Reinforcement Learning meets Federated Learning and Distributional Robustness

Yuejie Chi

Carnegie Mellon University

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Reinforcement learning (RL)

In RL, an agent learns by interacting with an environment.

- unknown environments
- maximize total rewards
- trial-and-error
- sequential and online



"Recalculating ... recalculating ..."

Recent successes in RL











RL holds great promise in the next era of artificial intelligence.

Sample efficiency

Collecting data samples might be expensive or time-consuming due to the enormous state and action space



clinical trials



autonomous driving



online ads

Sample efficiency

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clinical trials



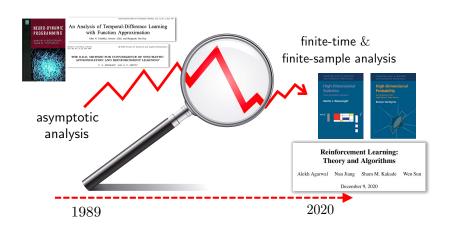
autonomous driving



online ads

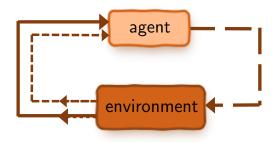
Calls for design of sample-efficient RL algorithms!

Statistical thinking in RL: non-asymptotic analysis



Non-asymptotic analyses are key to understand statistical efficiency in modern RL.

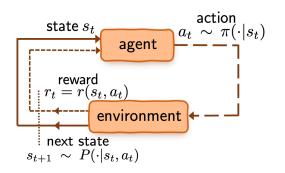
Recent advances in statistical RL



The playground: Markov decision processes



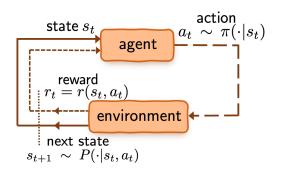
Backgrounds: Markov decision processes





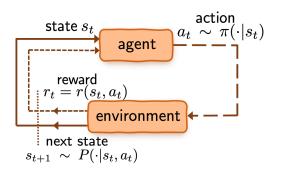
• \mathcal{S} : state space

A: action space



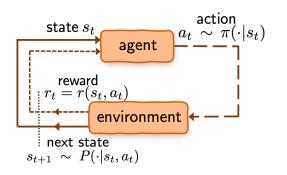


- S: state space A: action space
- $r(s, a) \in [0, 1]$: immediate reward





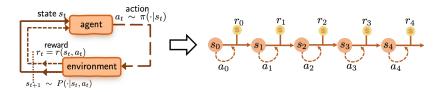
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- $\pi(\cdot|s)$: policy (or action selection rule)





- \mathcal{S} : state space \mathcal{A} : action space
- $r(s,a) \in [0,1]$: immediate reward
- $\pi(\cdot|s)$: policy (or action selection rule)
- $P(\cdot|s,a)$: transition probabilities

Value function



Value function of policy π :

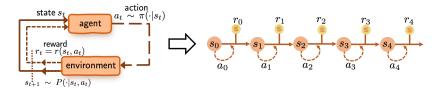
$$\forall s \in \mathcal{S}: \qquad V^{\pi}(s) := \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^{t} r_{t} \,\middle|\, s_{0} = s\right]$$

Q-function of policy π :

$$\forall (s, a) \in \mathcal{S} \times \mathcal{A}: \quad Q^{\pi}(s, a) := \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^{t} r(s_{t}, a_{t}) \,\middle|\, s_{0} = s, \frac{a_{0}}{a_{0}} = a\right]$$

8

Value function



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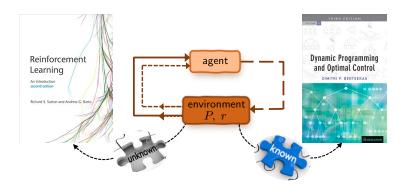
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- $\gamma \in [0,1)$ is the discount factor; $\frac{1}{1-\gamma}$ is effective horizon
- Expectation is w.r.t. the sampled trajectory under π

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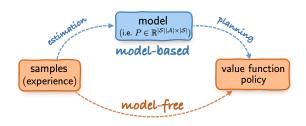
Searching for the optimal policy



Goal: find the optimal policy π^* that maximize $V^{\pi}(s)$

- optimal value / Q function: $V^\star := V^{\pi^\star}$, $Q^\star := Q^{\pi^\star}$
- optimal policy $\pi^{\star}(s) = \operatorname{argmax}_{a \in \mathcal{A}} Q^{\star}(s, a)$

