# Reconstruction of 3-D Dense Cardiac Motion from Tagged MRI Sequences

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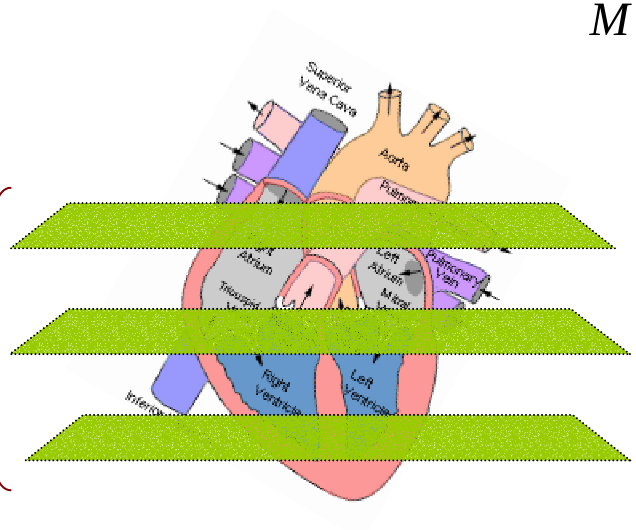
# Outline

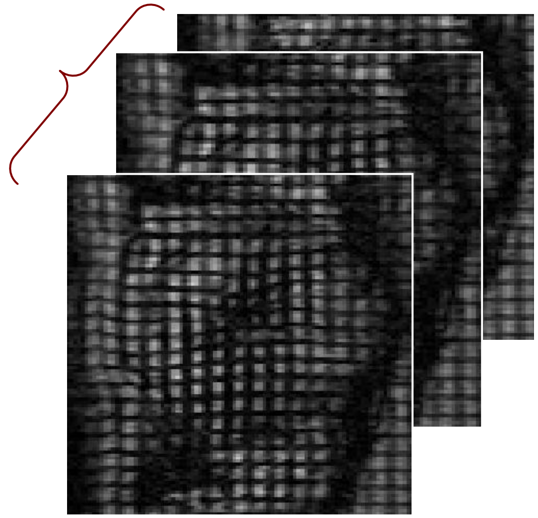
- **Introduction**
- **Methodology: Prior knowledge + MRI data**
  - Myocardial Fiber Based Structure
  - Continuum Mechanics
  - Constrained Energy Minimization
- **Results and Conclusions**

