18-643 Lecture 11: Memory Bound Designs

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Housekeeping

- Your goal today: see examples of customizing memory paths to algorithms, and vice versa
- Notices
  - Handout #5: lab 2, due noon, 10/9
  - Project status report due each Friday
- Readings (see lecture schedule online)
  - Kung, "Memory requirements for balanced computer architectures," ISCA 1986.
  - Williams, et al., "Roofline: an insightful . . .," 2008

Topic 1: Arithmetic Intensity

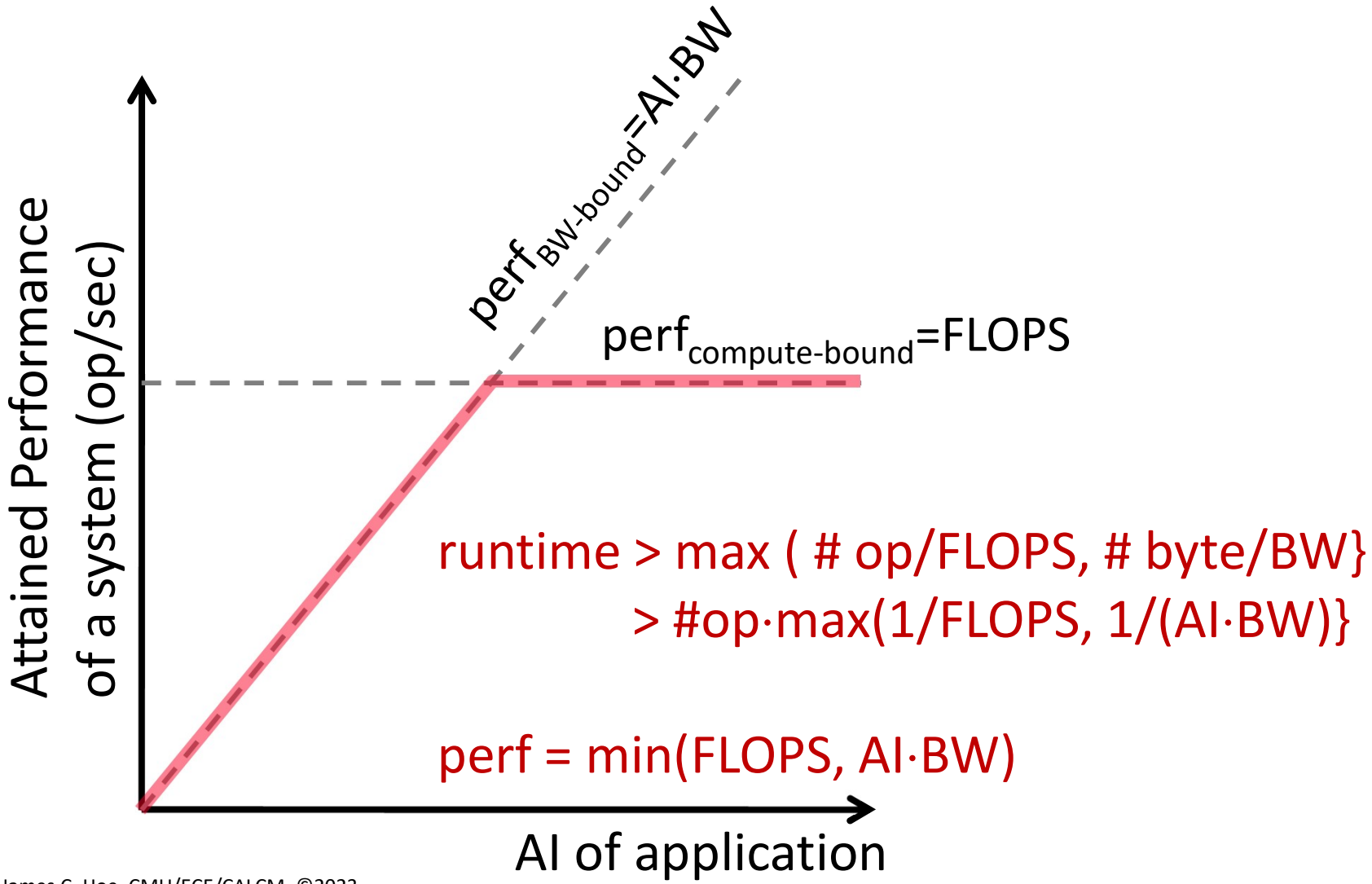
Arithmetic Intensity

- An algorithm has a cost in terms of operation count
  - $\text{runtime}_{\text{compute-bound}}$ = # operations / FLOPS
- An algorithm also has a cost in terms of number of bytes communicated (ld/st or send/receive)
  - $\text{runtime}_{\text{BW-bound}}$ = # bytes / BW
- Which one dominates depends on
  - ratio of FLOPS and BW of platform
  - ratio of ops and bytes of algorithm
- Average Arithmetic Intensity (AI)
  - how many ops performed per byte accessed
  - # operations / # bytes

