18-447 Lecture 23:
Illusiveness of Parallel Performance

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Housekeeping

- Your goal today
  - peel back simplifying assumptions to understand parallel performance (or the lack of)
- Notices
  - HW5, due Friday 4/26 midnight
  - get going on Lab 4, just 8 days left
  - **Final Exam,** May 3 Friday, 1-3:30pm
- Readings
  - P&H Ch 6
  - LogP: a practical model of parallel computation, Culler, et al. (advanced optional)

Format of Final Exam

- Comprehensive in coverage, HW, labs, assigned readings (from textbooks and papers)
- Types of questions
  - freebies: remember the materials
  - **>> probing: understand the materials <<**
  - applied: apply the materials in original interpretation
- ****150 minutes, 150 points****
  - point values calibrated to time needed
  - closed-book, three 8½x11-in^2 hand-written cribsheets
  - no electronics
  - use pencil or black/blue ink only
  - **No writing on the back of exam packet**

Continuing from Last Lecture

- Parallel Thread Code (Last Lecture)

```
void *sumParallel(void *_id) {
  long id=(long) _id;
  psum[id]=0;
  for(long i=0;i<(ARRAY_SIZE/p);i++)
     psum[id]+=A[id*(ARRAY_SIZE/p) + i];
}
```

- Assumed "+" takes 1 unit-time; **everything else** free

T_1=10,000

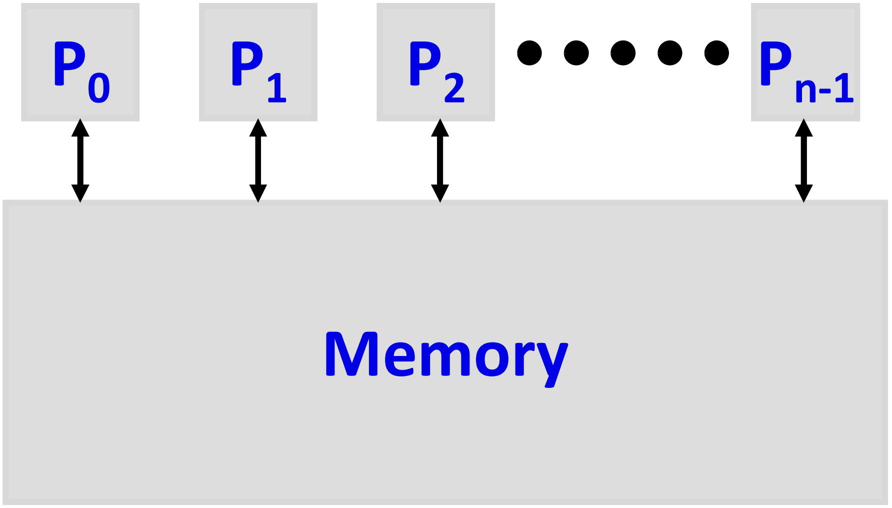
$T_\infty = \lceil \log_2 10{,}000 \rceil = 14$

$P_{average}$=714

What would you predict is the real speedup on the 16-core ECE server?

Need for more detailed analysis

- What cost were left out in **"everything else"**?
  - explicit cost: need to charge for all operations (branches, LW/SW, pointer calculations . . . .)
  - implicit cost: **communication and synchronization**
- PRAM model (Parallel Random Access Machine) capture cost/rate of parallel processing but assumes
  - **zero latency** and **infinite bandwidth** to share data between processors
  - **zero processor overhead** to send and receive



Useful when analyzing algorithms but not enough for performance tuning

Performance Scaling

Parallelism Defined

- T_1 (work measured in time):
  - time to do work with 1 PE
- T_∞ (critical path):
  - time to do work with infinite PEs
  - T_∞ bounded by dataflow dependence
- Average parallelism:

$$P_{avg} = T_1 / T_\infty$$

- For a system with **p** PEs

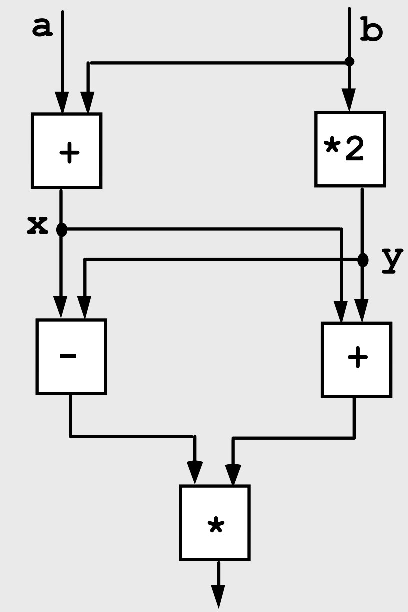
$$T_p \geq \max\{ T_1/p, T_\infty \}$$

- When $P_{avg} >> p$

$$T_p \approx T_1/p, \text{ aka "linear speedup"}$$

Recall

x = a + b;
y = b * 2
z =(x-y) * (x+y)