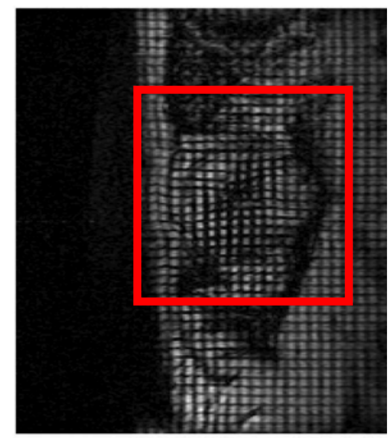
# 2-D Cardiac MRI Images

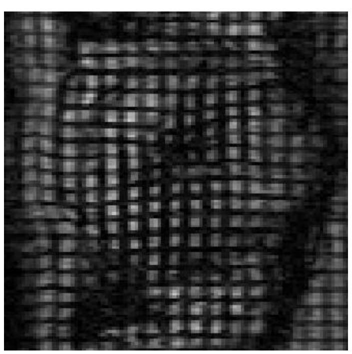
*N* slices

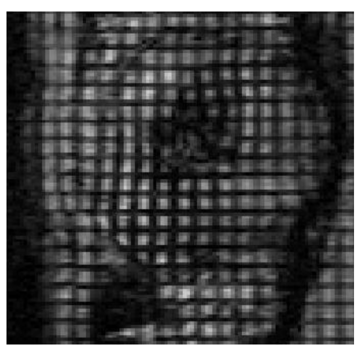


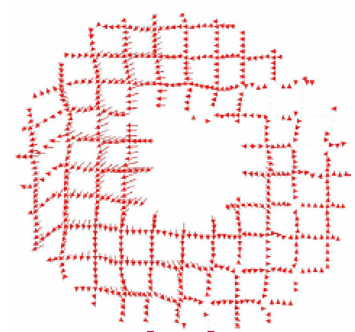

*M* frames per slice



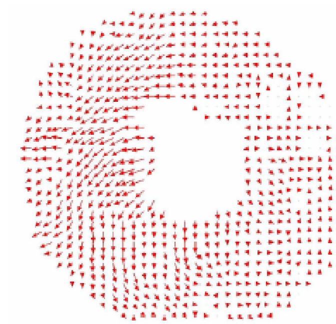








sparse displacements



dense displacements

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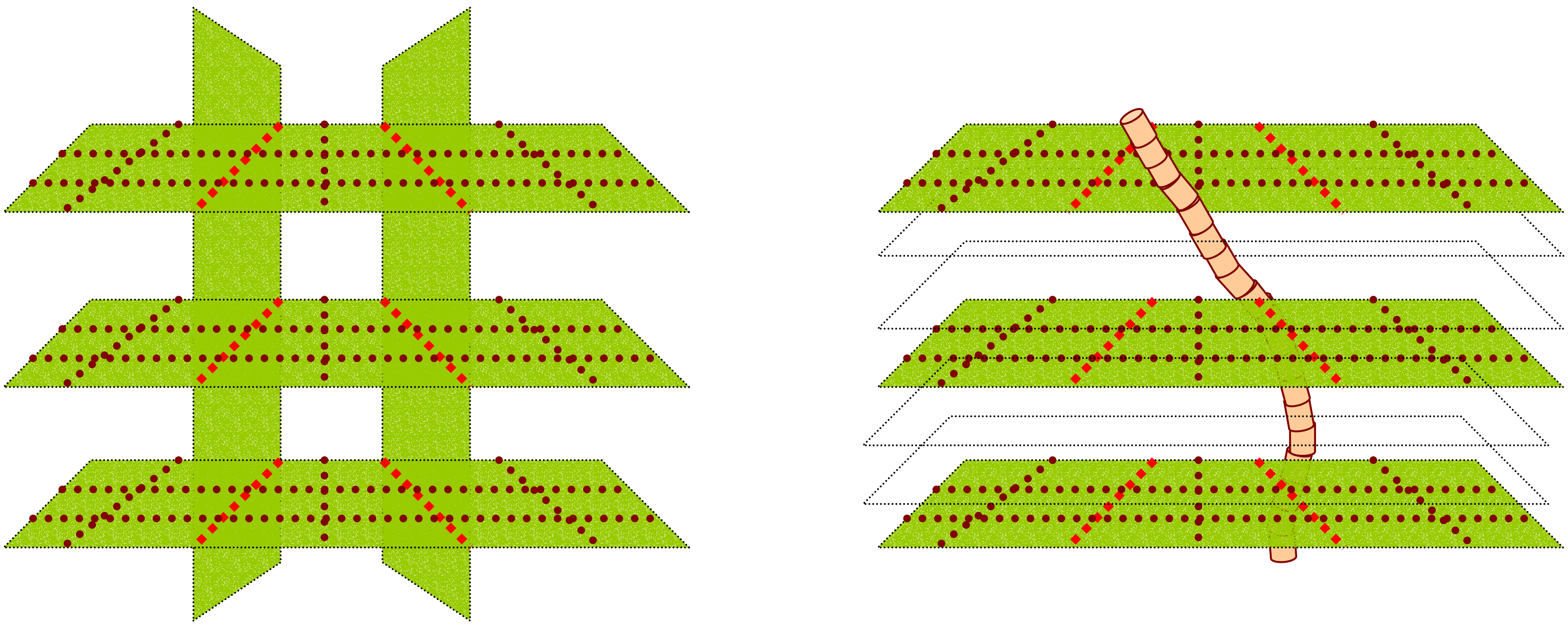
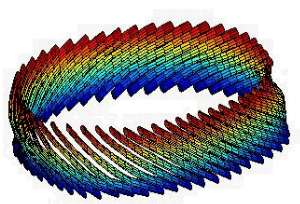
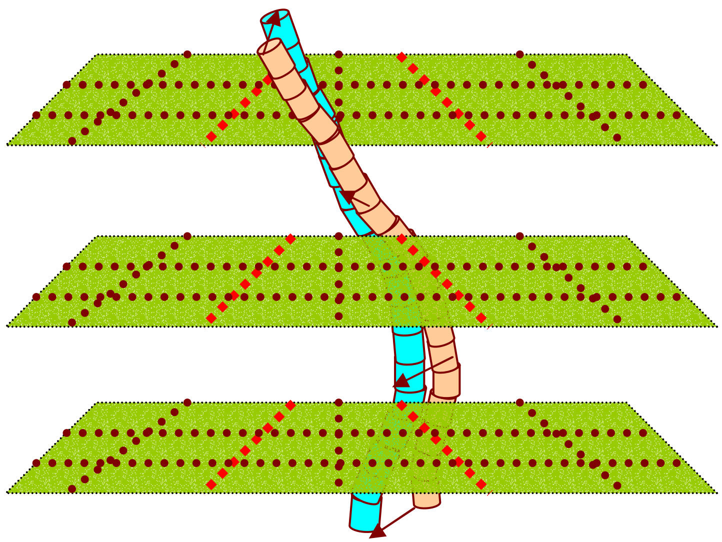
# 3-D Reconstruction: myocardial fiber model

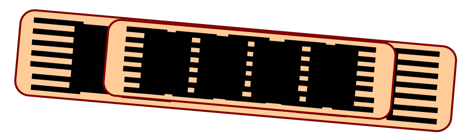


Use a fiber based model to find the correspondence between transversal slices.