Last Time

Roofline Performance Model

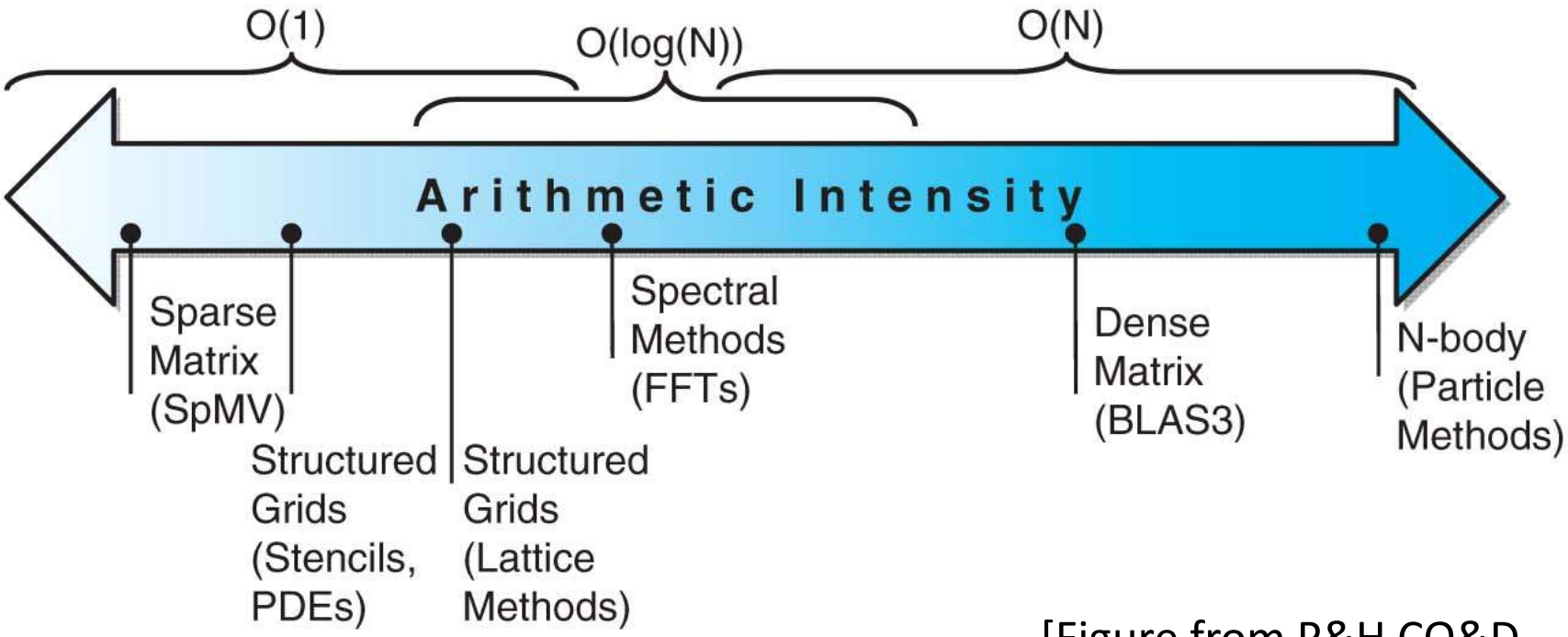
[Williams&Patterson, 2006]



AI and Algorithms

harder to speed up
& harder to scale up

easier



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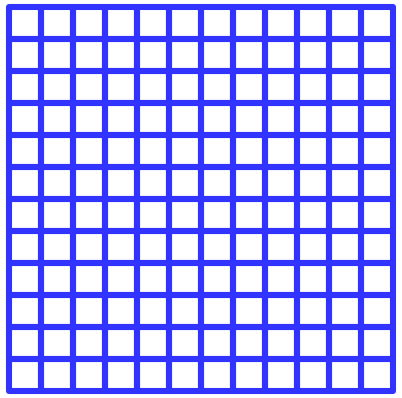
Simple AI Example: MMM

```
for(i=0; i<N; i++)
  for(j=0; j<N; j++)
    for(k=0; k<N; k++)
      C[i][j]+=A[i][k]*B[k][j];
```

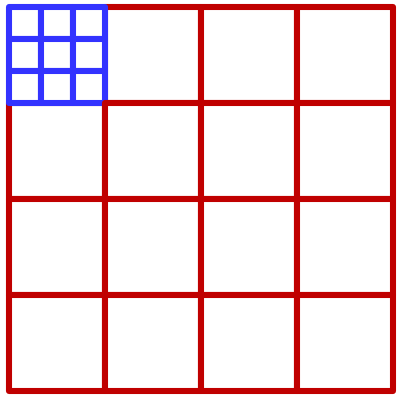


- N^2 data-parallel dot-product's
  - operation count: N^3 float-mult and N^3 float-add
- External memory access (assume 4-byte floats)
  - assume **N** is large *s.t.* 1 row/col too large for on-chip
  - $2N^3$ 4-byte reads (of **A** and **B**) from DRAM
  - . . . N^2 4-byte writes (of **C**) to DRAM . . .
- Arithmetic Intensity $\approx 2N^3/(4 \cdot 2N^3)=1/4$

GTX1080: 8 TFLOPS vs 320GByte/sec

Less Simple AI Example: MMM

```
for(i0=0; i0<N; i0+=Nb)
  for(j0=0; j0<N; j0+=Nb)
    for(k0=0; k0<N; k0+=Nb) {
      for(i=i0;i<i0+Nb;i++)
        for(j=j0;j<j0+Nb;j++)
          for(k=k0;k<k0+Nb;k++)
            C[i][j]+=A[i][k]*B[k][j];
    }
```