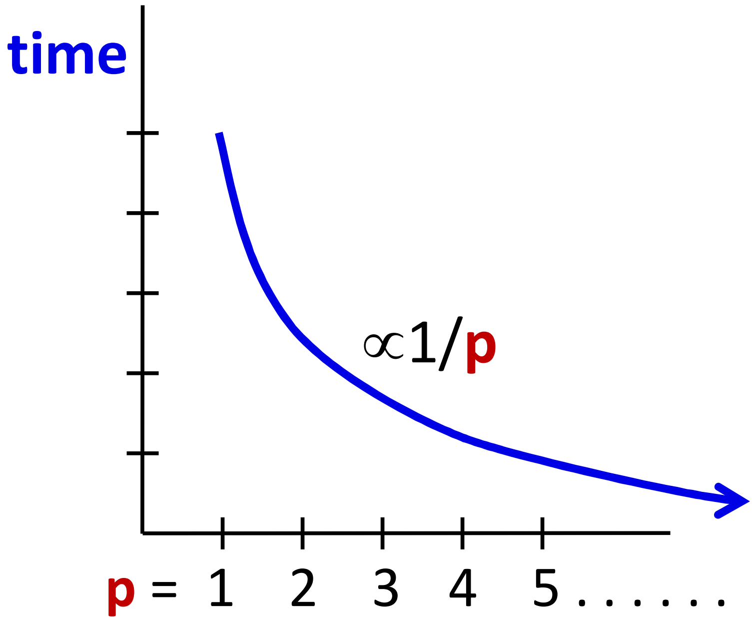


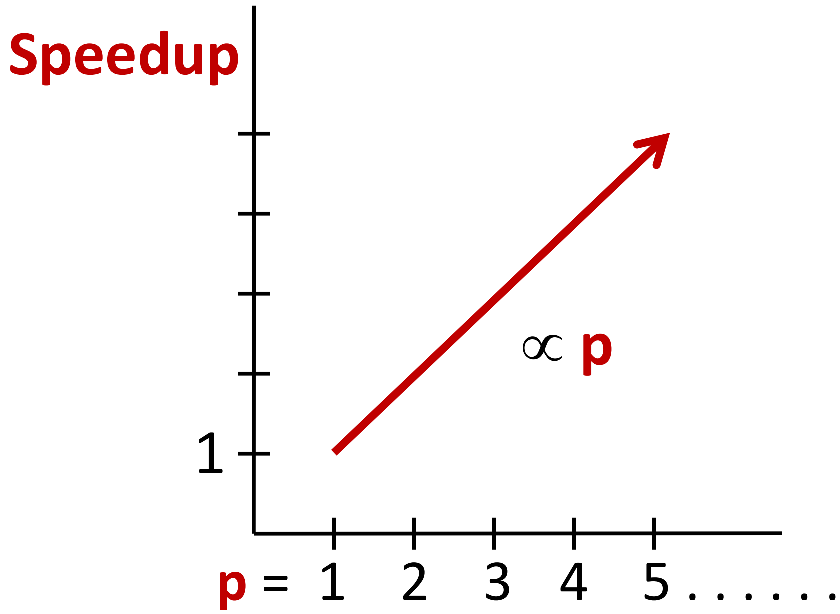
[Shiloach&Vishkin]

"Ideal" Linear Parallel Speedup

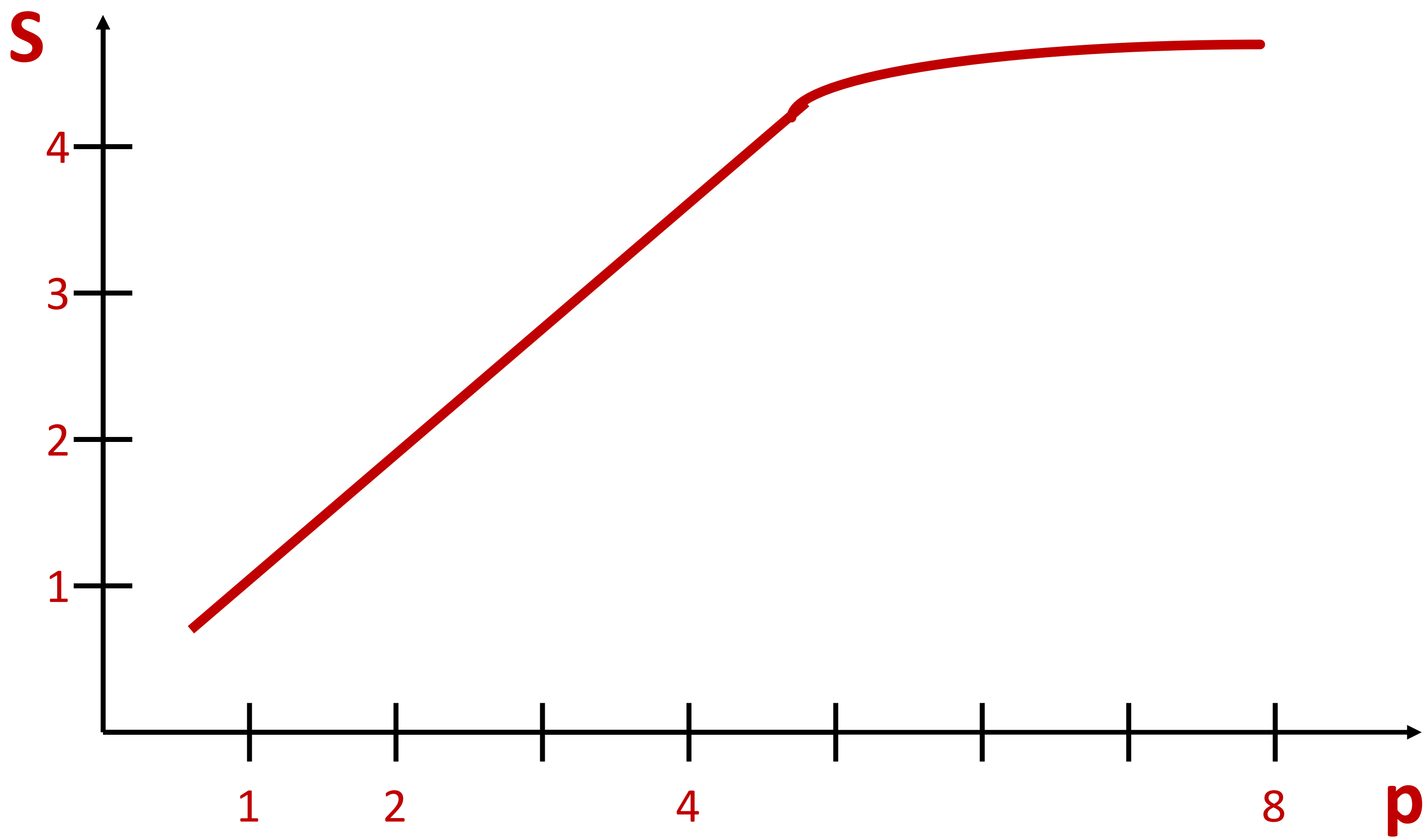
- Ideally, parallel speedup linear with **p**

$$\textbf{Speedup} = \frac{\textbf{time}_{\text{sequential}}}{\textbf{time}_{\text{parallel}}}$$





Non-Ideal Speed Up



Never get to high speedup regardless of **p***!!*

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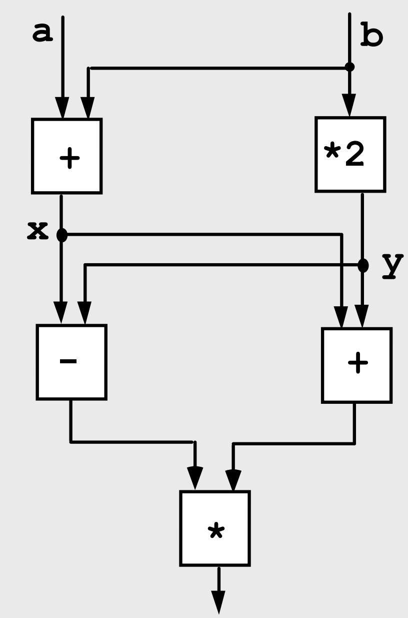
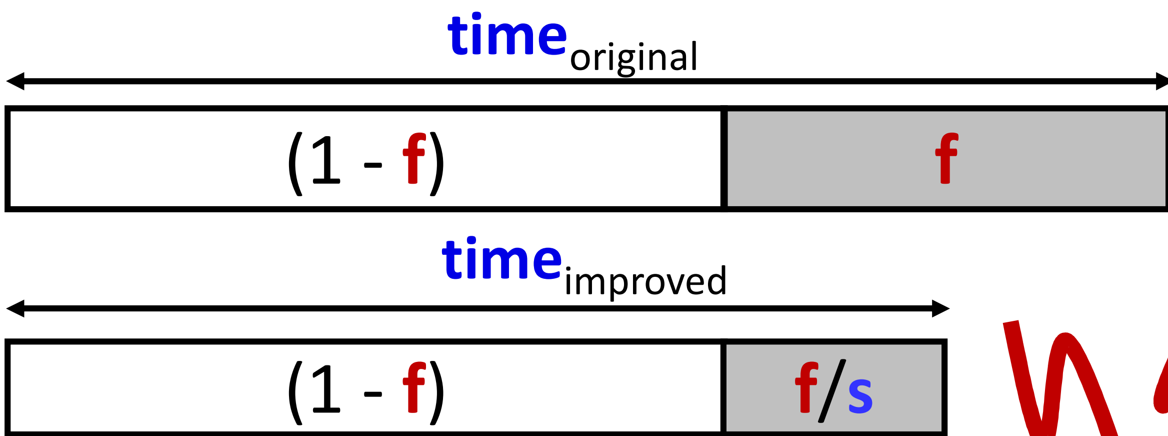