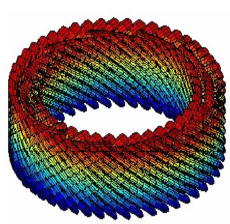
# 3-D Reconstruction: fiber deformation model



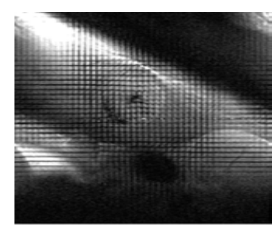




Use continuum mechanics to describe the motion of fibers.



Fit the model to MRI data by constrained energy minimization



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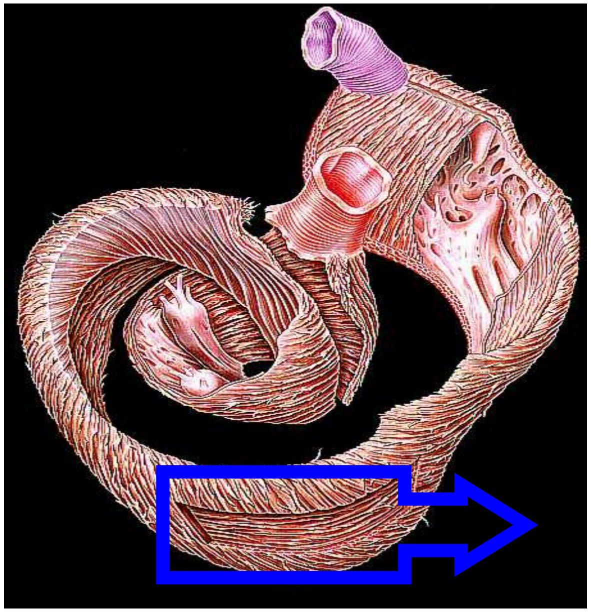
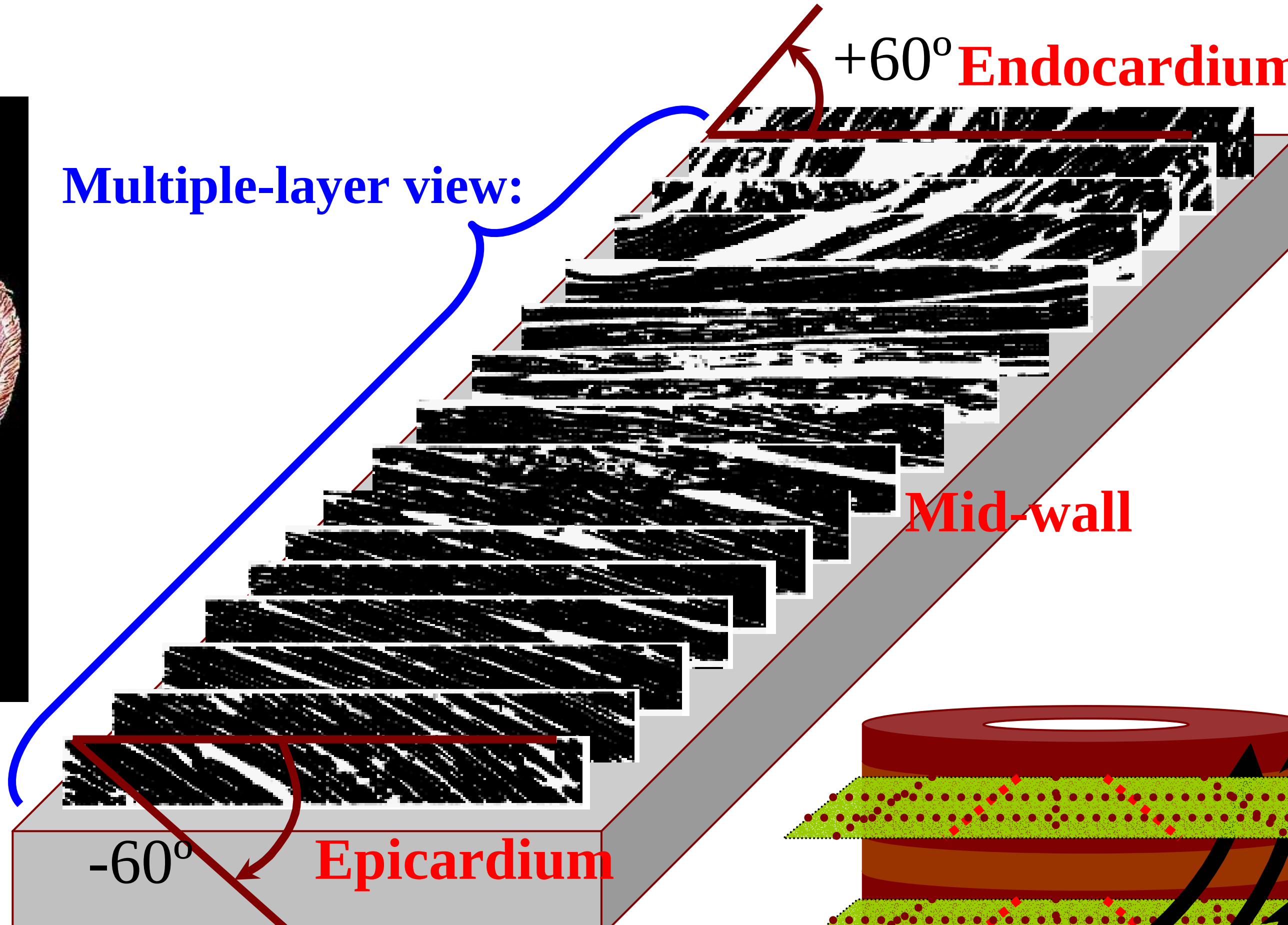
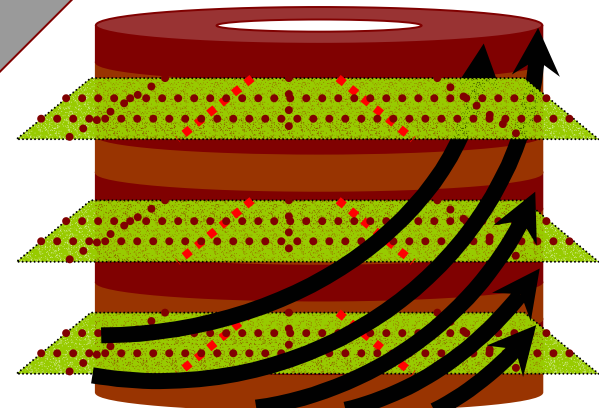
# Prior Knowledge: myocardial anatomy