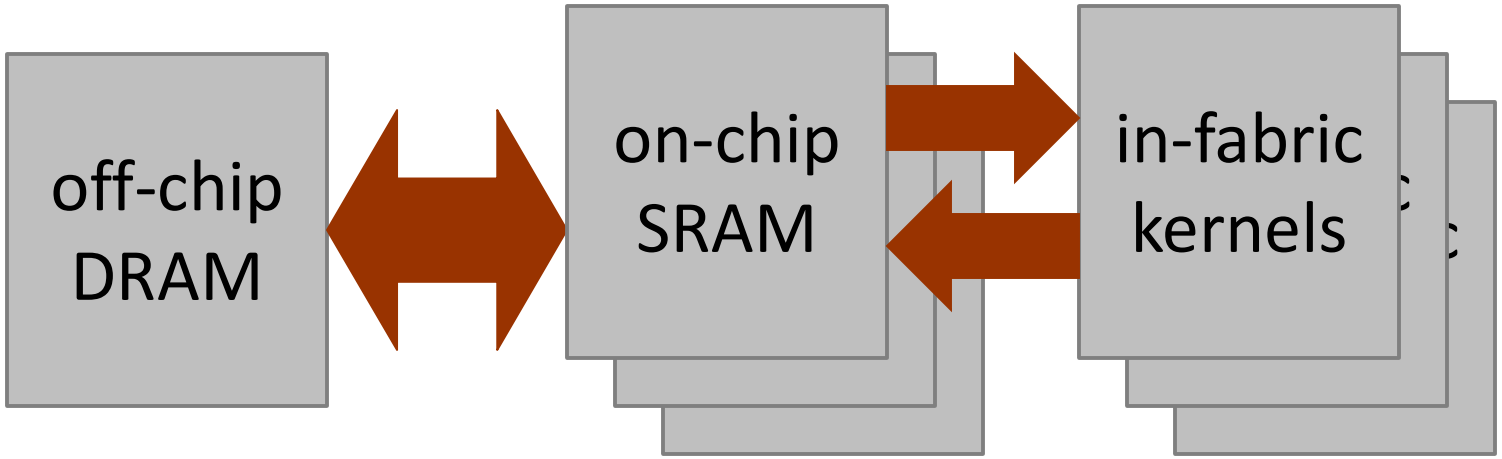
- Imagine a 'N/N_b'x''N/N_b' MATRIX of N_bxN_b matrices
  - inner-triple is straightforward matrix-matrix mult
  - outer-triple is MATRIX-MATRIX mult
- To improve AI, hold N_bxN_b sub-matrices on-chip for data-reuse

AI of blocked MMM Kernel (N_bxN_b)

```
for(i=i0;i<i0+Nb;i++)
  for(j=j0;j<j0+Nb;j++) {
    t=C[i][j];
    for(k=k0;k<k0+Nb;k++)
      t+=A[i][k]*B[k][j];
    C[i][j]=t;
  }
```

- Operation count: N_b^3 float-mult and N_b^3 float-add
- When **A**, **B** fit in scratchpad (2xN_b^2x4 bytes)
  - 2xN_b^3 4-byte on-chip reads (**A**, **B**) (fast)
  - 2xN_b^2 4-byte off-chip DRAM read **A**, **B** (slow)
  - 2xN_b^2 4-byte off-chip DRAM read/write of **C** (slow)
- Arithmetic Intensity = $2N_b^3/(4 \cdot 4N_b^2) = N_b/8$

The Performance Balancing Act



1. Kernels' **op/sec** requires some **byte/sec** — a function of **algorithm** and **kernel size**
2. On-chip SRAM "filters" kernel **byte/sec** down to DRAM **byte/sec** — a function of SRAM **capacity**
3. DRAM system offers some aggregate **byte/sec** — a function of **access pattern** (**algorithm**)

Some Hints on Lab 3

- Lab 3 kernel's op/sec just need to be fast enough to match memory-bound (op/byte × byte/sec)
- Lab 3 emphasis on improving memory-bound
  - size tiles and order outer loops for data reuse *(don't forget the batch loop!)*

    Mindful of buffer sizes and degree reuse
  - use memory resources efficiently *(fit bigger tiles)*
  - layout data in DRAM for sequential read *(don't forget to widen the read path)*
- Use DFX to tune the 2 layers differently

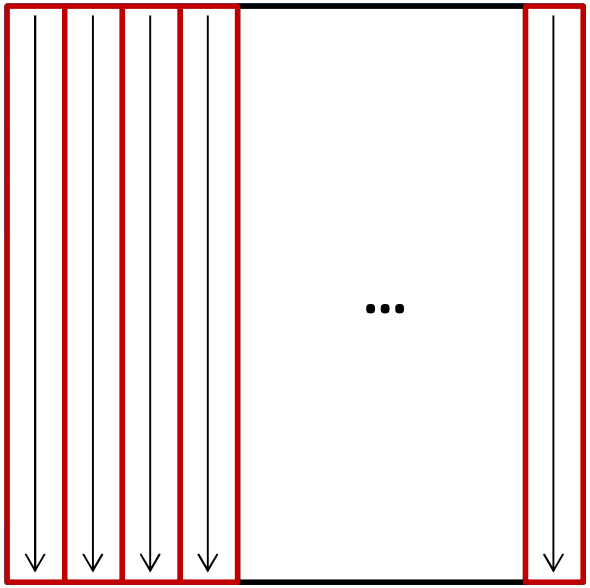
Topic 2:
Data Layout and Access Pattern

Data Layout and Access Pattern: 2D-FFT

- Row-column algorithm:

