CMU 18–746 **Storage Systems**
Spring 2009 **Solutions to Homework 1**

Problem 1 : Finding service time. [20 points]

With some simple formulae we can find for any LBN its surface, cylinder, and offset.

$$\begin{aligned} \text{surface} &= \frac{LBN}{200} \ \% \ 8 \\ \text{cylinder} &= \frac{\text{LBN}}{1600} \\ \text{offset} &= \text{LBN} \ \% \ 200 \end{aligned}$$

Now we can assign a physical location to each LBN:

LBN	surface	cylinder	offset
0	0	0	0
9600600	3	6000	0
4801400	7	3000	0
11200948	4	7000	148

We also note that a 10000 RPM disk takes $\frac{1}{10000} \times 60 \times 1000 = 6$ milliseconds to make a full rotation. Now we can apply the service time formula:

$$\begin{aligned} T_{service} &= T_{seek} + T_{rotation} + T_{transfer} \\ T_{seek} &= 0.0006 \times \text{cylinder\_distance} + 2 \\ T_{rotation} &= \frac{\text{sectors to rotate}}{200} \times 6\text{ms} \\ T_{transfer} &= \frac{\text{sectors to transfer}}{200} \times 6\text{ms (same as rotation!)} \end{aligned}$$

It is important to remember that during the seek, the disk continues to rotate and so we need to take into account the amount that the disk has rotated during the seek and add in any additional rotational latency this introduces.

time (ms)	time taken (ms)	LBN	cylinder	surface	offset	next event
0		0	0	0	0	transfer
0.12	0.12	4	0	0	4	6000 cylinder seek
5.72	5.60	9600790.67	6000	3	190.67	rotational latency
6.00	0.28	9600600	6000	3	0	transfer
6.24	0.24	9600608	6000	3	8	3000 cylinder seek
10.04	3.80	4801534.67	3000	7	134.67	rotational latency
12.00	1.96	4801400	3000	7	0	transfer
12.12	0.12	4801404	3000	7	4	4000 cylinder seek
16.52	4.40	11200950.67	7000	4	150.67	rotational latency
22.44	5.92	11200948	7000	4	148	transfer
22.92	0.48	11200964	7000	4	164	done

Problem 2 : Media bandwidth and data buffering. [5 points]

$\frac{1}{20 \text{ MB per second}} = 0.05$ seconds per MB
The 1 MB memory will fill in 50 ms.

Problem 3 : Capacity benefit of zoned recording. [20 points]

Zoned recording

$$\begin{aligned}
C &= 8*(195*(1344-0)+187*(2449-1345)+176*(3542-2450)+166*(4407-3543)+ \\
&\quad +155*(5224-4408)+145*(5957-5225)+131*(6581-5958)) \\
C &= 8947016 \quad \text{blocks} \\
C &= \frac{512*8947016}{1024^3} = \boxed{4.37 \text{ GB}}
\end{aligned}$$

Single Zone

Since the number of sectors per track is the same for all cylinders, there are only 131 sectors per track. Note that at the end of each zone, there is a spare cylinder.

$$\begin{aligned}
C &= 8*131*(6581-6)) \\
C &= 6890600 \quad \text{blocks} \\
C &= \frac{512*6890600}{1024^3} = \boxed{3.37 \text{ GB}}
\end{aligned}$$

Note: Zone-bit recording provides $100*(4.3/3.3-1) = 30\%$ capacity improvement at the expense of increased compexity in the disk firmware to handle multiple zones in the layout.

First Cylinder	Last Cylinder	Sectors per Track	Sectors w/ Zoned Rec.	Sectors w/o Zoned Rec.	
0	1343	195	2096640	1408512	
1345	2448	187	1651584	1156992	
2450	3541	175	1528800	1144416	
3543	4406	165	1140480	905472	
4408	5223	154	1005312	855168	
5225	5956	145	849120	767136	
5958	6580	131	652904	652904	
			8924840	6890600	Total sectors
			4357.83 MB	3364.55 MB	Total cap (1M=2^{20})
			30%		Zone increase

Problem 4 : Rotational Latency vs. Seek. [20 points]

The response time T consists of seeking to the appropriate location and then waiting, on average half a revolution, for the sectors to arrive underneath the head. Thus

$$T = T_{seek} + T_{rotlat} + T_{mxfer}$$

From the table, the LBN 0 is mappped to a sector on cylinder 0. The LBN 3720000 mapped onto the n-th cylinder.