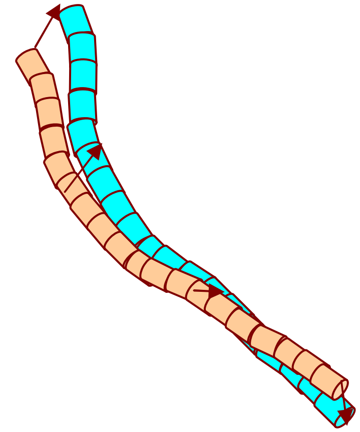




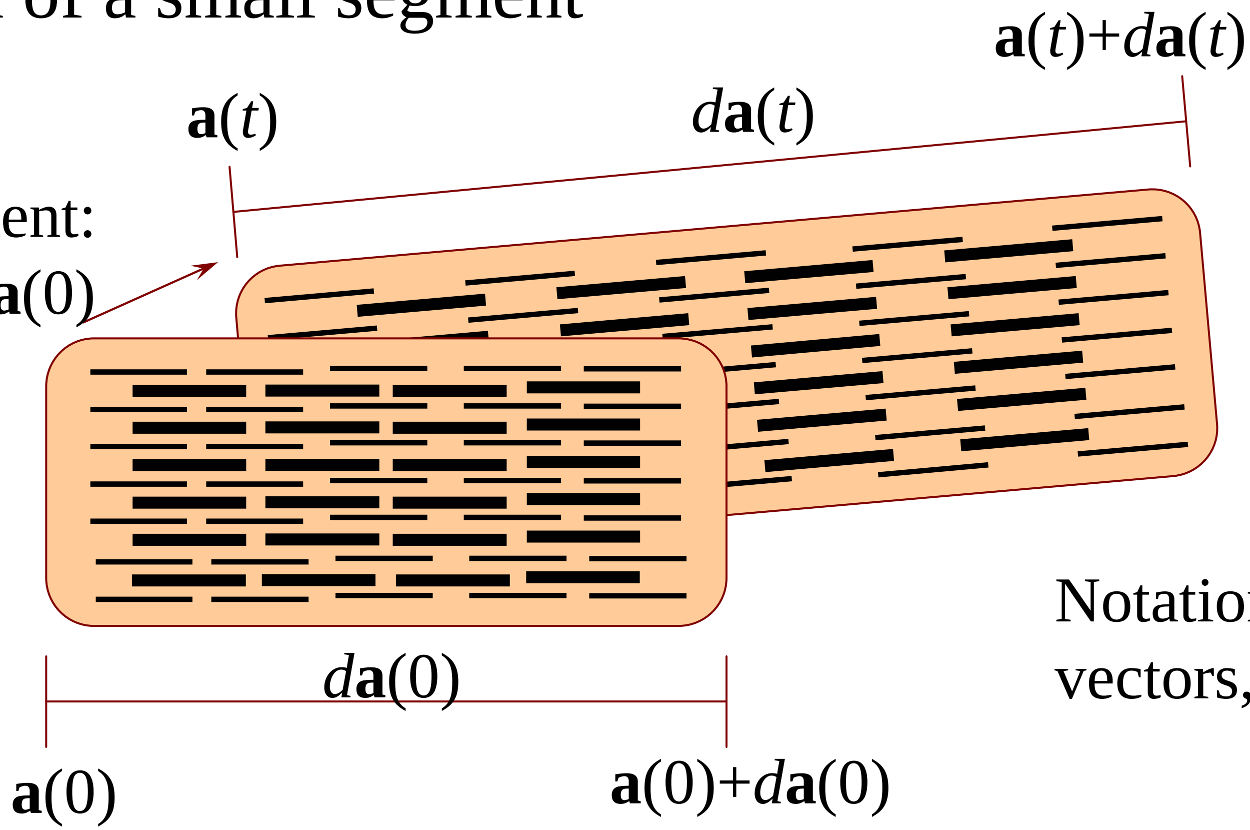
Streeter, in *Handbook of Physiology Volume 1: the Cardiovascular System*, American Physiological Society, 1979