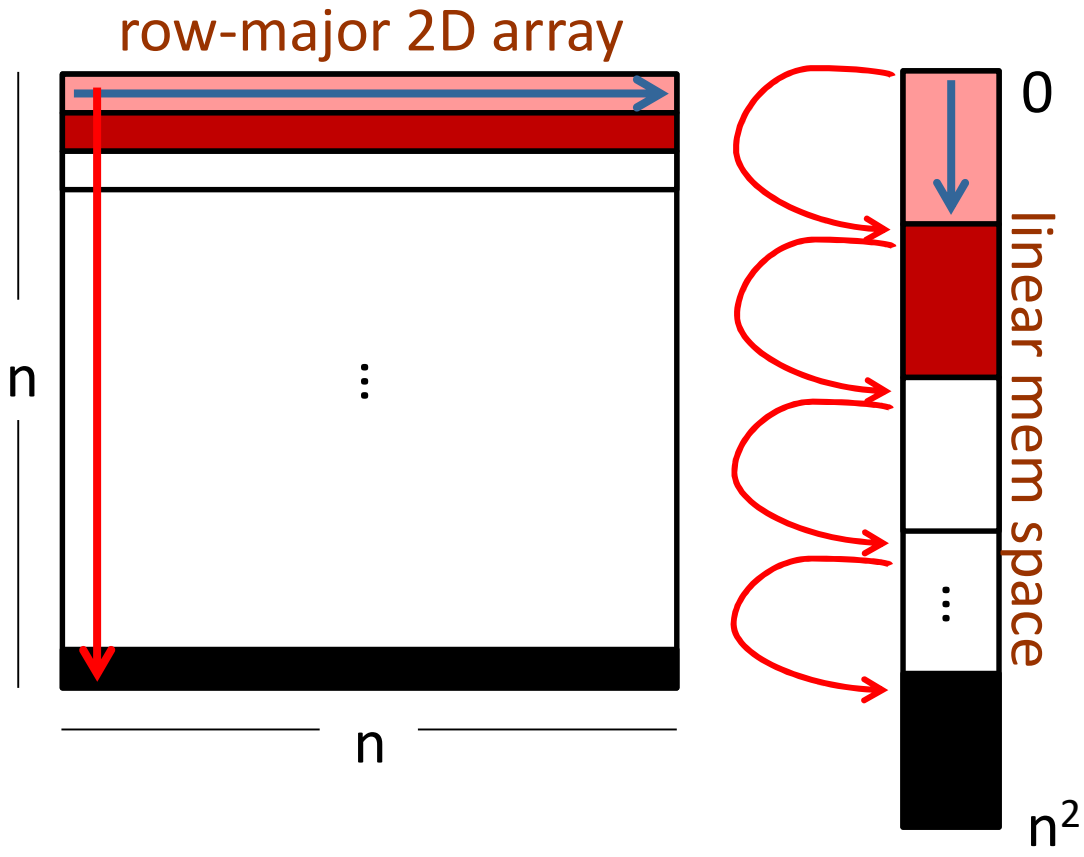
$$\text{2D-DFT}_{n\times n} = \underbrace{(\text{DFT}_n \otimes \text{I}_n)}_{\text{Column Stage}}\underbrace{(\text{I}_n \otimes \text{DFT}_n)}_{\text{Row Stage}}$$

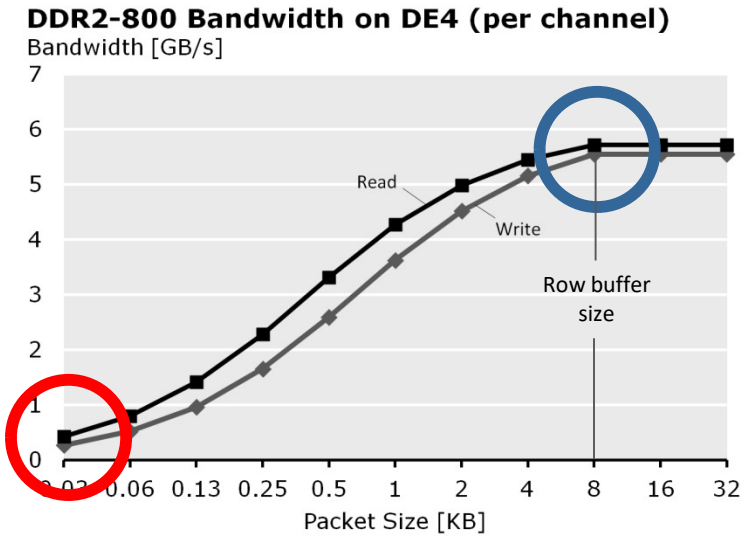
Dataset:
(Logical abstraction of the 2D dataset)



