times. Although the incremental approach with spatiotemporal correlation provide roughly the same gain in accuracy as the global one, the running time is clearly much shorter.

The ratio of running times when neglecting one or both of the spatial or temporal correlations is different for the global vs. incremental approach: for the global one, it is more expensive to consider temporal correlations, while in the incremental approach, the spatial correlations are more time consuming.

These observations were proved to be consistent in all our experiments on benchmark circuits. In the following, we give the error values only for PR inputs, using the incremental approach. In reporting the error, we compared our switching activity estimates with the results of logic simulation at every internal node and primary output.

Table 3: Benchmark circuits -PR inputs

Circuit	MAX	MEAN	RMS	STD	TIME
C17	0.0565	0.0119	0.0238	0.0226	0.35 s
C432	0.0716	0.0133	0.0222	0.0179	276.43 s
C499	0.0334	0.0047	0.0072	0.0055	519.56 s
C880	0.1131	0.0158	0.0306	0.0264	320.11 s
C1355	0.0393	0.0027	0.0051	0.0044	333.02 s
C1908	0.0353	0.0044	0.0082	0.0069	489.23 s
C3540	0.1765	0.0276	0.0429	0.0318	3307.11 s
C6288	0.2046	0.0204	0.0443	0.0396	4530.84 s
z4ml	0.0672	0.0106	0.0197	0.0168	7.85 s
duke2	0.3199	0.0272	0.0657	0.0609	350.75 s

To conclude, two important observations should be made. First, Markov Chains are useful in modelling input correlations. Second, the degree in which any type of correlations affects the overall quality of estimations, depends on the internal structure of the circuit and the correlations among the primary inputs. The best way to use this framework in practice would be to combine both approaches in a hierarchical manner.

4. Conclusions

We have proposed a novel approach for estimation of the switching activities in combinational logic circuits under pseudorandom or biased inputs. Using the zero-delay hypothesis, we have derived a probabilistic model based on lag-one Markov Chains and conditional probabilities. The main feature of our approach is the systematic way in which we can deal with complex dependencies that may appear in practice; more precisely, our model supports spatiotemporal correlations among the primary inputs and internal lines of the circuit under consideration. A comparative analysis and benchmark evaluations emphasize the superiority of our approach over the current existing techniques and show its practicality on large combinational circuits. Our future work will concentrate on heuristics for improving the accuracy and time/space complexity and also on extensions of this approach beyond the logic level.

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3. Experimental results and discussions

All experiments were performed in the SIS environment [14] on a SPARC II workstation with 64Mbytes of memory. The experimental setup is shown below:

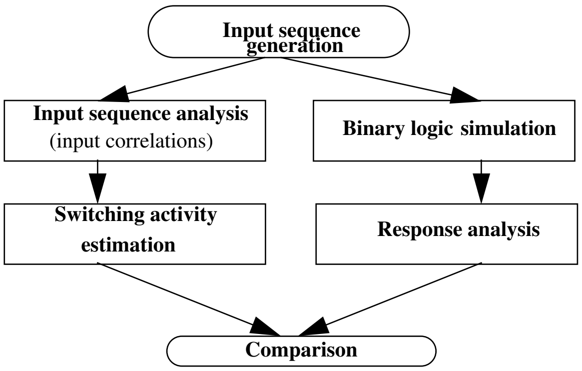

Fig.4

To generate pseudorandom (PR) inputs we have used as input generator a maximal-length linear feed-back shift register (LFSR) modified to include the all-zero pattern [13]. The average power consumption of a gate in a synchronous CMOS circuit, one can use the well-known formula $P_{avg} = 0.5\ (V_{dd}^2 / T_{cycle})\ C_{load}\ sw(x)$ where V_{dd} is the supply voltage, T_{cycle} is the clock cycle period, C_{load} is the load capacitance, x is the output of the gate and $sw(x)$ is computed as in (9). In our experiments, we were mainly interested to measure the accuracy of the model in estimating the switching activity locally (at each internal node of interest) and globally (for the entire circuit), given a set of inputs with spatiotemporal correlations.

