Superposition of power? (Answer: No)

In the 12:30 recitation section, we ran out of time on the question about finding power dissipation by superposition. We were working with the example below.

Suppose we are interested in finding the voltage \( V_R \) and current \( I_R \) of the resister (1\( \Omega \)). We are also interested in finding the power dissipated by the resister.

\[
\begin{align*}
\begin{array}{c}
+ \quad V_R - \\
\hline
\end{array} \\
\begin{array}{c}
\begin{array}{c}
\quad + \\
2V \\
\quad - \\
\hline
1V
\end{array} \\
\begin{array}{c}
\quad + \\
I_{\text{R}} \\
\quad - \\
\hline
\end{array}
\end{array}
\end{align*}
\]

On this simple circuit, you should be able to work out by inspection that \( V_R = 1V \) and \( I_R = 1A \), and therefore the power dissipated by the resister is \( P = 1W \).

Let’s try by superposition.

Step 1:

\[
\begin{align*}
\begin{array}{c}
+ \quad V_R' - \\
\hline
\begin{array}{c}
\quad + \\
2V \\
\quad - \\
\hline
\end{array} \\
\begin{array}{c}
\quad + \\
I_{\text{R'}} \\
\quad - \\
\hline
\end{array}
\end{array}
\end{align*}
\]

You should be able to work out by inspection that \( V_R' = 2V \) and \( I_{R'} = 2A \) and \( P' = 4W \).

Step 2:
Again, you should be able to work out by inspection that $V_{R''} = -1V$ and $I_{R''} = -1A$ and $P'' = 1W$.

Combining the two steps, indeed $V_R = V_R' + V_R'' = 2V + (-1V) = 1V$ and $I_R = I_R' + I_R'' = 2A + (-1A) = 1A$. Superposition works nicely for current and voltage.

$P$ however is not $P' + P''$. The problem is that power is not linear. (If you write an equation to solve for power somewhere in the circuit, it will involve the squares of voltage or current terms or the product of voltage and current terms. On the other hand, if you write an equation to solve for a voltage or current somewhere in the circuit, it will only involve linear combinations of voltage and current terms.)

Therefore, solve for total voltage and current using superposition, and only afterwards use the total voltage and current to compute power.