# Prior Knowledge: fiber dynamics



Motion of a small segment

Displacement:
$\mathbf{u}(t)=\mathbf{a}(t)-\mathbf{a}(0)$




$$d\mathbf{a}(t)=\frac{\partial \mathbf{a}(t)}{\partial \mathbf{a}(0)}d\mathbf{a}(0)$$

Notations are column vectors, ex:

$$\mathbf{a}(t)=\begin{bmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \end{bmatrix}$$

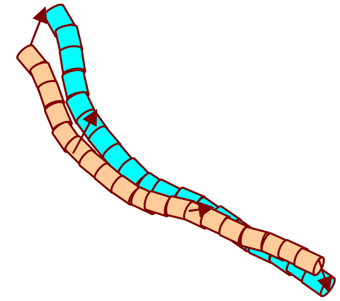
# Deformation Gradient Matrix

$$\mathbf{F}(t) = \frac{\partial \mathbf{a}(t)}{\partial \mathbf{a}(0)} = \begin{bmatrix} \frac{\partial a_1(t)}{\partial a_1(0)} & \frac{\partial a_1(t)}{\partial a_2(0)} & \frac{\partial a_1(t)}{\partial a_3(0)} \\ \frac{\partial a_2(t)}{\partial a_1(0)} & \frac{\partial a_2(t)}{\partial a_2(0)} & \frac{\partial a_2(t)}{\partial a_3(0)} \\ \frac{\partial a_3(t)}{\partial a_1(0)} & \frac{\partial a_3(t)}{\partial a_2(0)} & \frac{\partial a_3(t)}{\partial a_3(0)} \end{bmatrix}$$

$$= \mathbf{I} + d\mathbf{F}(t) = \mathbf{I} + \frac{\partial \mathbf{u}(t)}{\partial \mathbf{a}(0)} = \mathbf{I} + \begin{bmatrix} \frac{\partial u_1(t)}{\partial a_1(0)} & \frac{\partial u_1(t)}{\partial a_2(0)} & \frac{\partial u_1(t)}{\partial a_3(0)} \\ \frac{\partial u_2(t)}{\partial a_1(0)} & \frac{\partial u_2(t)}{\partial a_2(0)} & \frac{\partial u_2(t)}{\partial a_3(0)} \\ \frac{\partial u_3(t)}{\partial a_1(0)} & \frac{\partial u_3(t)}{\partial a_2(0)} & \frac{\partial u_3(t)}{\partial a_3(0)} \end{bmatrix}$$

**Deformation gradient $\mathbf{F}(t)$ is a function of displacement $\mathbf{u}(t)$.**

# Strain



- Strain is the displacement per unit length, and is written mathematically as $\mathbf{S} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$

$$\mathbf{S} = \frac{1}{2}\left[(\mathbf{I} + d\mathbf{F})^T(\mathbf{I} + d\mathbf{F}) - \mathbf{I}\right] = \frac{1}{2}\left[d\mathbf{F}^T + d\mathbf{F} + d\mathbf{F}^T d\mathbf{F}\right]$$