Inefficient DRAM Access Patterns

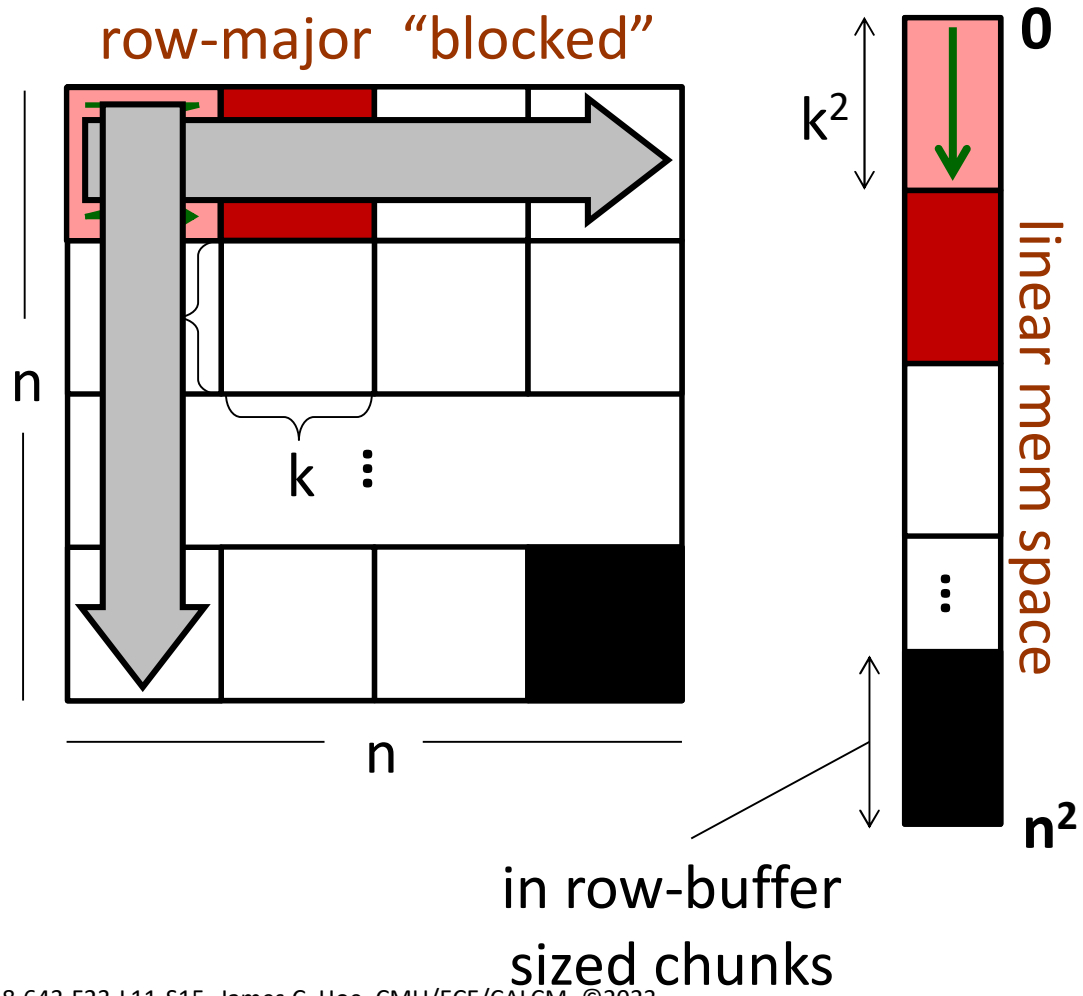
- Row-wise traversal -> Sequential accesses
- Column-wise traversal -> Large strided accesses

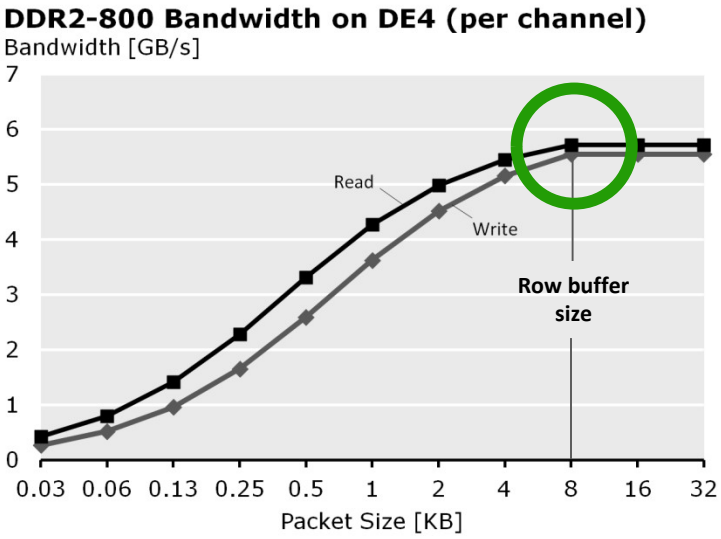




Gather-Scatter

Tiled Layout and Access Patterns





What if you only have (k/2)·n on-chip buffer?

Design Generator w/ Tensor Formalism

column stage row stage

$$\text{2D-DFT}_{n\times n} = (\overbrace{\text{DFT}_n \otimes \text{I}_n}^{\text{column stage}})(\overbrace{\text{I}_n \otimes \text{DFT}_n}^{\text{row stage}})$$

row-column algorithm

$$= \prod_{i=0}^{1} \left(\text{L}_n^{n^2} (\text{I}_n \otimes \text{DFT}_n)\, \text{I}_{n^2} \right)$$