To bound the error during the propagation procedure, we used two mechanisms:

- One based on the paradigm in fig.2, that is we calculate the signal probabilities independently and use these values as a more reliable measure for correcting the values of transition probabilities that fall out of range [0, 1]; more precisely, we normalize conditional probabilities such that relations (4) hold at each step;

- The other based on limiting the *TC* values, that is we normalize the values of coefficients using the set of equations (15).

To assess the impact of spatiotemporal correlations on switching activity estimations, we considered the **f51m** benchmark circuit and performed the following set of experiments:

- a PR experiment where the inputs were generated with the polynomial $p(x) = 1 \oplus x \oplus x^2 \oplus x^7 \oplus x^8$;

- a biased experiment where the switching activities of the inputs were $sw(i) = 0.25$, $i = 1, 2..., 7$ and $sw(8) = 0.375$.

In addition to the difference in switching activities of the circuit inputs, the biased input stream shows a higher amount of spatial and temporal correlations.

To compare our model with different other approaches reported in the literature, we analyzed exhaustively this circuit for the switching activity at primary outputs and all internal nodes. Comparing our estimations with the exact **logic simulation** results, we report in Tables 1 & 2, the usual measures for accuracy: maximum error (MAX), mean error (MEAN), root-mean square (RMS) and standard deviation (STD).

Table 1:f51m -PR inputs

	Global approach			
	W/ spatial correlations		W/o spatial correlations	
Error	W/ temporal correlations	W/o temporal correlations	W/ temporal correlations	W/o temporal correlations
MAX	0.0078	0.1949	0.1949	0.1949
MEAN	0.0003	0.0464	0.0463	0.0464
RMS	0.0014	0.0699	0.0699	0.0699
STD	0.0013	0.0526	0.0527	0.0526
TIME	254.16 s	1.64 s	7.31 s	0.42 s
	Incremental approach			
	W/ spatial correlations		W/o spatial correlations	
Error	W/ temporal correlations	W/o temporal correlations	W/ temporal correlations	W/o temporal correlations
MAX	0.1615	0.2062	0.2265	0.2264
MEAN	0.0131	0.0464	0.0473	0.0474
RMS	0.0289	0.0701	0.0714	0.0714
STD	0.0258	0.0528	0.0538	0.0537
TIME	44.57 s	4.16 s	0.64 s	0.43 s

Table 2:f51m - biased inputs

	Global approach			
	W/ spatial correlations		W/o spatial correlations	
Error	W/ temporal correlations	W/o temporal correlations	W/ temporal correlations	W/o temporal correlations
MAX	0.0767	0.2893	0.1714	0.3110
MEAN	0.0111	0.0939	0.0380	0.1049
RMS	0.0205	0.1164	0.0511	0.1239
STD	0.0174	0.0692	0.0344	0.0663
TIME	266.23 s	1.73 s	6.81 s	0.44 s
	Incremental approach			
	W/ spatial correlations		W/o spatial correlations	
Error	W/ temporal correlations	W/o temporal correlations	W/ temporal correlations	W/o temporal correlations
MAX	0.1517	0.2885	0.1714	0.3110
MEAN	0.0158	0.0930	0.0344	0.0972
RMS	0.0296	0.1162	0.0480	0.1166
STD	0.0252	0.0702	0.0337	0.0648
TIME	48.86 s	4.22 s	0.69 s	0.41 s

For PR inputs, global approaches with spatiotemporal correlations are almost 50 times more accurate than the approaches that do not account for any of these dependencies. Incremental approaches that consider both types of correlations are on average 3 times more accurate than the ones that neglect any of these. The price we have to pay in terms of accuracy is justified by a significant computational speed-up of incremental method vs. the global one. It is worthwhile to note that taking into account any of the spatial or temporal correlations by itself does not really improve the accuracy of the estimations.