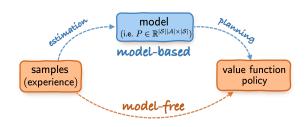
Two approaches to RL



Model-based approach ("plug-in")

- 1. build an empirical estimate \widehat{P} for P
- 2. planning based on empirical \widehat{P}

Two approaches to RL



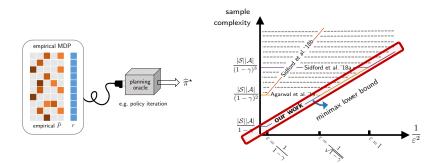
Model-based approach ("plug-in")

- 1. build an empirical estimate \widehat{P} for P
- 2. planning based on empirical \widehat{P}

Model-free approach

- 1. learning w/o constructing model explicitly
- 2. memory-efficient

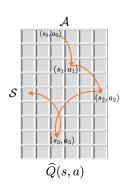
Recent advances in model-based RL

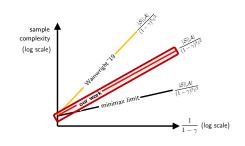


Plug-in estimators are minimax-optimal

(Sidford et al., 2018; Agarwal et al., 2019; Wang 2019; Li et al., 2020)

Recent advances in model-free RL

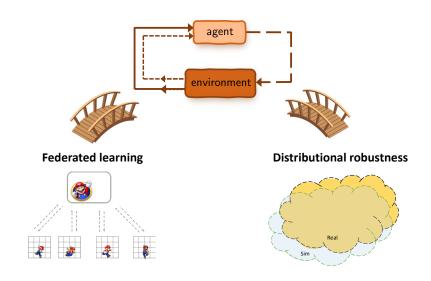




Q-learning is not minimax-optimal

(Even-Dar and Mansour, 2013; Wainwright, 2019; Chen et al., 2020; Li et al., 2021)

This talk: beyond standard MDP



Reinforcement learning meets federated learning: linear speedup and beyond



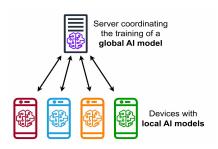
Jiin Woo CMU



Gauri Joshi CMU

"The Blessing of Heterogeneity in Federated Q-Learning: Linear Speedup and Beyond," arXiv:2305.10697, short version at ICML 2023.

Federated learning



IBM Federated Learning Research - Extracting Machine Learning Models From Multiple Data Pools

Kevin Krewell Contributor
Tirias Research Contributor Group ©

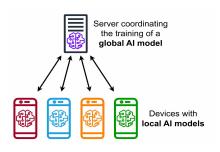
FORRES > INNOVATION > AL



How Apple personalizes Siri without hoovering up your data
The techjant is using privacy-preserving machine learning to improve its voice assistant while keeping your data on your phone.

By Karen Hao
December II. 2019

Federated learning



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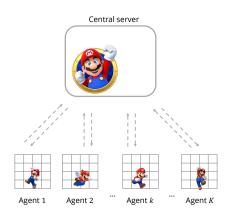
How Apple personalizes Siri without hoovering up your data

The tech giant is using privacy-preserving machine learning to mprove its voice assistant while keeping your data on your phone.

December 11, 2019

Can we harness the power of federated learning for RL?

RL meets federated learning



Federated reinforcement learning: enables multiple agents to collaboratively learn a global policy without sharing datasets.

Questions

Understand the sample complexity of Q-Learning in federated settings.

Linear speedup:

Can we achieve linear speedup when learning with multiple agents?

Communication efficiency:

Can we perform multiple local updates to save communication?

Taming heterogeneity:

How to combine heterogeneous local updates to accelerate learning?

Q-learning: a classical model-free algorithm





Chris Watkins

Peter Dayan

Stochastic approximation for solving the Bellman equation

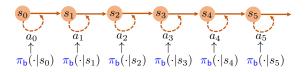
Robbins & Monro, 1951

$$Q^{\star} = \mathcal{T}(Q^{\star})$$

where

$$\mathcal{T}(Q)(s,a) := \underbrace{r(s,a)}_{\text{immediate reward}} + \gamma \mathop{\mathbb{E}}_{s' \sim P(\cdot \mid s,a)} \left[\underbrace{\max_{a' \in \mathcal{A}} Q(s',a')}_{\text{next state's value}} \right].$$