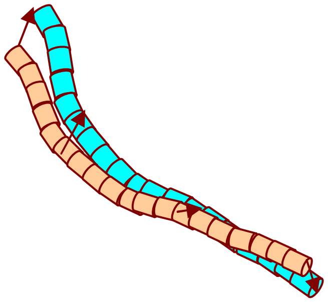
Ref: Y.C. Fung, *A First Course in Continuum Mechanics*, 3rd ed., Prentice-Hall, New Jersey, 1994

- When strain is small, it is approximated as

$$\mathbf{S} \approx \frac{1}{2}\left[d\mathbf{F}^T + \mathbf{I} + d\mathbf{F} + \mathbf{I}\right] - \mathbf{I} = \frac{1}{2}(\mathbf{F}^T + \mathbf{F}) - \mathbf{I}$$

(**Note:** **S** is symmetric)

# Linear Strain Energy Model



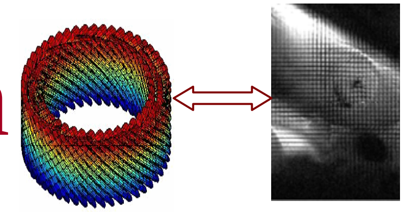
- **S** is symmetric, so we vectorize the entries at upper triangle.

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ & S_{22} & S_{23} \\ & & S_{33} \end{bmatrix} \rightarrow \mathbf{s} = [S_{11}, S_{22}, S_{33}, S_{12}, S_{13}, S_{23}]^T$$

- Let **C** describe the material properties. It can be shown the linear strain energy is $e = \mathbf{s}^T \mathbf{C} \mathbf{s} = e(\mathbf{u})$
- The entire energy of the heart:

$$E(\mathbf{U}) = \sum_{\forall \text{fibers}} \sum_{\forall \text{segments}} e(\mathbf{u}) = \sum_{\forall \text{fibers}} \sum_{\forall \text{segments}} \mathbf{s}^T \mathbf{C} \mathbf{s}$$

# Constrained Energy Minimization



$$E(\mathbf{U},\lambda) = \gamma_1 E_{int}(\mathbf{U}) + \gamma_2 E_{ext}(\mathbf{U}) + \lambda E_{con}(\mathbf{U})$$

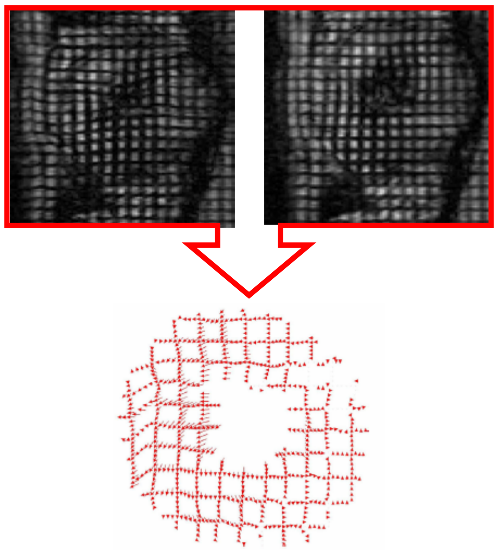
$$E_{int}(\mathbf{U}) = \sum_{\forall \text{fibers}} \sum_{\forall \text{segments}} \mathbf{s}^{\mathrm{T}}\mathbf{C}\mathbf{s}$$

$$E_{ext}(\mathbf{U}) = \left\| \mathbf{I}(t) - \mathbf{I}(t+1) \right\|^2$$

- Internal energy: continuum mechanics governs the fibers to move as smooth as possible.
- External energy: pixel intensities of fibers should be kept similar across time.

# 2-D Displacement Constraints

$$E(\mathbf{U},\lambda) = \gamma_1 E_{int}(\mathbf{U}) + \gamma_2 E_{ext}(\mathbf{U}) + \lambda E_{con}(\mathbf{U})$$



**D**: 2-D displacements of the taglines

**ΘU**: picks the entries of **U** corresponding to **D**

2-D displacement constraints: **ΘU=D**

λ **:** Lagrange multiplier

$$E(\mathbf{U},\lambda) = \gamma_1 E_{int}(\mathbf{U}) + \gamma_2 E_{ext}(\mathbf{U}) + \lambda \left\| \mathbf{\Theta U} - \mathbf{D} \right\|^2$$