symmetric algorithm

symmetric algorithm with tiling

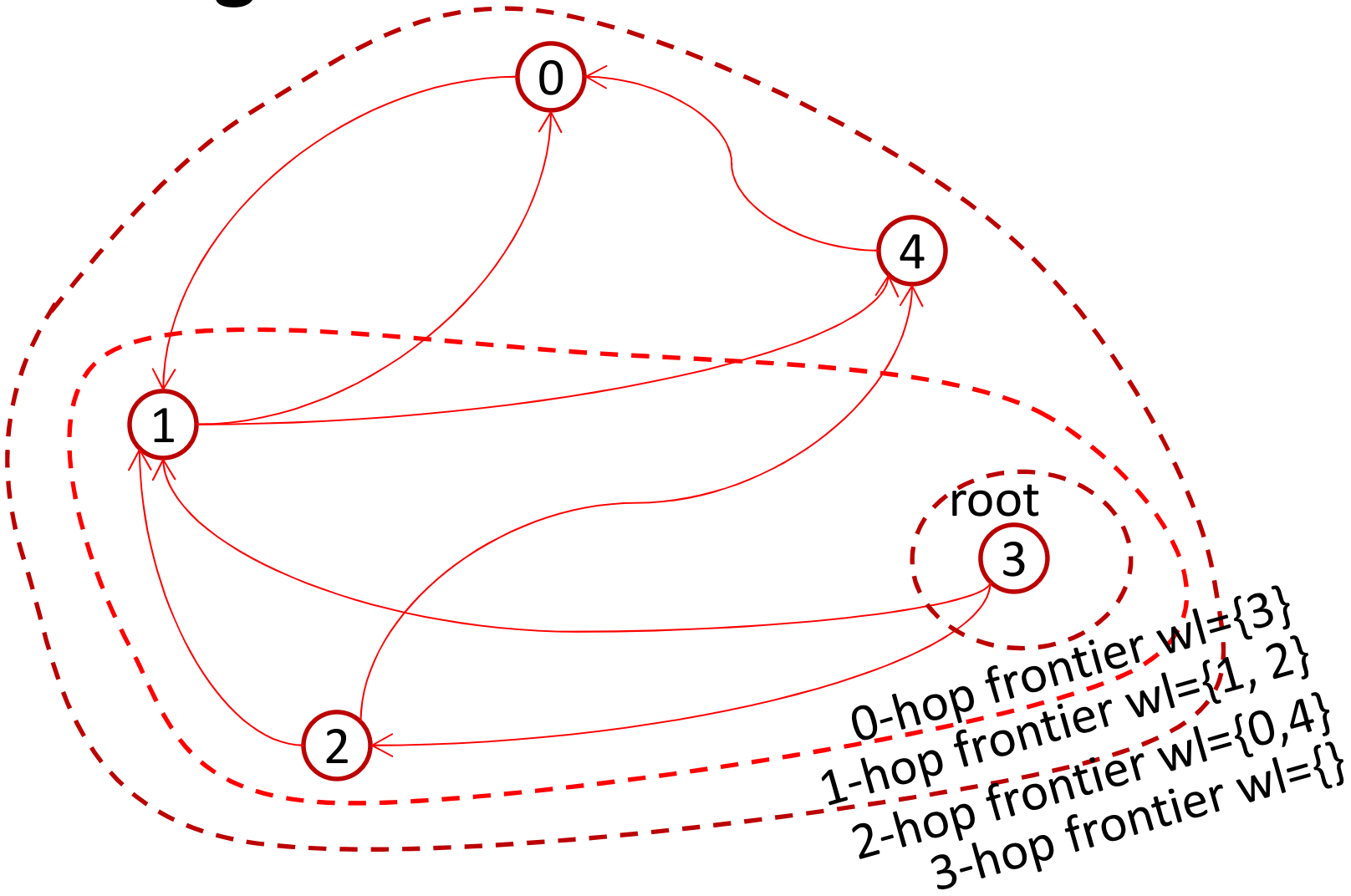
$$= \prod_{i=0}^{1} ((\underbrace{\text{L}_n^{n^2/k} \otimes \text{I}_k}_{\text{write tiles column-wise}})(\underbrace{\text{I}_{n/k} \otimes \text{L}_n^{nk}}_{\text{transpose and re-tile on-chip}})(\underbrace{\text{I}_{n/k} \otimes (\text{I}_k \otimes \text{DFT}_n)}_{\text{FFT processing}})(\underbrace{\text{I}_{n/k} \otimes (\text{L}_k^n \otimes \text{I}_k)}_{\text{linearize on-chip}})(\underbrace{\text{I}_{n/k} \otimes (\text{L}_{n/k}^n \otimes \text{I}_k)}_{\text{read tiles row-wise}}))$$

not on midterm

[Akin, et al., FCCM 2012]

Topic 3: Irregular

Irregular: Breadth First Search



Large graph has more than millions of nodes with may be handful edges per node

Breadth-First Search (Pseudo Code)

```
foreach (node n in graph) n.dist=∞;

worklist = {root}; root.dist=0;

foreach (node n in worklist) {
  foreach (neighbor of n) {
    if (n.dist + 1 < neighbor.dist) {
        neighbor.dist = n.dist + 1;
        add neighbor to worklist;
    }
  }
}
```

Has Parallelism?
Yes, not perfect,
input-dependent

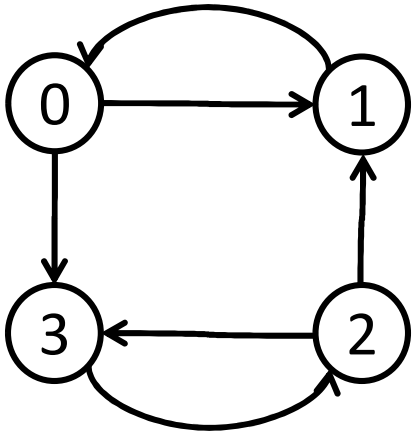
(see http://iss.ices.utexas.edu/?p=projects/galois/benchmarks/bread_first_search)

Real Code with CSR Memory Accesses

```
while(wl.mHowmany) { // worklist not empty
   // repeat for each node on frontier
   int curr=wl.mList[wl.mDeq];                    // S0
   int myDist=graph->mPerNode[curr].dist;         // S1
   int numEdges=graph->mPerNode[curr].fanout;     // S1
   int scan=graph->mPerNode[curr].edges;          // S1
   { ... dequeue from worklist ...}
   while (numEdges--) {
     // repeat for each neighbor
     int dest=graph->mPerEdge[scan].dest;         // S2
     int destDist=graph->mPerNode[dest].dist;     // S3
     if ((myDist+1)<destDist) {                   // S4
       graph->mPerNode[dest].dist=myDist+1;       // S4
       { ...enqueue dest to worklist...}          // S5
     }
     scan++;
   }
 }
```