For biased inputs, the global approach using both spatial and temporal correlations is 6 times more accurate than the one that ignores both dependencies; on the other hand, the incremental approach provides a gain in accuracy of 4

only the OBDD of f and by using a dynamic programming approach, compute the transition probabilities more efficiently.

Based on the above representation, the event 'f switching from value i to value j' ($i, j = 0, 1$) may be written as:

$$f_{i \to j} = \sum_{\pi \in \Pi_i} \sum_{\pi' \in \Pi_j} \prod_{k=1}^{n} x_{k_{i_k \to j_k}} \tag{17}$$

where i_k, j_k are the values of variable x_k on paths π and π' respectively (i_k, j_k = 0, 1, 2, where 2 stands for don't care values) for each k = 1, 2,..., n. Thus the probability that f switches from i to j may be expressed as:

$$p(f_{i \to j}) = p\left(\sum_{\pi \in \Pi_i} \sum_{\pi' \in \Pi_j} \prod_{k=1}^{n} x_{k_{i_k \to j_k}}\right) \tag{18}$$

Applying the property of disjoint events (that is satisfied by the collection of paths in the OBDD representation), the above formula becomes:

$$p(f_{i \to j}) = \sum_{\pi \in \Pi_i} \sum_{\pi' \in \Pi_j} p\left(\prod_{k=1}^{n} x_{k_{i_k \to j_k}}\right) \tag{19}$$

However, since the variables x_k may *not* be spatially independent of one another, the probability of a path to 'switch' from (i_1, i_2,..., i_n) to (j_1, j_2,..., j_n) may *not* be expressed as the product of transition probabilities for individual variables. If relation (16) is true for any three signals from the set $\{x_1, x_2,..., x_n\}$, then:

$$p\left(\prod_{k=1}^{n} x_{k_{i_k \to j_k}}\right) = \prod_{k=1}^{n}\left(p(x_{k_{i_k \to j_k}}) \prod_{1 \le k < l \le n} TC^{x_k x_l}_{i_k i_l, j_k j_l}\right) \tag{20}$$

We therefore obtain the following result:

Proposition 9 The transition probability of a signal f from state i to state j ($i, j = 0, 1$) is:

$$p(f_{i \to j}) = \sum_{\pi \in \Pi_i} \sum_{\pi' \in \Pi_j} \prod_{k=1}^{n}\left(p(x_{k_{i_k \to j_k}}) \prod_{1 \le k < l \le n} TC^{x_k x_l}_{i_k i_l, j_k j_l}\right) \tag{21}$$

❒

Although this expression seems to be very complicated, its complexity is within reasonable bounds; it is not necessary to enumerate all *pairs* of paths in the OBDD (which would provide a quadratic complexity in the number of paths in the OBDD), but for a fixed path in Π_i the computation may be done in linear time in the number of OBDD nodes.

For the incremental approach, we need a mechanism not only for computing the transition probabilities, but also for propagating the *TC*'s through the boolean network. For a given node in the circuit, it is only necessary to propagate the *TC* of the output with respect to the signals on which the inputs depend.

b) Propagation of the transition correlation coefficients

Let f be a node with immediate inputs x_1, x_2,..., x_n and x a signal on which at least one of the inputs x_1, x_2,..., x_n depends. According to the definition of the *TC*, for every i, j, p, q = 0, 1 possible values of f and x respectively, we have:

$$TC^{fx}_{ip, jq} = \frac{p(f_{i \to j} x_{p \to q})}{p(f_{i \to j}) p(x_{p \to q})} \tag{22}$$

Since the transition probabilities for f and x are already computed at this point, the only problem is to compute the probability of both f and x switching from i to j and from p to q, respectively. We get the following important result:

Proposition 10 The TC between signals f and x, for any values i, j, p, q = 0, 1 may be expressed as:

$$TC^{fx}_{ip, jq} = \frac{\sum_{\pi \in \Pi_i} \sum_{\pi' \in \Pi_j} \prod_{k=1}^{n}\left(TC^{x_k x}_{i_k p, j_k q}\, p(x_{k_{i_k \to j_k}}) \prod_{1 \le k < l \le n} TC^{x_k x_l}_{i_k i_l, j_k j_l}\right)}{p(f_{i \to j})} \tag{23}$$