[Shiloach&Vishkin]

Recall

Amdahl's Law: a lesson on speedup

- If only a fraction **f** (of time) is speedup by **s**



$$\text{time}_{\text{improved}} = \text{time}_{\text{original}} \cdot \left((1-f) + f/s \right)$$

$$S_{\text{effective}} = 1 / \left((1-f) + f/s \right)$$

  – if **f** is small, **s** doesn't matter
  – even when **f** is large, diminishing return on **s**; eventually "1-**f**" dominates

Recall

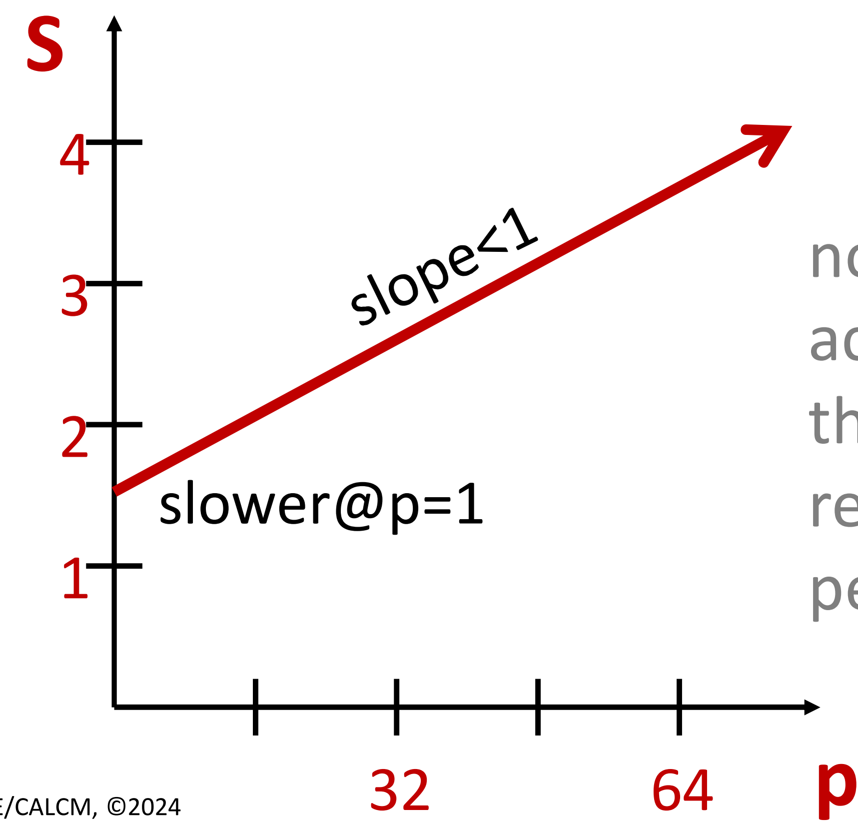
Non-Ideal Speed Up

Cheapest algo may not be the most scalable, s.t.

$\textbf{time}_{\text{parallel-algo@}\mathbf{p}\text{=1}} = \mathbf{K}\cdot\text{time}_{\text{sequential-algo}}$ where **K**>1