Cheetah 4LP

$$\begin{aligned}
3720000 &= 8 \times (195 \times \text{zone1} + 187 \times \text{zone2} + 176 \times \text{zone3}) \\
3720000 &= 8 \times (195 \times (1344 - 0) + 187 \times (n - 1345) \\
\lfloor n \rfloor &= 2431
\end{aligned}$$

$$\begin{aligned}
T_{seek} &= \text{8.8ms for the distance of 2431 cylinders} \\
T_{rotlat} &= \frac{1}{2}\frac{60}{10033} \times 10^3 = \frac{1}{2} \times 5.98\text{ms} = 2.99\text{ms} \\
T_{mxfer} &= 2 \times \frac{1}{187} \times 5.98\text{ms} = 0.06\text{ms}
\end{aligned}$$

Thus $T = 8.8 + 2.99 + 0.06$ms = $\boxed{11.85\text{ms}}$.

IBM Ultrastar 18ES

$$\begin{aligned}
3720000 &= 5 \times (390 \times \text{zone1} + 374 \times \text{zone2} + 364 \times \text{zone3}) \\
3720000 &= 5 \times (390 \times (378 - 0) + 374 \times (1264 - 378) + 364 \times (n - 1264 + 1)) \\
\lfloor n \rfloor &= 1993
\end{aligned}$$

$$\begin{aligned}
T_{seek} &= \text{5.9ms for the distance of 1993 cylinders} \\
T_{rotlat} &= \frac{1}{2}\frac{60}{7200} \times 10^3 = \frac{1}{2} \times 8.33\text{ms} = 4.17\text{ms} \\
T_{mxfer} &= 2 \times \frac{1}{364} \times 8.33\text{ms} = 0.05\text{ms}
\end{aligned}$$

Thus $T = 5.9 + 4.17 + 0.05$ms = $\boxed{10.12\text{ms}}$.

Note: The problem statement is somewhat ambiguous in describing the request stream. In these solutions, we don't include rotational latency for the first request. If you do include a half-a-rotation delay in the first request, your results will be different, but the conclusions will be the same.

Even though the Cheetah 4LP is a 10K RPM disk, the access time is better for the 7200 RPM Ultrastar 18ES disk. This is because of two main reasons:

- The IBM Ultrastar 18ES disk has better seek profile than the Seagate Cheetah 4LP, thus seeks of the same cylinder distance take shorter time.
- The IBM Ultrastar 18ES has bigger bit density. Notice the first zone has 390 sectors per track whereas on the Cheetah there are only 195 sectors.

Note: This is not an entirely fair comparison since the Cheetah 4LP disk is about 1–2 generations older than the Ultrastar 18ES (1996 vs. 1998) disk, but the point is show that RPM is not the only metric that matters.

Problem 5 : Zero-latency access. [20 points]

Non-zero latency disk The head can be positioned with probability $\frac{1}{N}$ above any sector of the track. Thus with equal probability we have to wait 0, 1, or up to $(N-1)$ sectors before we can access the first sector of S. Once the first sector arrives under the head, it will take $\frac{S}{N}$ revolutions to access the data. Therefore, the time to read S sectors is

$$T(N,S) = \frac{1}{N}\left(0+\frac{1}{N}+\frac{2}{N}+\ldots+\frac{N-1}{N}\right)+\frac{S}{N} = \frac{1}{N^2}\sum_{i=0}^{N-1} i+\frac{S}{N} = \frac{N-1}{2N}+\frac{S}{N}$$

Zero-latency disk A zero-latency disk can access the S sectors on the track as soon as ony of the sectors is under the read/write head. Therefore, the sectors can be read out of order. Then they are put into an intermediate buffer on the disk controller, reordered, and sent to the host in the correct order. As a consequence, a zero-latency disk will spend at most 1 revolution accessing S sectors on the track.