❒

c) Complexity issues

In order to assess the complexity claimed above, let us define the following notation:

$$f_{\pi \to j} = \sum_{\pi' \in \Pi_j} \prod_{k=1}^{n} x_{k_{i_k \to j_k}} \tag{24}$$

where π is a fixed path in Π_i. Thus, using the disjointedness property, the corresponding probability is:

$$p(f_{\pi \to j}) = \sum_{\pi' \in \Pi_j} p\left(\prod_{k=1}^{n} x_{k_{i_k \to j_k}}\right)$$

Since path π is fixed, the above probability may be computed on the OBDD in the same way as a signal probability. The idea is that, using Shannon decomposition, the signal probability (and hence the above probability) may be computed in linear time in the number of the OBDD-nodes [6]. Thus, $f_{\pi \to j}$ may be decomposed as follows:

$$f_{\pi \to j} = x_{k_{i_k \to 0}} f_{\pi \to j}^{\bar{x}_k} + x_{k_{i_k \to 1}} f_{\pi \to j}^{x_k} \tag{25}$$

where $f_{\pi \to j}^{\bar{x}_k}$, $f_{\pi \to j}^{x_k}$ are the cofactors with respect to $\bar{x}_k$ and x_k, respectively. Based on this recursive decomposition, we may also write a similar relation for the corresponding probabilities, taking also into account the possible existing correlations:

$$p(f_{\pi \to j}) = p(x_{k_{i_k \to 0}}) p(f_{\pi \to j}^{\bar{x}_k}) \prod_{1 \le k < l \le n} TC^{x_k x_l}_{i_k i_l, 0 j_l} + p(x_{k_{i_k \to 1}}) p(f_{\pi \to j}^{x_k}) \prod_{1 \le k < l \le n} TC^{x_k x_l}_{i_k i_l, 1 j_l} \tag{26}$$

Having computed this probability for each path π, we immediately get the corresponding transition probabilities and hence the switching activity.Thus, for a fixed path π, the complexity is $O(n^2 N)$ where n is the number of variables and N is the number of nodes in the OBDD. The n^2 factor comes from the necessity of taking into account the correlations: in addition to the transition probabilities, we have to keep track of the *TC*'s involved on each path. There is a number of C_n^2 factors in the product, thus the complexity is quadratic in the number of variables.

Hence, overall, the time complexity is $O(n^2 NP)$ where P is the number of paths in the OBDD. In the incremental approach, this is within reasonable limits since n does not exceed 5 or 6 variables in the immediate fanin of the node.

Definition 3: Conditional probability $p_{a,\,b}$ is defined as:

$$p_{a,\,b} = p(x(t) = k \wedge y(t) = l | x(t-\delta) = i \wedge y(t-\delta) = j) \quad (9)$$

where a, b = 0, 1, 2, 3, a is encoded as ij and b as kl.
Ercolani et al. consider in [5] structural dependencies between any two signals in a circuit by using the signal correlation coefficients (*SC*); these coefficients can be expressed as:

$$SC_{ij}^{xy} = \frac{p(x = i \wedge y = j)}{p(x = i)p(y = j)} \quad (10)$$

where i, k = 0,1. Assuming that higher order correlations of two signals to a third one can be neglected, they approximated the correlation coefficient among three signals as:

$$SC_{ijk}^{xyz} = SC_{ij}^{xy}SC_{ik}^{xz}SC_{jk}^{yz} \quad (11)$$

Our approach is more general in that we capture the spatial correlations between signals, for each pair of signals (x,y) and for all possible transitions between them as described next:

Definition 4: Transition correlation coefficients (*TC*) for two signals x, y is defined as:

$$TC_{ij,\,kl}^{xy} = \\ = \frac{p(x(t-\delta) = i \wedge x(t) = k \wedge y(t-\delta) = j \wedge y(t) = l)}{p(x(t-\delta) = i \wedge x(t) = k)p(y(t-\delta) = j \wedge y(t) = l)} \quad (12)$$

where i,j, k, l = 0, 1.