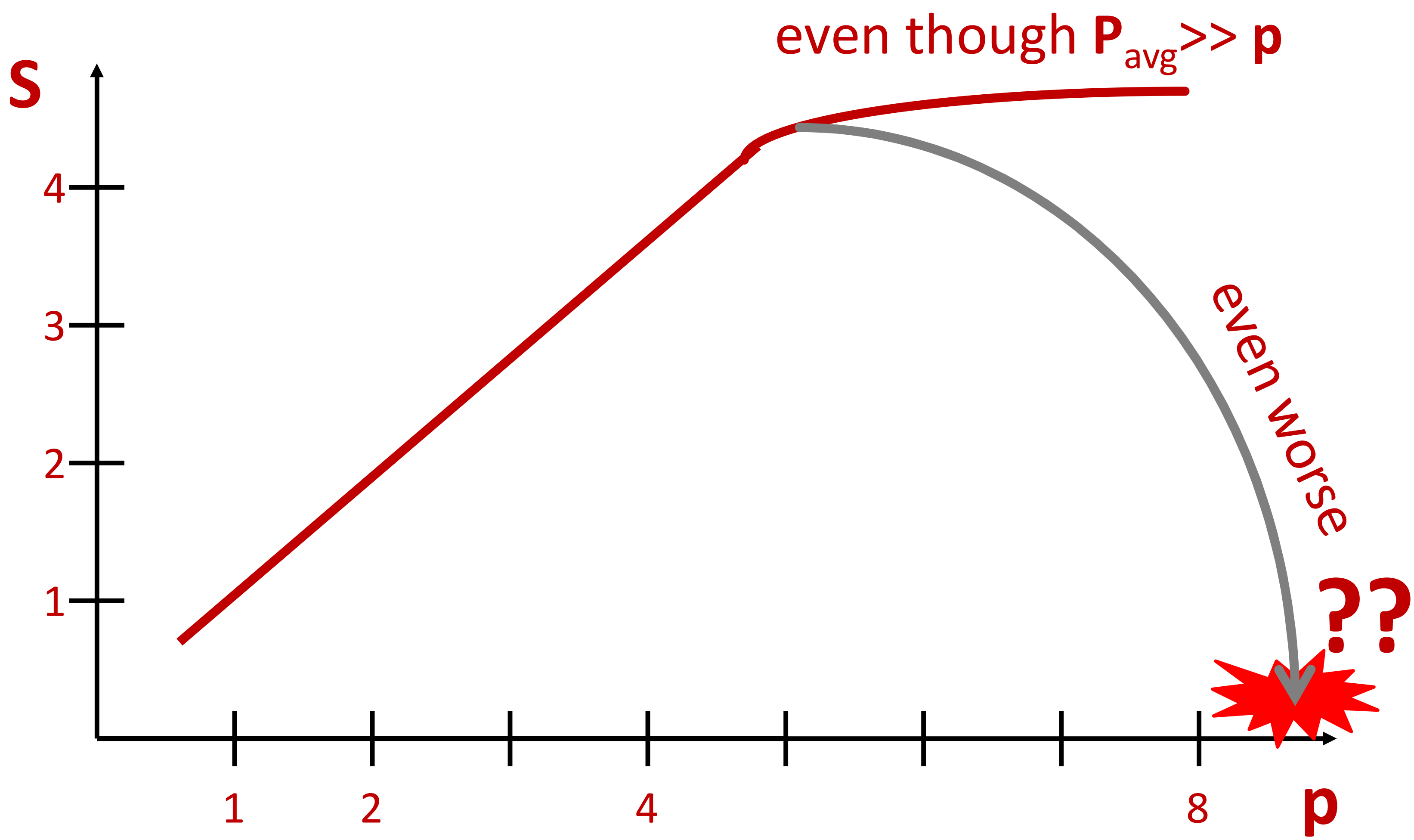
and

Speedup = **p**/**K**



not efficient but acceptable if it is the only way to reach required performance

Very Non-Ideal Speed Up



Communication not Free

- PE may spend extra time
  - in the act of sending or receiving data
  - waiting for data to be transferred from another PE
    - latency: data coming from far away
    - bandwidth: data coming thru finite channel
  - waiting for another PE to get to a particular point of the computation (a.k.a. synchronization)

How does communication cost grow with T_1?

How does communication cost grow with p?

Strong vs. Weak Scaling

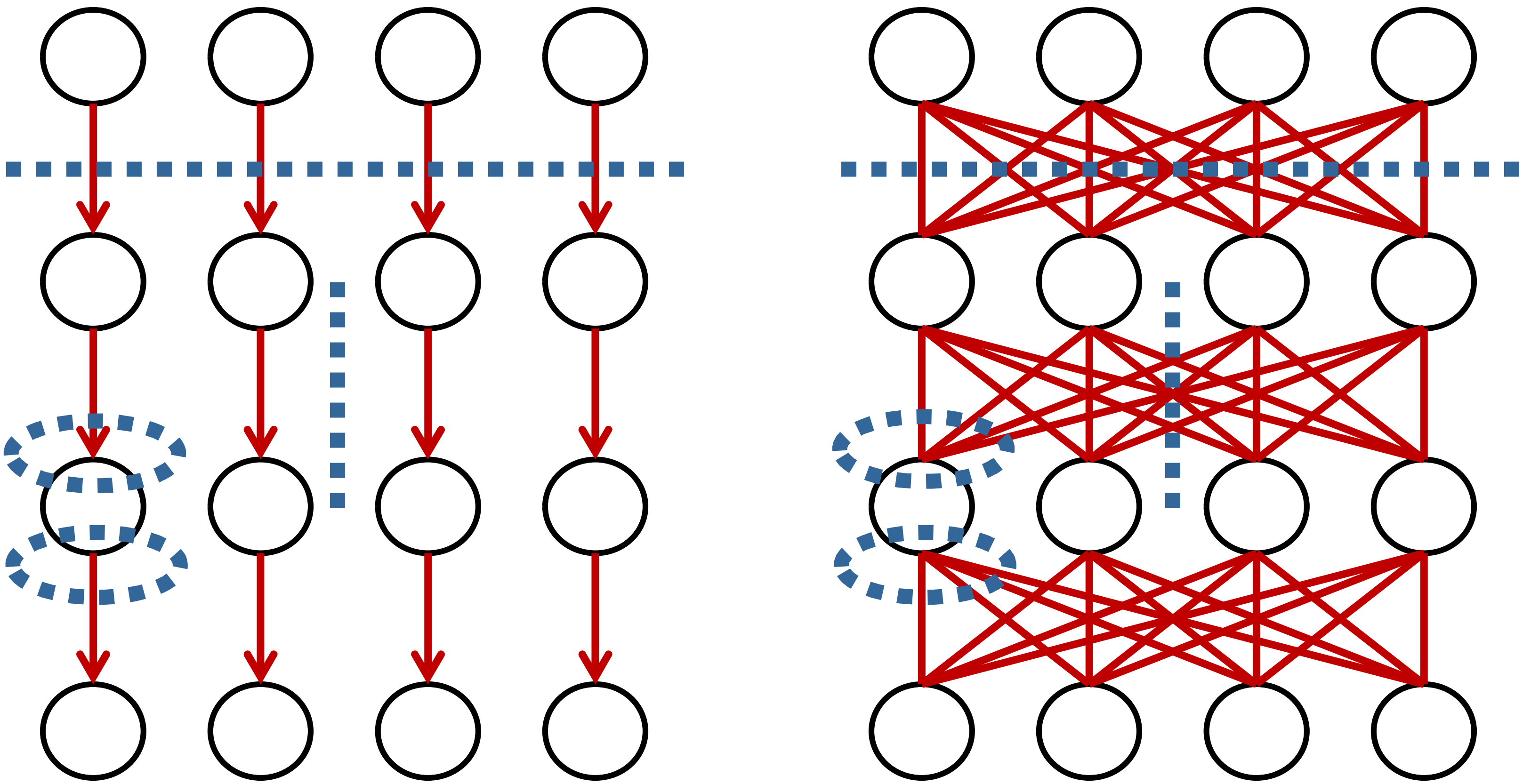
- **Strong Scaling**
  - what is S_p as **p** increases for constant work, T_1

    run same workload faster on new larger system
  - harder to speedup as (1) **p** grows toward P_{avg} and (2) communication cost increases with **p**
- **Weak Scaling**
  - what is S_p as **p** increases for larger work, $T_1'=p \cdot T_1$

    run a <u>larger</u> workload faster on new larger system
  - $S_p = \mathbf{time}_{sequential}(p \cdot T_1)/\mathbf{time}_{parallel}(p \cdot T_1)$
- Which is easier depends on
  - how P_{avg} scales with work size T_1'
  - relative scaling of bottlenecks *(storage, BW, etc)*