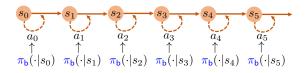
Asynchronous Q-learning



Stochastic approximation for solving Bellman equation $Q^\star = \mathcal{T}(Q^\star)$ using samples collected from a behavior policy π_{b} :

$$\underbrace{Q_{t+1}(s_t, a_t) = (1 - \eta)Q_t(s_t, a_t) + \eta \mathcal{T}_t(Q_t)(s_t, a_t)}_{\text{only update } (s_t, a_t) \text{-th entry}}, \quad t \geq 0$$

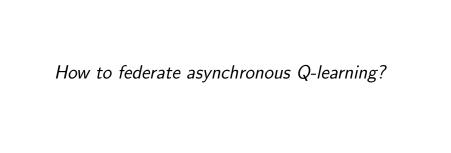
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$$\begin{split} \mathcal{T}_t(Q)(s_t, a_t) &= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a') \\ \mathcal{T}(Q)(s, a) &= r(s, a) + \gamma \mathop{\mathbb{E}}_{s' \sim P(\cdot \mid s, a)} \left[\max_{a'} Q(s', a') \right] \end{split}$$

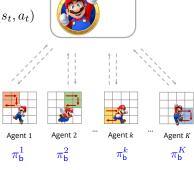


Federated asynchronous Q-learning with local updates

• The agent k performs τ rounds of local Q-learning updates:

$$Q_{t+1}^k(s_t, a_t) \leftarrow (1 - \eta)Q_t^k(s_t, a_t) + \eta \mathcal{T}_t(Q_t^k)(s_t, a_t)$$

and sends it to the server.



Central server

Federated asynchronous Q-learning with local updates

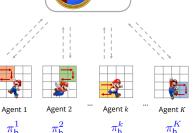
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and sends it to the server.

 The server averages the local updates and communicates it back to agents:

$$Q_t = \frac{1}{K} \sum_{k=1}^K Q_t^k$$



Central server

Federated asynchronous Q-learning with local updates

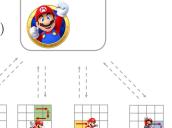
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Agent k

Central server

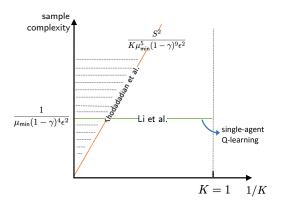
Can we achieve faster convergence with heterogeneous local behavior policies with low communication complexity?

Agent 1

Agent 2

Agent K

Prior art

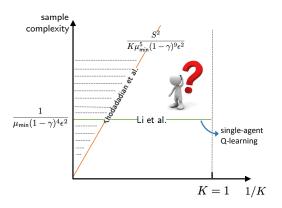


Key quantity: minimum state-action occupancy probability

$$\mu_{\min} := \min_{i,s,a} \ \underbrace{\mu_{\pi_{\mathrm{b}}^i}(s,a)}_{\text{stationary distribution}}$$

The benefit of linear speedup only becomes effective $K\gg \frac{S^2}{\mu_{\min}^4(1-\gamma)^5}$

Prior art



Key quantity: minimum state-action occupancy probability

$$\mu_{\min} := \min_{i,s,a} \ \underbrace{\mu_{\pi_{\mathrm{b}}^i}(s,a)}_{\text{stationary distribution}}$$

Can we improve the dependency on the salient parameters?

Our theorem

Theorem (Jiin, Joshi, Chi, ICML 2023)

For sufficiently small $\epsilon>0$, federated asynchronous Q-learning yields $\|\widehat{Q}-Q^\star\|_\infty \leq \epsilon$ with sample complexity at most

$$\widetilde{O}\left(\frac{C_{\mathsf{het}}}{K\mu_{\mathsf{min}}(1-\gamma)^5\epsilon^2}\right)$$

ignoring the burn-in cost that depends on the mixing times, where

$$C_{\mathsf{het}} = K \max_{k,s,a} \frac{\mu_{\mathsf{b}}^k(s,a)}{\sum_{k=1}^K \mu_{\mathsf{b}}^k(s,a)}.$$

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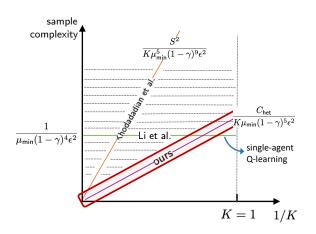
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$$C_{\text{het}} = K \max_{k,s,a} \frac{\mu_{\text{b}}^{k}(s,a)}{\sum_{k=1}^{K} \mu_{\text{b}}^{k}(s,a)}.$$

- $1 \leq C_{\rm het} \leq \frac{1}{\mu_{\rm min}}$ measures the heterogeneity of local behavior policies.
- Near-optimal linear speedup when the local behavior policies are similar, $C_{\rm het} \approx 1$.