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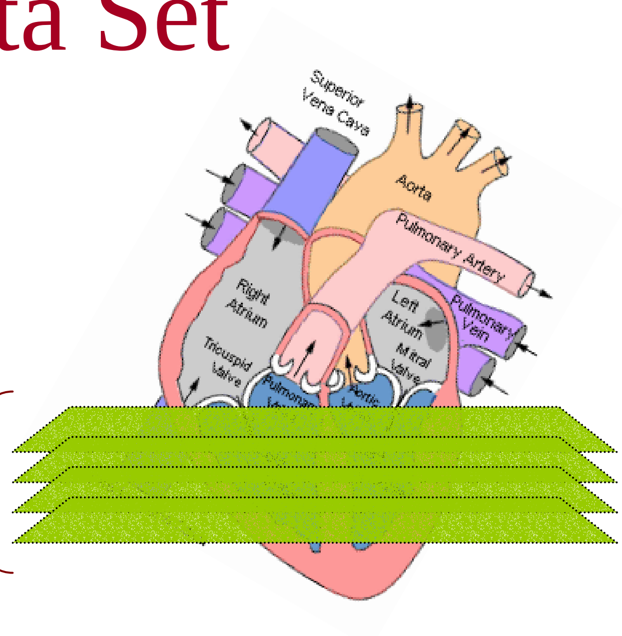
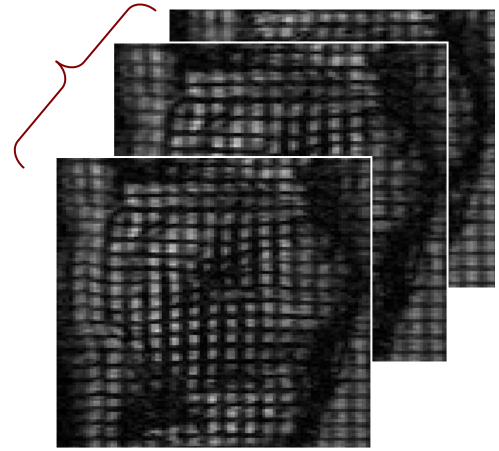
# Data Set

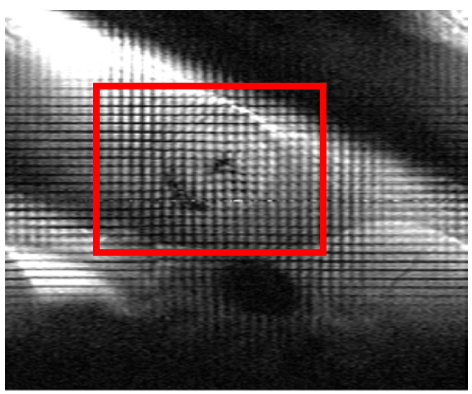
*4* slices



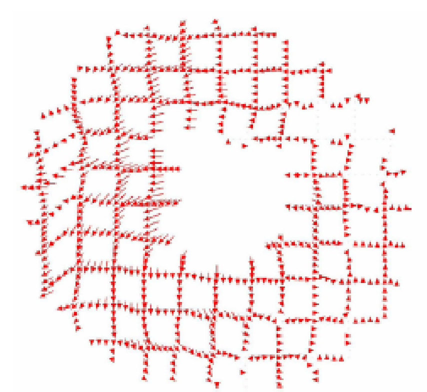

*10* frames per slice



256×256 pixels per image



- Transplanted rats with heterotropic working hearts.
- MRI scans performed on a Bruker AVANCE DRX 4.7-T system



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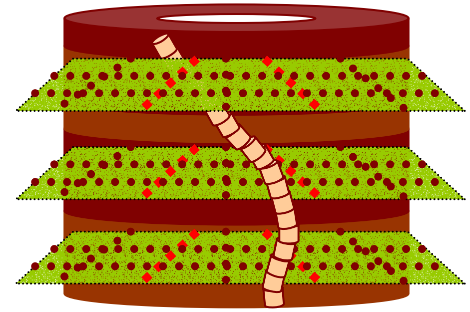
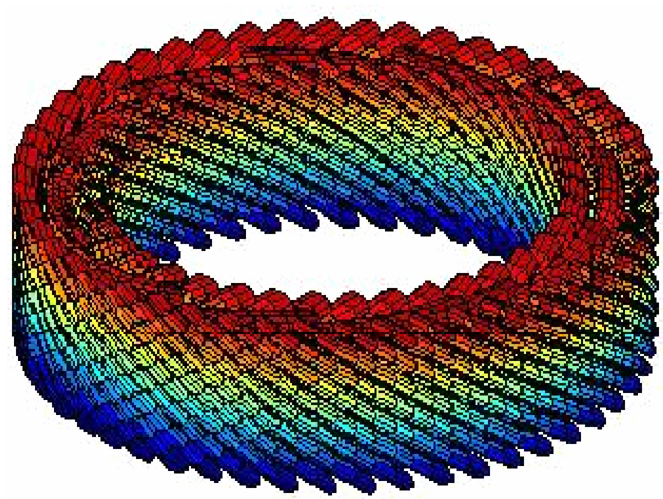
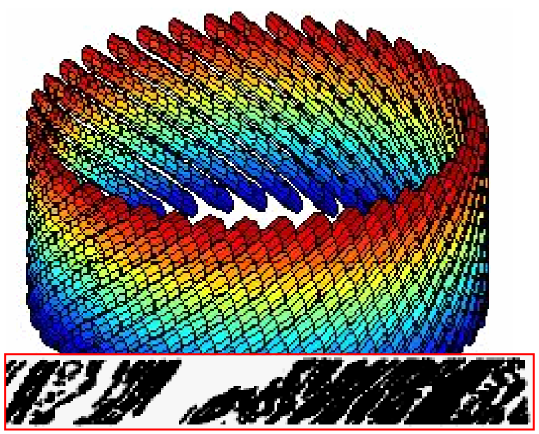
# Fiber Based Model