Arithmetic Intensity: Modeling Communication as "Lump" Cost

Not All Parallelism Created Equally



Arithmetic Intensity

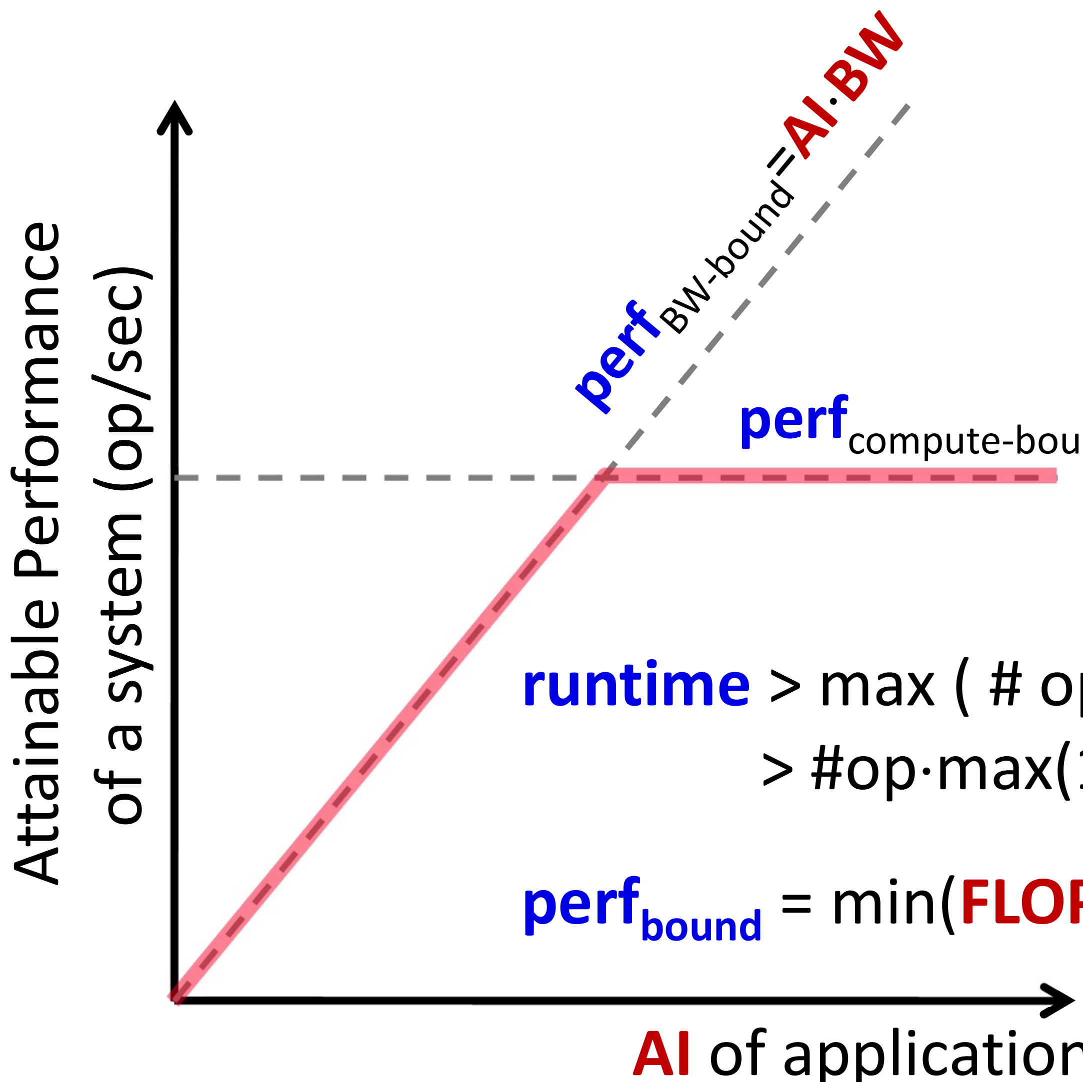
- An algorithm has a cost in terms of operation count
  - **runtime**$_{\text{compute-bound}}$ = # operations / **FLOPS**
- An algorithm also has a cost in terms of number of bytes communicated (ld/st or send/receive)
  - **runtime**$_{\text{BW-bound}}$ = # bytes / **BW**
- Which one dominates depends on
  - ratio of **FLOPS** and **BW** of platform
  - ratio of ops and bytes of algorithm
- Average **Arithmetic Intensity** (**AI**)
  - how many ops performed per byte accessed
  - # operations / # bytes

FLOPS=floating-point operations per second

Roofline Performance Model

[Williams&Patterson, 2006]

cost for CPU to communicate with memory



runtime > max (# op/**FLOPS**, # byte/**BW**}

> #op·max(1/**FLOPS**, 1/(**AI**·**BW**)}

perf$_{\text{bound}}$ = min(**FLOPS**, **AI**·**BW**)

Parallel Sum Revisited with AI

- Last lecture we said
  - 100 threads perform 100 +'s each in parallel, and
  - between 1~7 (plus a few) +'s each in the parallel reduction
  - T_{100}= 100 + 7
  - S_{100}= 93.5
- Now we see *(assume 1 op per cycle per thread)*
  - **AI** is a constant, 1 op / 8 bytes (for doubles)
  - Let $\mathbf{BW_{cyc}}$ be total bandwidth (byte/cycle) shared by threads on a multicore

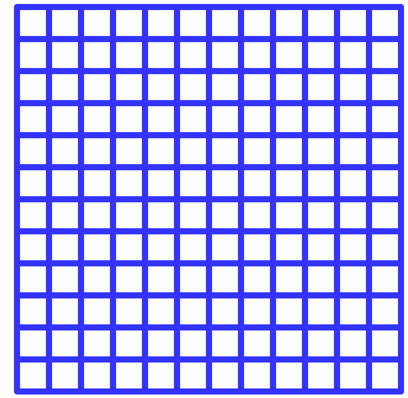
$$Perf_p < \min\{\ \mathbf{p}\ \text{ops/cycle},\ \mathbf{AI}*\mathbf{BW_{cyc}}\ \}$$

  - useless to parallelize beyond $\mathbf{p} > \mathbf{BW_{cyc}}/8$

What about a multi-socket system?

Interesting AI Example: MMM

```
for(i=0; i<N; i++)
  for(j=0; j<N; j++)
    for(k=0; k<N; k++)
      C[i][j]+=A[i][k]*B[k][j];
```

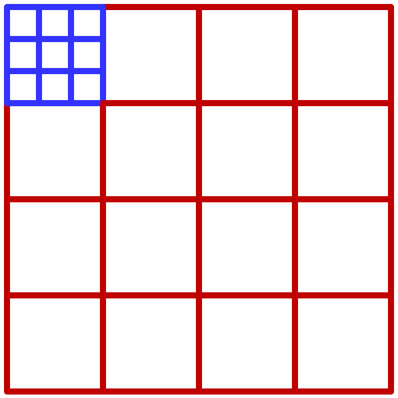


- N^2 data-parallel dot-product's
- Assume N is large *s.t.* 1 row/col too large for on-chip
- Operation count: N^3 float-mult and N^3 float-add
- External memory access (assume 4-byte floats)
  - $2N^3$ 4-byte reads (of A and B) from DRAM
  - . . . N^2 4-byte writes (of C) to DRAM . . .
- Arithmetic Intensity $\approx 2N^3/(4 \cdot 2N^3)=1/4$

GTX1080: 8 TFLOPS vs 320GByte/sec

More Interesting AI Example: MMM

```
for(i0=0; i0<N; i0+=Nb)
  for(j0=0; j0<N; j0+=Nb)
    for(k0=0; k0<N; k0+=Nb) {
      for(i=i0;i<i0+Nb;i++)
        for(j=j0;j<j0+Nb;j++)
          for(k=k0;k<k0+Nb;k++)
            C[i][j]+=A[i][k]*B[k][j];
    }
```