There is $\frac{N-S}{N}$ probability that the head is not positioned above any of the S sectors we want to access and $\frac{1}{N}$ probability it is exactly above the first sector of S. Then, there is $\frac{1}{N-S+1}$ probability we are 0, 1, 2, or up to $N-S$ sectors away from the first sector in S. Since the time to access S sectors is $\frac{S}{N}$ once we arrive at the start of the sectors S, the time T_1 to read S sectors is

$$T_1 = \frac{1}{N-S+1}\left(0+\frac{1}{N}+\frac{2}{N}+\ldots+\frac{N-S}{N}\right)+\frac{S}{N} = \frac{1}{N(N-S+1)}\sum_{i=0}^{N-S} i+\frac{S}{N}$$

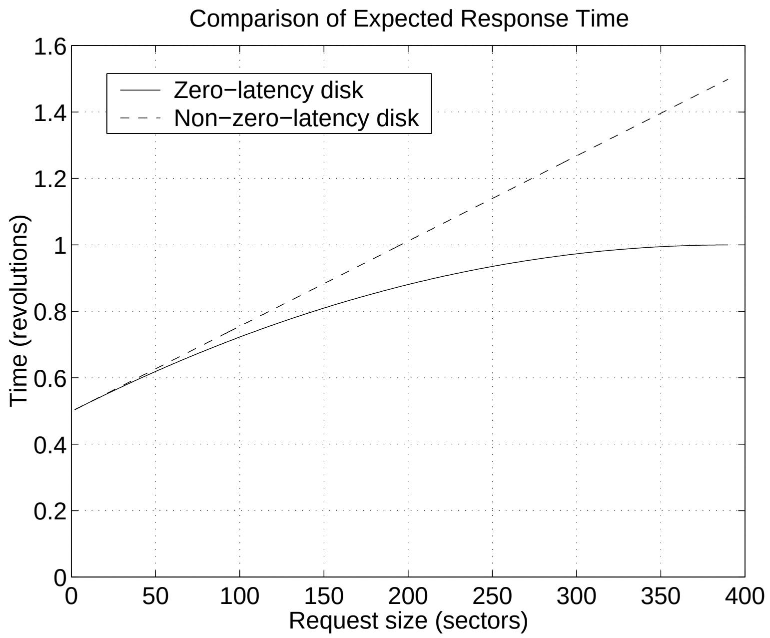
With probability of $\frac{S-1}{N}$ the head is anywhere within the S sectors not including the first sector of S. Thus the total access time is one revolution and can be expressed as

$$T_2 = \frac{1}{S-1}(1+\ldots+1) = \frac{1}{S-1}\sum_{i=1}^{S-1} 1 = 1$$

Combining the terms T_1 and T_2 and their respective probabilities, we can express the expected time for accessing S sectors without a track switch

$$\begin{aligned}
T(N,S) &= \frac{N-S+1}{N}T_1+\frac{S-1}{N}T_2 \\
&= \frac{N-S+1}{N}\left(\frac{1}{N(N-S+1)}\sum_{i=0}^{N-S} i+\frac{S}{N}\right)+\frac{S-1}{N} \\
&= \frac{N-S+1}{N^2}\left(\frac{N-S}{2}+S\right)+\frac{S-1}{N} \\
&= \frac{(N-S+1)(N+S)}{2N^2}+\frac{S-1}{N}
\end{aligned}$$

From the graph you can see, that, as expected, the zero-latency disk takes at most 1 revolution to service a request, whereas the non-zero-latency one approximately 1.5 revolutions for $S=N$.



Problem 6 : A glimpse into the future. [15 points]

We hope you used these opportunities to get a feel for where the disk drive industry might be going, and to learn a little more about a subject that you didn't understand as well before.