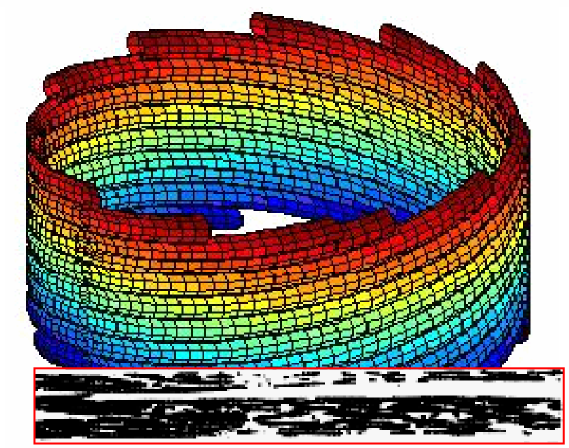




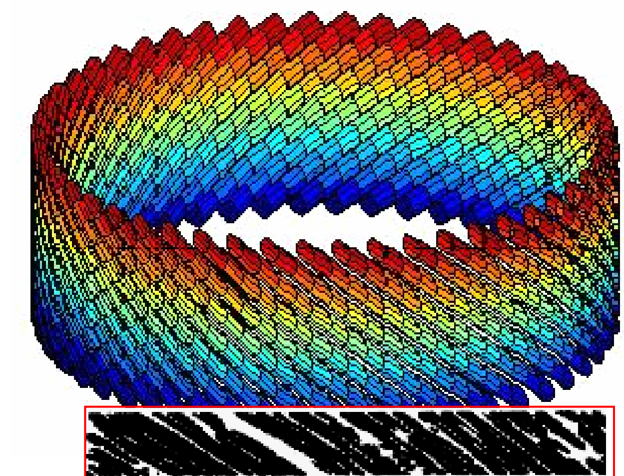
Whole left ventricle



endocardium

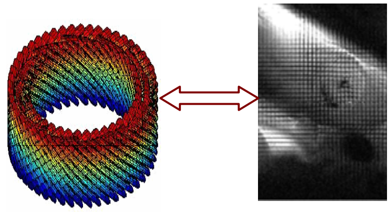


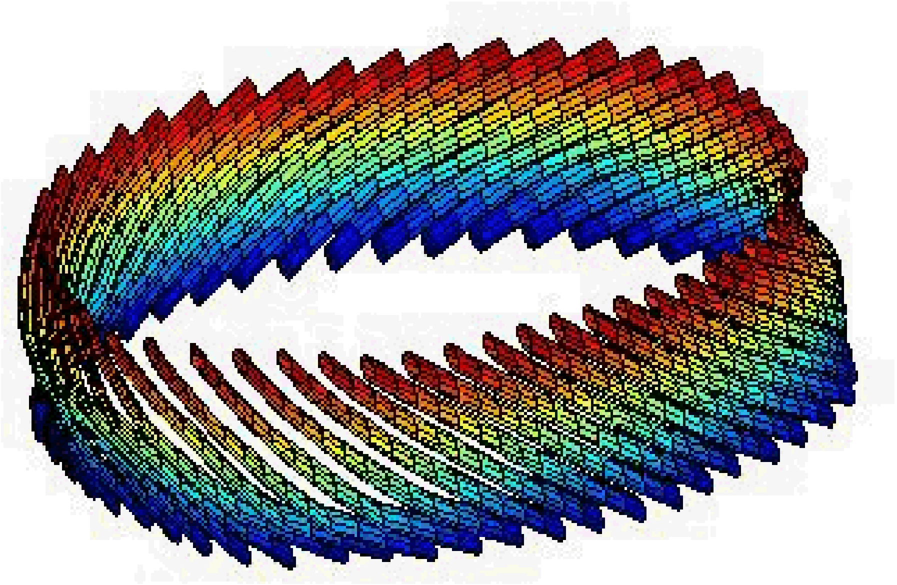
mid-wall



epicardium

# 3-D Reconstruction of the Epicardium





# Conclusions

- Take into account the *myocardial fiber based structure*.
- Adopt the *continuum mechanics* framework.
- Implement *constrained energy minimization* algorithms.