- Imagine a ‘N/N_b’x’‘N/N_b’ MATRIX of N_bxN_b matrices
  - inner-triple is straightforward matrix-matrix mult
  - outer-triple is MATRIX-MATRIX mult
- To improve **AI**, hold N_bxN_b sub-matrices on-chip for data-reuse

need to copy block (not shown)

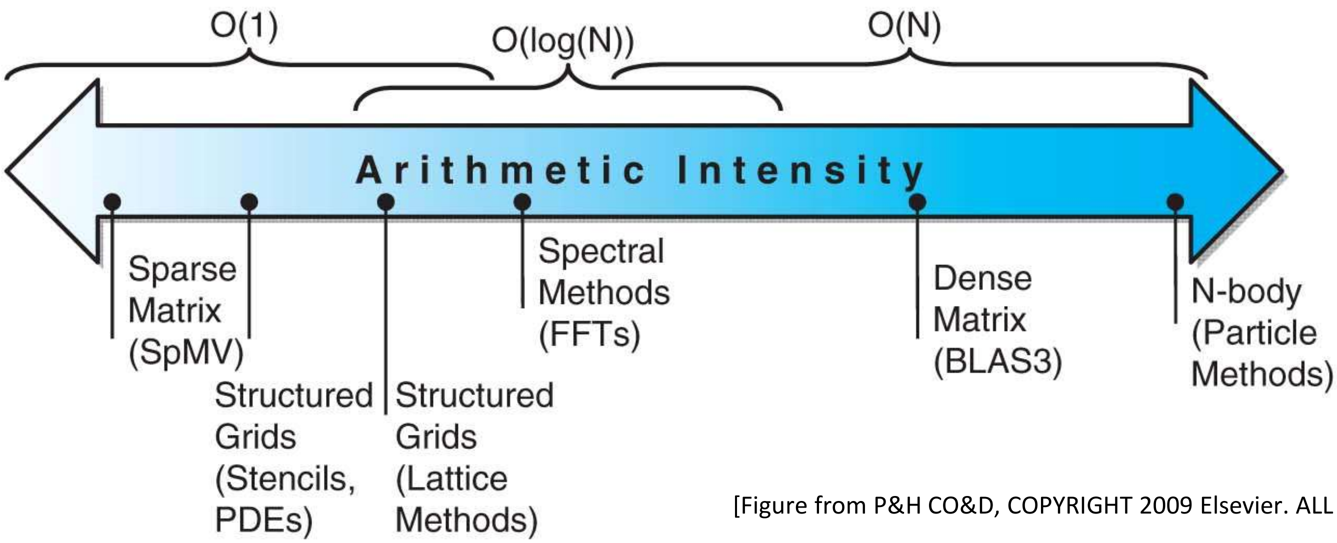
AI of blocked MMM Kernel (N_bxN_b)

```
for(i=i0;i<i0+Nb;i++)
  for(j=j0;j<j0+Nb;j++) {
    t=C[i][j];
    for(k=k0;k<k0+Nb;k++)
      t+=A[i][k]*B[k][j];
    C[i][j]=t;
  }
```

need to copy block (not shown)

- Operation count: N_b^3 float-mult and N_b^3 float-add
- When **A**, **B** fit in scratchpad (2xN_b^2x4 bytes)
  - $2N_b^3$ 4-byte on-chip reads (**A**, **B**) (fast)
  - $3N_b^2$ 4-byte off-chip DRAM read **A**, **B**, **C** (slow)
  - N_b^2 4-byte off-chip DRAM writeback **C** (slow)
- Arithmetic Intensity = $2N_b^3/(4{\cdot}4N_b^2)=N_b/8$

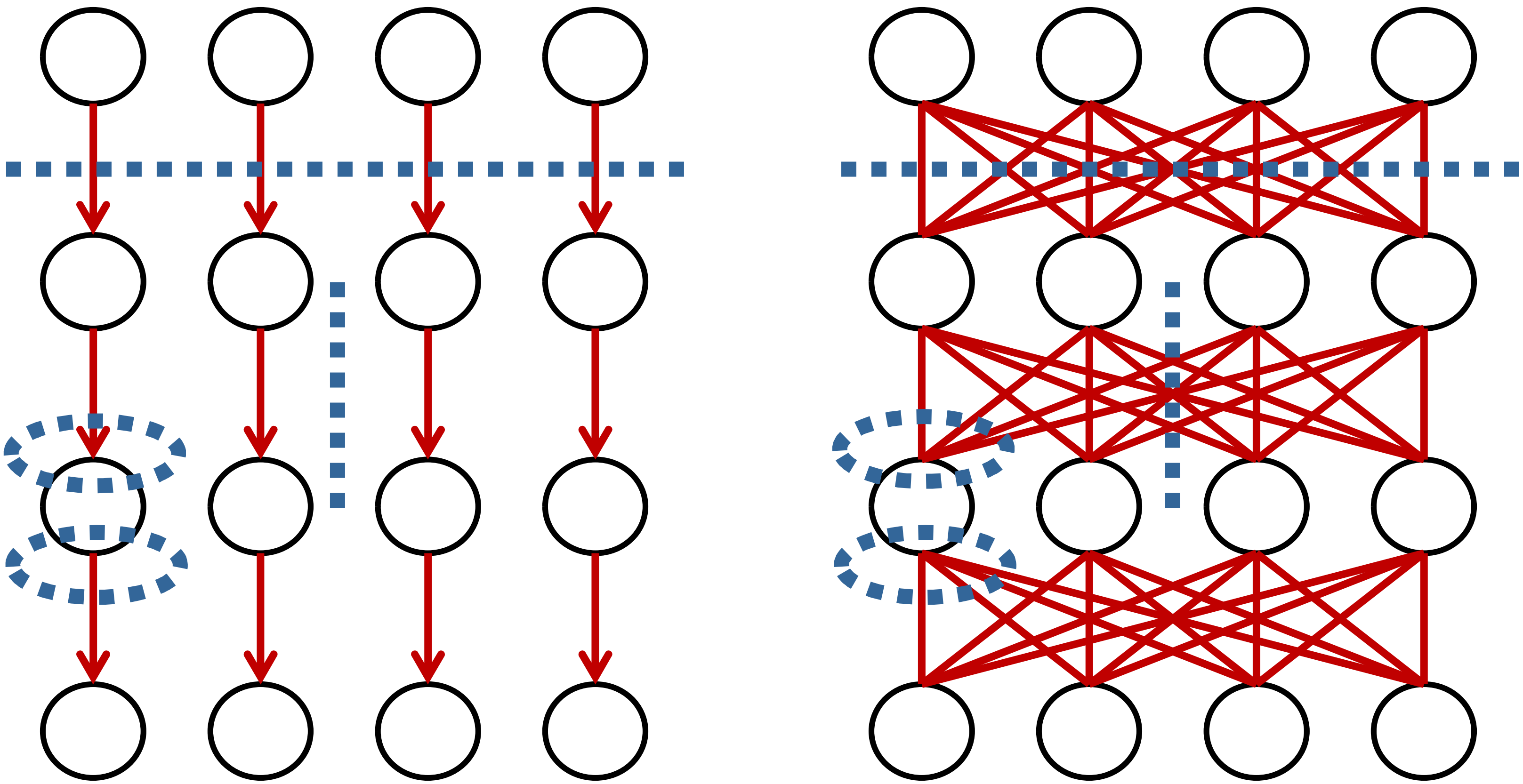
AI and Scaling



- **AI** is a function of algorithm and problem size
- Higher **AI** means more work per communication and therefore easier to scale
- Recall strong vs. weak scaling
  – strong=increase perf on fixed problem sizes
  – weak=increase perf on proportional problem sizes
  – weak scaling easier if **AI** grows with problem size

