i^1 \tag{3.4a}$$

$$F_i^2 = AF_i^1 \tag{3.4b}$$

Using the linearized longitudinal dynamics of section 2.3.1, an engine manifold airflow measurement is given by the first element of the system output (2.7). Therefore, any fault in the engine manifold airflow sensor can be modeled as an additive term in the measurement equation as in (3.3)

$$y = Cx + Du + E_{y_m}\mu_{y_m}$$

where

$$E_{y_m} = \begin{bmatrix} 1, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}^T$$

and where μ_{y_m} is an arbitrary time-varying real scalar. An equivalent two-dimensional fault F_{y_m} found by solving (3.4) is

$$F_{y_m} = \begin{bmatrix} 0.1145 & 0.0232 \\ 0.9439 & 8.0234 \\ 0.3365 & -94.5225 \\ -2.8676 & 32.1570 \\ 3.5965 & 30.7195 \\ 21.5405 & 73.7392 \\ 15.5799 & -179.3062 \end{bmatrix}$$

Other vehicle sensor fault directions are found in the same way.

3.2 Actuator Fault Models

A fault in a control input is modeled as an additive term in the system dynamics. In the case of a fault appearing at the input of an actuator, that is the actuator command, the fault has the same direction as the associated column of the system B matrix. A fault appearing at the output of an actuator, the actuator position, has the same direction as the associated column of the system A matrix.

For the vehicle longitudinal dynamics developed in section 2.3, the actuator dynamics are relatively fast and, in an approximation, are removed from the system model. Thus, the control inputs are applied directly to the car dynamics through a column of the B matrix and to the sensor outputs through a column of the feedforward D matrix. So, for this system a control input fault has three directions. One fault direction is the B matrix column. The other two directions come from treating the D matrix column as if it were a sensor fault which is explained above.

The engine throttle control is the first element of the system input so one direction of an engine throttle control fault is the first column of the B matrix from (2.7)

$$F_\alpha^1 = \begin{bmatrix} 0.9509 \\ 2.8861 \\ 2.9813 \\ -0.4068 \\ -0.2001 \\ -0.1069 \\ -0.1389 \end{bmatrix} \tag{3.5}$$

Because the linear model (2.7) has a control feedforward term, a throttle control fault also shows up directly in the system outputs in a direction given by the first column of the D matrix, that is,

$$E_\alpha = \begin{bmatrix} 1.1894e-07 \\ -4.6361e-04 \\ 1.0324e-03 \\ -3.6799e-05 \\ 2.0846e-06 \\ -9.9046e-05 \\ 1.3114e-03 \end{bmatrix}$$

As with a sensor fault, this direction E_α leads to a two-dimensional dynamics fault direction given by solving (3.4). Together with (3.5), an engine throttle fault is modeled as a three-dimensional dynamics fault

$$F_\alpha = \begin{bmatrix} 0.9509 & -0.0023 & -0.0002 \\ 2.8861 & -0.0021 & -0.0462 \\ 2.9813 & -0.0074 & -0.0911 \\ -0.4068 & -0.0220 & 0.0911 \\ -0.2001 & 0.0300 & 0.1458 \\ -0.1069 & 0.1758 & 0.5580 \\ -0.1389 & 0.1273 & -1.5206 \end{bmatrix}$$

A fault model for the brake torque is developed in the same way and is given by

$$F_\beta = \begin{bmatrix} -3.4075 & 0.0423 & 0.0004 \\ -1.0743 & 0.0370 & 0.0831 \\ 0.8178 & 0.1236 & 0.1275 \\ 1.1552 & 0.3200 & -0.1217 \\ 1.8522 & -0.4697 & -0.2077 \\ 0.3980 & -2.8685 & -0.9034 \\ 1.0870 & -2.0732 & 2.4854 \end{bmatrix}$$