Proposition 4: For every pair of signals (x,y) and all possible values i, j, k, l = 0, 1, the following holds:

$$SC_{ij}^{xy} = \sum_{k,\,l\,=\,0,\,1} TC_{ij,\,kl}^{xy}\frac{p(x_{i\to k})p(y_{j\to l})}{p(x = i)p(y = j)} \quad (13)$$

❒

Proposition 5: For every pair of signals (x,y) and all possible values i, j = 0, 1, the following equations hold:

$$\sum_{j\,=\,0,\,1} SC_{ij}^{xy}p(y = j) = 1 \qquad \forall\, i = 0, 1;$$
$$\sum_{i\,=\,0,\,1} SC_{ij}^{xy}p(x = i) = 1 \qquad \forall\, j = 0, 1. \quad (14)$$

❒

Proposition 6: For every pair of signals (x,y) and all possible values i, j, k, l = 0, 1 the following equations hold:

$$\sum_{j,\,l\,=\,0,\,1} TC_{ij,\,kl}^{xy}p(y_{j\to l}) = 1 \qquad \forall\, i, k = 0, 1;$$
$$\sum_{i,\,k\,=\,0,\,1} TC_{ij,\,kl}^{xy}p(x_{i\to k}) = 1 \qquad \forall\, j, l = 0, 1. \quad (15)$$

❒

We provide in the following other two useful results:

Proposition 7: The set of 4 equations and 4 unknowns SC_{ij}^{xy}, i, j = 0, 1 in Proposition 5 is indeterminate. Moreover, the matrix of the system has the rank 3 in the non-trivial cases (the trivial case is when any one of the signal probabilities is either 0 or 1).

❒

Proposition 8: The set of 8 equations and 16 unknowns $TC_{ij,kl}^{xy}$, i, j, k, l = 0, 1 is indeterminate; the matrix of the system has the rank 7 in the non-trivial cases (the trivial case is when any three of the transition probabilities are zero).

❒

The last two propositions are very important from a practical point of view. The set of equations involving *SC*'s may be solved knowing only SC_{11}^{xy}, for example, and that was the approach taken by Ercolani et al. in [5] (although, no similar analysis appeared in the original paper). In the more complex case involving *TC*'s, we need to know 9 out of 16 coefficients in order to deduce all values.

2.3. Propagation mechanisms

In what follows we ignore higher order correlations, that is, the correlation between any number of signals is expressed only in terms of pairwise correlation coefficients; the same assumption was used in [5], but only for signal correlation coefficients.

Definition 5: We define the *TC* among three signals as:

$$TC_{ijk,\,lmn}^{xyz} = \frac{p(x_{i\to l}y_{j\to m}z_{k\to n})}{p(x_{i\to l})p(y_{j\to m})p(z_{k\to n})}$$

Neglecting higher order correlations, we therefore assume that the following holds for any signals x, y, z and any values i, j, k, l, m, n = 0, 1:

$$TC_{ijk,\,lmn}^{xyz} = TC_{ij,\,lm}^{xy}TC_{jk,\,mn}^{yz}TC_{ik,\,ln}^{xz} \quad (16)$$

Definition 5 and relation (16) may be easily extended to any number of signals. Based on the above assumption, we use an OBDD-based procedure for computing the transition probabilities and for propagating the *TC*'s through the network. The main reason for using the OBDD representation [8] for a signal is that it is a compact and canonical representation of a Boolean function and that it offers a disjoint cover which is essential for our purposes. Depending on the set of signals with respect to which we represent a node of the boolean network, two approaches may be used:

- **The global approach** - for each node, we build the OBDD in terms of the primary inputs of the circuit;
- **The incremental approach** - for each node, we build the OBDD in terms of its immediate fanin and propagate the transition probabilities and the *TC*'s through the circuit.