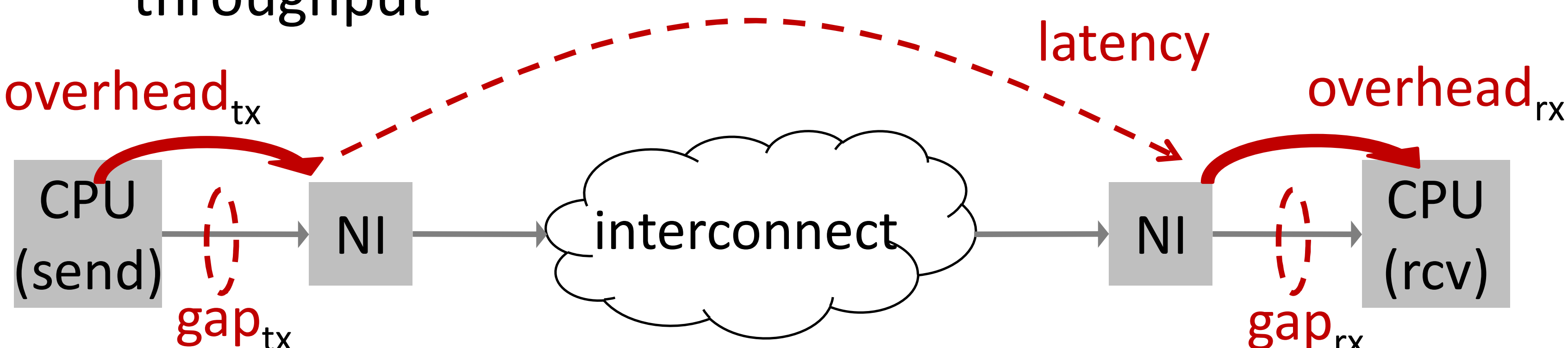
LogP Model:
Components of Communication Cost

Not All Parallelism Created Equally



LogP

- A parallel machine model with explicit communication cost
  - **Latency**: transit time between sender and receiver
  - **overhead**: time used up to setup a send or a receive (cycles not doing computation)
  - **gap**: wait time in between successive data units sent or received due to limited transfer bandwidth
  - **Processors**: number of processors, i.e., computation throughput



Message Passing Example

```
if (id==0)          //assume node-0 has A initially
   for (i=1;i<p;i=i+1)
      SEND(i, &A[SHARE*i], SHARE*sizeof(double));
else
   RECEIVE(0,A[])   //receive into local array

sum=0;
for(i=0;i<SHARE;i=i+1) sum=sum+A[i];

remain=p;
do {
    BARRIER();
    half=(remain+1)/2;
    if (id>=half&&id<remain) SEND(id-half,sum,8);
    if (id<(remain/2)) {
       RECEIVE(id+half,&temp);
       sum=sum+temp;
    }
    remain=half;
} while (remain>1);
```

SHARE=HOWMANY/p

Review

[based on P&H Ch 6 example]

Parallel Sum Revisited with LogP

How long?

```
if (id==0)
    for (i=1;i<100;i=i+1)
        SEND(i, &A[100*i], 100*sizeof(double));
else RECEIVE(0, A[])
```

- o • assuming no back-pressure, **node-0** finishes sending to **node-99** after 99× overhead of **SEND**( )
- L • first byte arrives at **node-99** some network latency later
- g • the complete message arrives at **node-99** after
  100*sizeof(double)/network_bandwidth
- o • **node-99** finally ready to compute after the overhead to **RECEIVE**( )

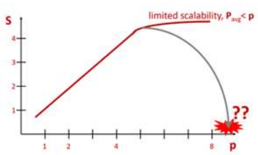
What if 100*sizeof(double)/network_bandwidth greater than the overhead to **SEND**( )?