Compressed Sparse Row (CSR) Adjacency Matrix

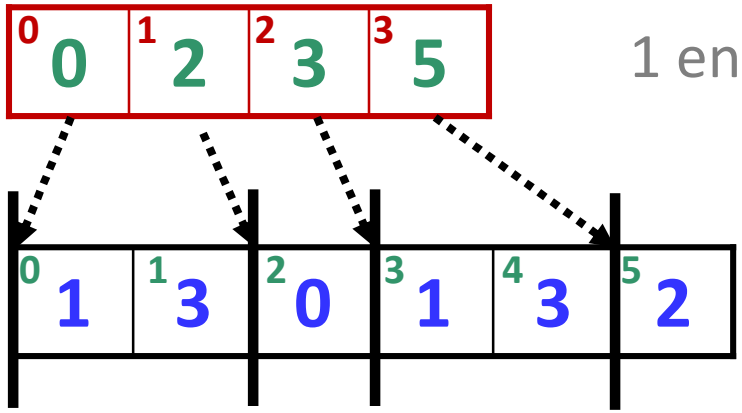
src↓ dest→	0	1	2	3
0	0	**1**	0	**1**
1	**1**	0	0	0
2	0	**1**	0	**1**
3	0	0	**1**	0



dense

sparse

array indexed by row/src idx
(holds offset into element array)



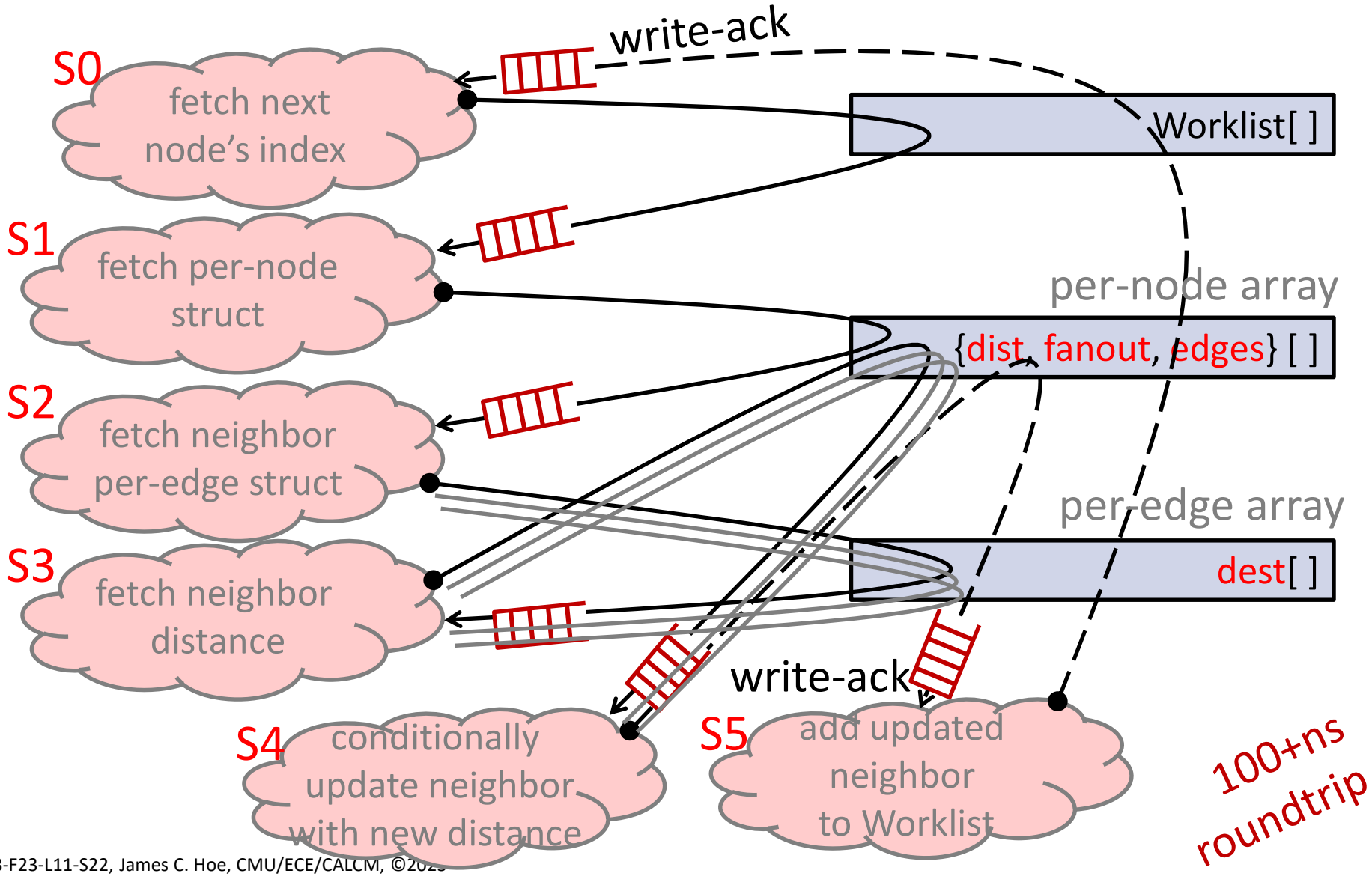
1 entry per-node

1 entry per-edge

array of all non-0 elements
in row-order
(holds col/dest index)