The first approach is more accurate, but requires much more memory and running time; indeed, for many large circuits, it is nearly impractical. The second one, offers accurate enough results whilst being more efficient as far as memory requirement and running time are concerned.

a) Computation of the transition probabilities

Let f be a node in the boolean network represented in terms of n (immediate or primary input) variables $x_1, x_2,..., x_n$. f may be defined through the following two sets of OBDD paths:

- $\prod_1$ - the set of all paths in the ON-set of f
- $\prod_0$ - the set of all paths in the OFF-set of f

Some of the approaches reported in the literature (e.g. [9]), use the XOR-OBDD of f at two consecutive time steps to compute the transition probabilities. We consider instead

During the application of the input vectors, x may be 0 or 1, so that if we define its state at time n by random variable x_n, then the behavior of line x can be described as a lag-one Markov Chain $\{x_n\}_{n>1}$ over the state set S = {0,1} through the transition matrix Q [12]:

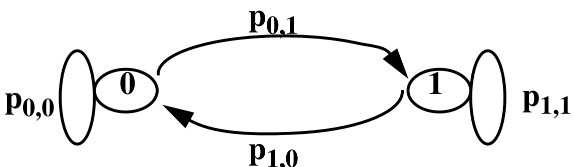

Fig.1

$$x_n = \begin{cases} 0 \text{ if x} = 0 \\ 1 \text{ if x} = 1; \end{cases} \qquad Q = \begin{bmatrix} p^x_{0,0} & p^x_{1,0} \\ p^x_{0,1} & p^x_{1,1} \end{bmatrix} \quad (1)$$

Every entry $p_{i,j}$ in the Q matrix represents a conditional probability and may be viewed as the one-step transition probability to state i at step n from state j at step n-1. The expressions for these conditional probabilities are:

$$\begin{aligned} p^x_{0,0} &= p((x(t) = 0) | (x(t-\delta) = 0)) \\ p^x_{0,1} &= p((x(t) = 1) | (x(t-\delta) = 0)) \\ p^x_{1,0} &= p((x(t) = 0) | (x(t-\delta) = 1)) \\ p^x_{1,1} &= p((x(t) = 1) | (x(t-\delta) = 1)) \end{aligned} \quad (2)$$

In the Q matrix, every column adds to unity, i.e:

$$p^x_{0,0} + p^x_{0,1} = 1 \qquad p^x_{1,0} + p^x_{1,1} = 1 \quad (3)$$

A lag-one Markov Chain has the property that one-step transition probabilities do not depend on the 'history', i.e they are the same irrespective of the number of previous steps. If the process $\{x_n\}_{n>1}$ is homogenous, then the probability distribution of the chain is $\mathcal{P} = (Q)^n \mathcal{P}_0$ where $\mathcal{P}_0$ is the initial distribution vector. If we assume the stationarity of the process $\{x_n\}_{n>1}$, then the previous relation becomes $\mathcal{P} = Q\mathcal{P}$. Based on this, we get the following (all the proofs can be found in [11]):

Proposition 1: The signal probabilities may be expressed in terms of conditional probabilities as follows:

$$p(x=0) = \frac{p^x_{1,0}}{p^x_{1,0} + p^x_{0,1}} \qquad p(x=1) = \frac{p^x_{0,1}}{p^x_{1,0} + p^x_{0,1}} \quad (4)$$

❒

Definition 1: Transition probabilities are defined as:

$$\begin{aligned} p(x_{0\to0}) &= p((x(t) = 0) \wedge (x(t-\delta) = 0)) \\ p(x_{0\to1}) &= p((x(t) = 1) \wedge (x(t-\delta) = 0)) \\ p(x_{1\to0}) &= p((x(t) = 0) \wedge (x(t-\delta) = 1)) \\ p(x_{1\to1}) &= p((x(t) = 1) \wedge (x(t-\delta) = 1)) \end{aligned} \quad (5)$$