Parallel Sum Revisited with LogP

How long?

```
sum=0;
for(i=0;i<100;i=i+1) sum=sum+A[i];
```

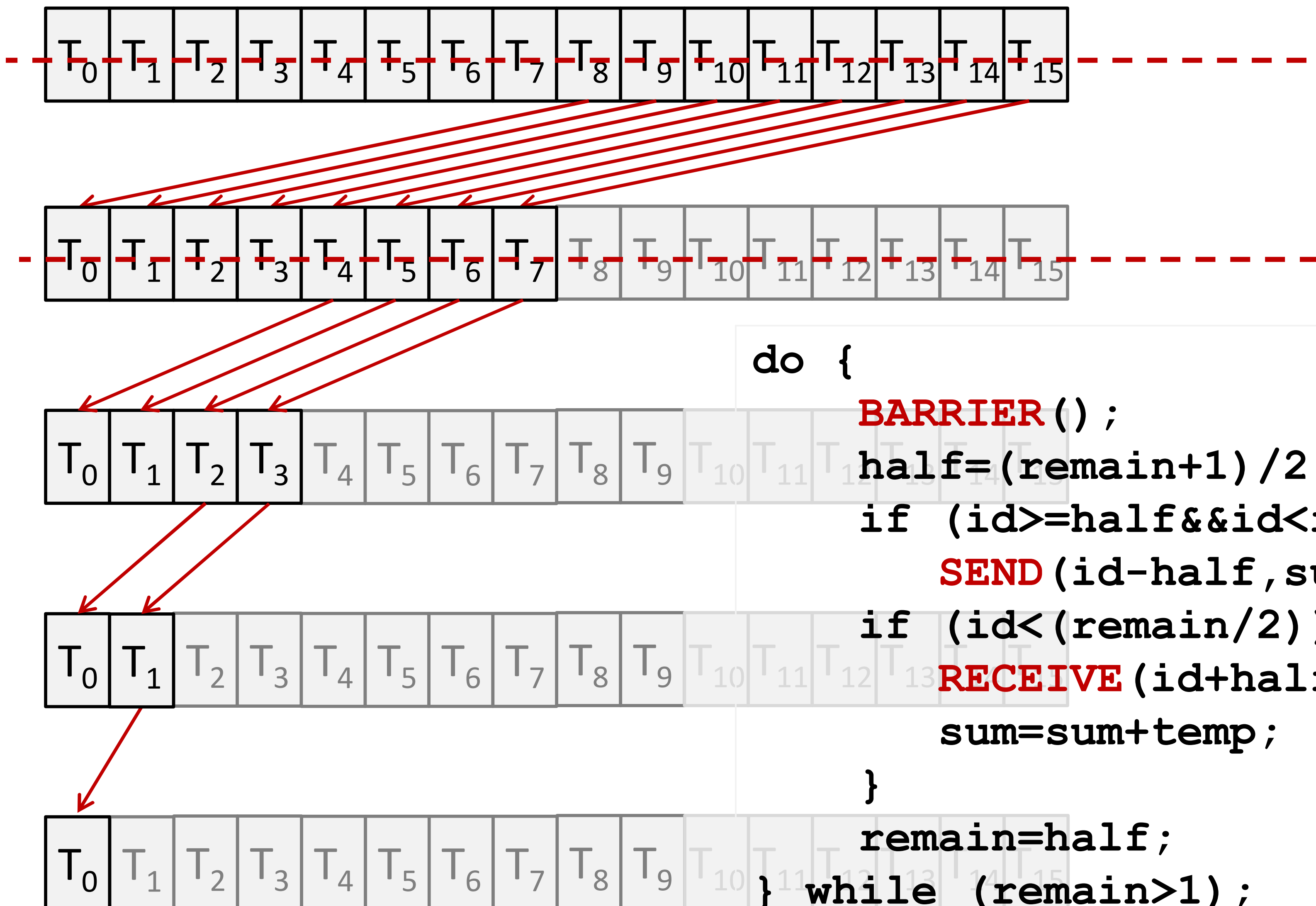
- ideally, this step is computed **p**=100 times faster than summing 10,000 numbers by one processor
- big picture thinking, e.g.,
  - is the time saved worth the data distribution cost?
  - if not, actually faster if parallelized less



- fine-tooth comb thinking, e.g.,
  - **node-1** begins work first; **node-99** begins work last ⇒ minimize overall finish time by assigning more work to **node-1** and less work to **node-99**
  - maybe latency and bandwidth are different to different nodes

Performance tuning is a craft

Parallel Sum Revisited with LogP



```
do {
   BARRIER();
   half=(remain+1)/2;
   if (id>=half&&id<remain)
      SEND(id-half,sum,8);
   if (id<(remain/2)) {
      RECEIVE(id+half,&temp);
      sum=sum+temp;
   }
   remain=half;
} while (remain>1);
```

Parallel Sum Revisited with LogP

```
do {
    BARRIER();
    half=(remain+1)/2;
    if (id>=half&&id<remain) SEND(id-half,sum,8);
    if (id<(remain/2)) {
        RECEIVE(id+half,&temp);
        sum=sum+temp;
    }
    remain=half;
} while (remain>1);
```

- do we need to synchronize each round?

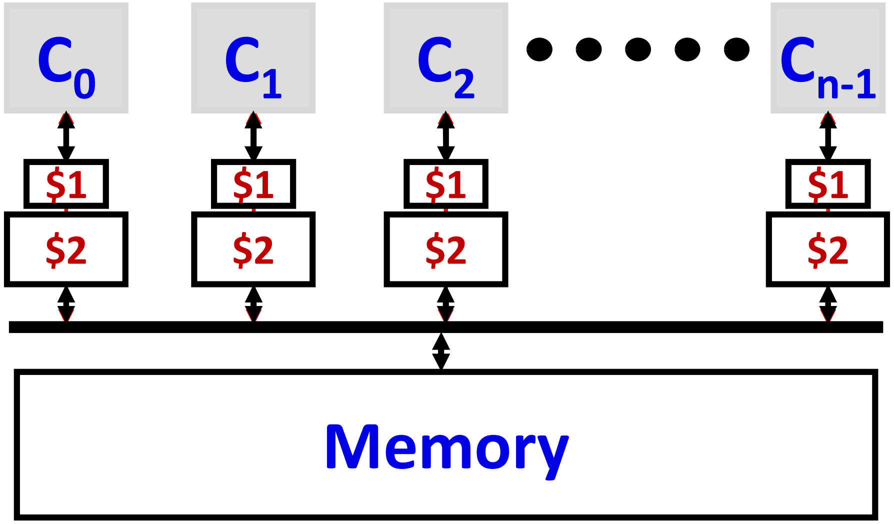
  *how does one build a **BARRIER()**?*

- is this actually faster than if all nodes sent to node-0?

  *What if **p** is small? What if **p** is very large?*

  Real answer is a combination of techniques

LogP applies to shared memory too




```
do {
  pthread_barrier_wait(…);

  half=(remain+1)/2;
  if (id<(remain/2))
     psum[id]=psum[id]+
              psum[id+half];
  remain=half;
} while (remain>1);
```

- When C_0 is reading `psum[0+half]`, the value originates in the cache of $C_{\text{“half”}}$
  - **L**: time from C_0's cache miss to when data retrieved from the cache of $C_{\text{“half”}}$ *(via cache coherence)*
  - **g**: there is a finite bandwidth between C_0 and $C_{\text{“half”}}$
  - **o**: as low as a LW instruction but also pay for stalls

Implications of Communication Cost

- Large **g**—*can't exchange a large amount of data*
  - must have lots of work per byte communicated
  - only scalable for applications with high **AI**
- Large **o**—*can't communicate frequently*
  - can only exploit coarse-grain parallelism
  - if DMA, amount of data not necessarily limited
- Large **L**—*can' t send data at the last minute*
  - must have high average parallelism *(more work/time between production and use of data)*
- High cost in each category limits
  - the kind of applications that can speed up, and
  - how much they can speed up

Parallelization not just for Performance

- Ideal parallelization over ***N*** CPUs
  - $T = Work \,/\, (k_{perf} \cdot \boldsymbol{N})$
  - $E = (k_{switch} + k_{static} \,/\, k_{perf}) \cdot Work$

    N-times static power, but ***N***-times faster runtime
  - $P = \boldsymbol{N}\,(k_{switch} \cdot k_{perf} + k_{static})$
- Alternatively, forfeit speedup for power and energy reduction by $s_{freq}=1/\boldsymbol{N}$ (assume $s_{voltage} \approx s_{freq}$ below)
  - $T = Work \,/\, k_{perf}$
  - $E'' = (k_{switch} \,/\, \boldsymbol{N}^2 + k_{static} \,/\, (k_{perf}\boldsymbol{N})) \cdot Work$
  - $P'' = k_{switch} \cdot k_{perf} \,/\, \boldsymbol{N}^2 + k_{static} \,/\, \boldsymbol{N}$
- Also works with using ***N*** slower-simpler CPUs

Don't forget