Large graph has millions or more nodes each with may be handful edges per node

Elastic HW Processing Pipeline

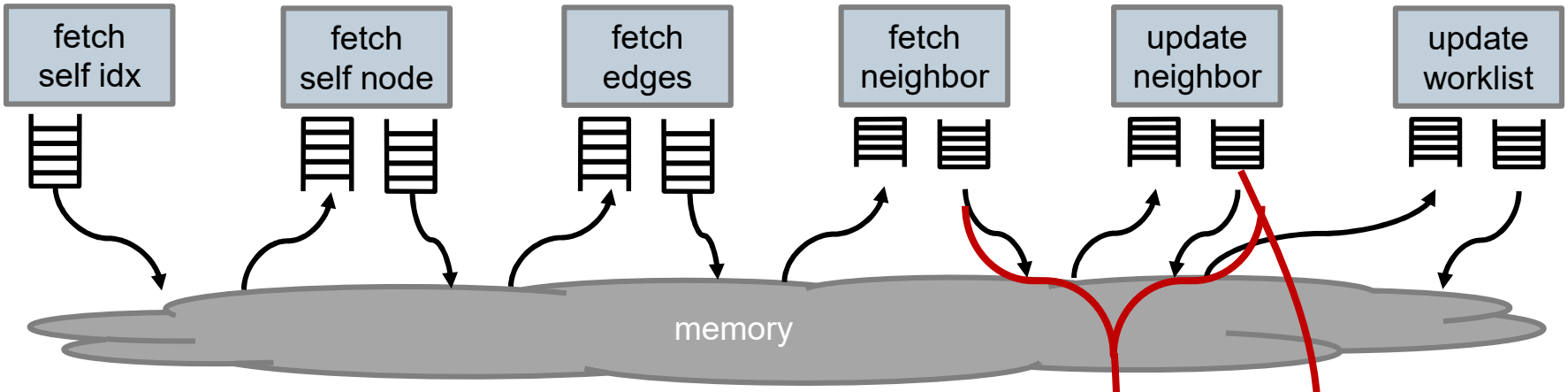


BFS Irregular Access Pattern

- Irregular and graph dependent
  - S0 read worklist: spatial locality, non-temporal
  - S1 read node array (self): no locality
  - S2 read edge array: some spatial locality, non-temporal
  - S3 read node array (neighbor): no locality
  - S4 write node array (neighbor): temporal with S3
  - S5 write worklist: spatial locality, non-temporal
- S3 most problematic of all
  - S1 and S3 lack locality but S3 repeated per neighbor
  - same number of S2 and S3 but S2 has spatial locality
  - BTW, S3 and S4 could have RAW hazard
  - BTW, all read/write granularity is multi-word

Can caching help?

How can "Caching" Help?

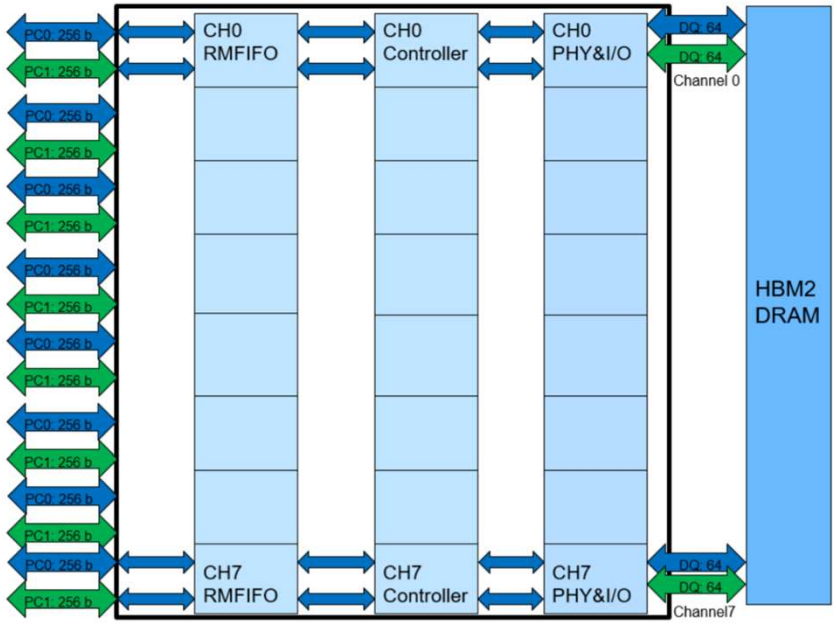


- Custom cache for only neighbor distance in node array (read in S3 written in S4)
- RAW hazard interlock when nodes in pipeline have same neighbor (stall S3 until conflict-free)
- Coalesces neighbor updates (collect partial writes to multi-word DRAM block)

Organized and operated unlike an ABC cache!!

How can HBM BW help?

- HBM gives you 512GB/sec
  - data partitioned 32 ways
  - 100s nsec latency
  - 32 byte per fetch
- Partition graph data into 32 banks s.t. maximizing non-conflict concurrent accesses across channel
- Per channel, prefetch many 10s of outstanding read requests to cover latency *(Little's Law)*
- Re-index nodes in graph s.t. maximizing spatial locality of each 32B fetch



Complexity of Sparse Algorithms

- Graph processing expressible using linear algebra primitives: SpMV, SpMSpV, SpMM, . . .

  Simplicity of form belies performance difficulties

- Basic challenges in large data set, low arithmetic intensity, and irregular access pattern
- graph-dependent behavior requires multiple implementations of same primitive depending on:
  - size and sparsity
  - structured?
  - compressed format: CSR, CSC, COO, . . .

  Each combination a different optimal design

Parting Thoughts

- When scaling data size and performance, memory design quickly become the PROBLEM
  - capacity, bandwidth, latency
- FPGAs specialization is an asset
  - balance memory throughput and compute throughput
  - have data to the right place at the right time
  - alter algorithm to memory constraints
- Designing “memorypath” as important as designing “datapath”