Proposition 2: Transition probabilities may be expressed in terms of conditional probabilities as:

$$\begin{aligned} p(x_{0\to0}) &= \frac{p^x_{1,0} p^x_{0,0}}{p^x_{1,0} + p^x_{0,1}} \qquad p(x_{0\to1}) = \frac{p^x_{1,0} p^x_{0,1}}{p^x_{1,0} + p^x_{0,1}} \\ p(x_{1\to0}) &= \frac{p^x_{1,0} p^x_{0,1}}{p^x_{1,0} + p^x_{0,1}} \qquad p(x_{1\to1}) = \frac{p^x_{1,1} p^x_{0,1}}{p^x_{1,0} + p^x_{0,1}} \end{aligned} \quad (6)$$

❒

Proposition 3: Conditional probabilities may be expressed in terms of transition probabilities as:

$$\begin{aligned} p^x_{0,0} &= \frac{p(x_{0\to0})}{p(x_{0\to0}) + p(x_{0\to1})} \qquad p^x_{0,1} = \frac{p(x_{0\to1})}{p(x_{0\to0}) + p(x_{0\to1})} \\ p^x_{1,0} &= \frac{p(x_{1\to0})}{p(x_{1\to0}) + p(x_{1\to1})} \qquad p^x_{1,1} = \frac{p(x_{1\to1})}{p(x_{1\to0}) + p(x_{1\to1})} \end{aligned} \quad (7)$$

❒

Relying on Propositions 1-3, the relationship between all kinds of probabilities can be illustrated as below:

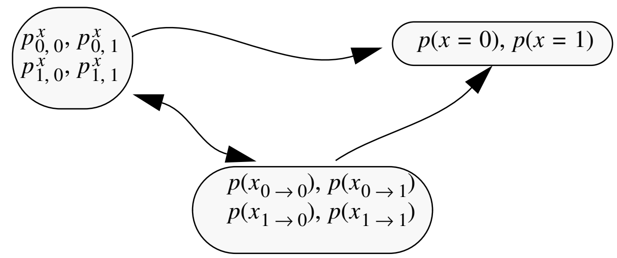

Fig.2

As we can see, we need less information to compute the signal probabilities, but the ability to derive anything else is severely limited; on the other side, once we get either conditional or transition probabilities we have all we need for that particular signal.

Definition 2: For any given line x, the switching activity is:

$$sw(x) = p(x_{0\to1}) + p(x_{1\to0}) = 2\frac{p^x_{1,0} p^x_{0,1}}{p^x_{1,0} + p^x_{0,1}} \quad (8)$$

2.2. Spatial correlations

This type of correlations has two important sources:

- Structural dependencies due to reconvergent fan-out (RFO);
- Pattern dependencies, that is, normally independent signal lines that become correlated due to a particular sequence of inputs.

To take into account the exact correlations is practically impossible even for small circuits. To make this problem more tractable, we allow only pairwise correlated signals, which is undoubtedly an approximation but provides good results in practice. Consequently, we consider the correlations for all 16 possible transitions of a pair of signals (x,y) and model it as a lag-one Markov Chain with 4 states (states 0, 1, 2, 3 which stand for encodings 00, 01, 10, 11 of (x,y)):

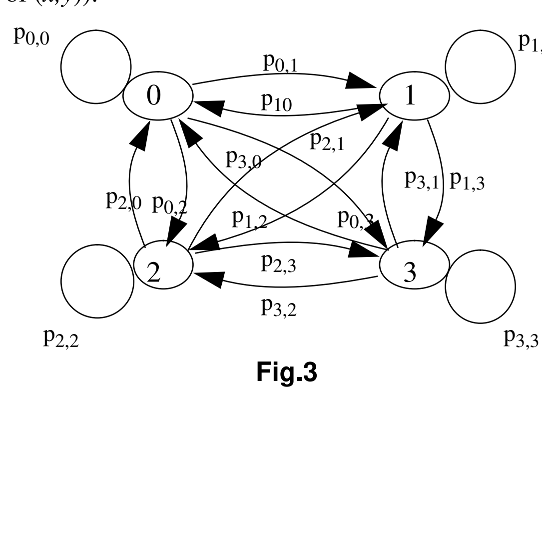

Fig.3

Switching Activity Analysis Considering Spatiotemporal Correlations *

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Abstract

This work presents techniques for computing the switching activities of all circuit nodes under pseudorandom or biased input sequences and assuming a zero delay mode of operation. Complex spatiotemporal correlations among the circuit inputs and internal nodes are considered by using a lag-one Markov Chain model. Evaluations of the model and a comparative analysis presented for benchmark circuits demonstrates the accuracy and the practicality of the method. The results presented in this paper are useful in power estimation and low power design.

1. Introduction

In estimating the power consumption in a digital circuit, knowledge about the average switching activity in the circuit plays a significant part because most of the power in CMOS circuits is consumed during charging and discharging of the load capacitance. To estimate the power consumption, one has to calculate the switching activity factors of the internal nodes of the circuit. The key issue in switching activity estimation is to account for various dependencies, irrespective of the particular way in which the inputs and the target circuits are described.

Common digital circuits exhibit a lot of dependencies; by far, the most known one is the dependency due to reconvergent fan-out among different signal lines, but even structurally independent lines may have dependencies (induced by the sequence of inputs applied to the circuit) that cannot be neglected. Accounting for all kinds of dependencies is impossible even for small circuits; consequently, for real-size circuits, only some of the dependencies have been considered and even then, only heuristics have been proposed. The main reason for this situation is the difficulty in managing complex data dependencies at acceptable levels of computational work.

Methods of estimating the activity factor at a circuit node involve estimation of signal probability. Computing signal probabilities has attracted much attention One of the earliest works in computing the signal probabilities in a combinational network is presented in [1]. For tree circuits which consists of simple gates, the exact signal probabilities can be computed during a single post-order traversal of the network [2]. Alternatively, one may use a graph-based algorithm to compute the exact values of signal probabilities using Shannon's expansion [3]. The cutting algorithm, which computes lower and upper bounds on the signal probability of reconvergent nodes was developed and presented in [4]. Also, the Ordered Binary Decision Diagram representation (OBDD) was used for computing the signal probability in [6] and [7].

The spatial correlations among different signals are modelled in [5] where a procedure is described for propagating signal probabilities from the circuit inputs toward the circuit outputs using only pairwise correlations between signals and ignoring higher order correlation terms.

None of the above mentioned methods adequately capture s*patiotemporal correlations*, that is correlations among logic transitions on two or more circuit lines. The approach proposed in this paper improves the state-of-the-art by a new analytical model which accounts for this kind of correlations. Its mathematical foundation is probabilistic in nature and consists of using lag-one Markov Chains to capture different kinds of depedencies in combinational circuits under a zero-delay model. For the first time to our knowledge, we have considered in a systematic way different kinds of dependencies in large combinational modules for both pseudorandom and biased input streams.

The results presented in this paper are useful in power estimation and low power design. Our approach provides a sound framework for efficiently and accurately estimating the effects of different transformations/optimizations on the power consumption of the circuits under complex spatiotemporal correlations.

The paper is organized as follows. In section 2 we present in detail our model for switching activity estimation and provide a measure of its complexity. In section 3 we give some discussions and our experimental results on benchmark circuits. Finally, we summarize our main results and indicate possible extensions in section 4.

2. An analytical model for dependencies

2.1. Temporal correlations

We treat the sequence that corresponds to different values of a signal line x as a discrete process where time units $1,2,...,n$ represent the time instances when the input vectors $V_1,V_2,...,V_n$ are applied to the circuit under consideration.

* This research was supported in part by the NSF's Research Initiation Award under contract No. MIP-9211668.