Comparison with prior art



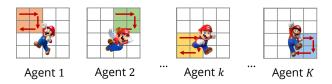
Linear speedup with near-optimal parameter dependencies!

• Curse of heterogeneity? performance degenerates when local behavior policies are heterogeneous (i.e. $C_{\rm het}\gg 1$).

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- Full coverage: require full coverage of every agent over the entire state-action space (i.e. $\mu_{\min} > 0$).

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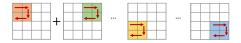
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- Full coverage: require full coverage of every agent over the entire state-action space (i.e. $\mu_{\min} > 0$).



Is it possible to alleviate these requirements?

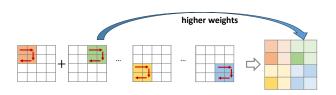
Importance averaging

Key observation: not all updates are of same quality due to limited visits induced by the behavior policy.



Importance averaging

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Importance averaging: the server averages the local updates based on importance via

$$Q_{t}(s, a) = \frac{1}{K} \sum_{k=1}^{K} \alpha_{t}^{k}(s, a) Q_{t}^{k}(s, a),$$

where

$$\alpha_t^k = \frac{(1-\eta)^{-N_{t-\tau,t}^k(s,a)}}{\sum_{k=1}^K (1-\eta)^{-N_{t-\tau,t}^k(s,a)}}, \quad N_{t-\tau,t}^k(s,a) = \quad \text{number of visits} \quad \text{in the sync period} \ .$$

Our theorem

Theorem (Jiin, Joshi, Chi, ICML 2023)

For sufficiently small $\epsilon>0$, federated asynchronous Q-learning with importance averaging yields $\|\widehat{Q}-Q^\star\|_\infty \leq \epsilon$ with sample complexity at most

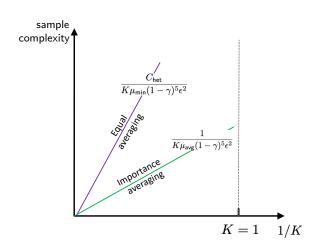
$$\widetilde{O}\left(\frac{1}{K\mu_{\mathsf{avg}}(1-\gamma)^5\epsilon^2}\right)$$

ignoring the burn-in cost that depends on the mixing times, where

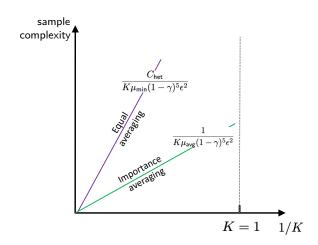
$$\mu_{\text{avg}} = \min_{s,a} \frac{1}{K} \sum_{k=1}^{K} \mu_{\text{b}}^{k}(s,a) \ge \mu_{\text{min}}.$$

 Linear speedup without requiring local behavior policies to cover the entire state-action space, as long as they collectively cover the entire state-action space.

Equal averaging versus importance averaging

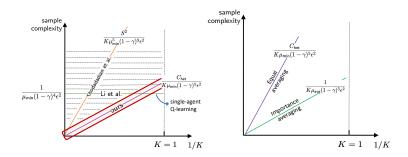


Equal averaging versus importance averaging



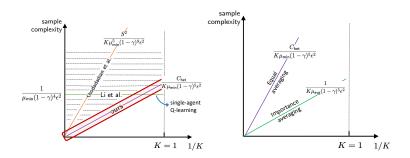
Importance averaging does not require full coverage of individual agents!

Summary



Provable benefits of federated Q-learning: near-optimal linear speedup!

Summary



Provable benefits of federated Q-learning: near-optimal linear speedup!

Ongoing and future work:

- Other problems in RL such as policy evaluation and offline RL.
- Multi-task RL: heterogeneous environments across agents.

RL meets distributional robustness: towards minimax-optimal sample complexity



Laixi Shi Caltech



Gen Li UPenn



Yuxin Chen UPenn



Yuting Wei UPenn



Matthieu Geist Google

"The Curious Price of Distributional Robustness in Reinforcement Learning with a Generative Model," arXiv:2305.16589.

Safety and robustness in RL

(Zhou et al., 2021; Panaganti and Kalathil, 2022; Yang et al., 2022;)



Training environment



Test environment

Safety and robustness in RL

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Training environment

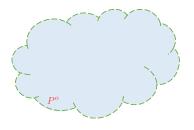


Test environment

Sim2Real Gap: Can we learn optimal policies that are robust to model perturbations?

Uncertainty set of the nominal transition kernel P^o :

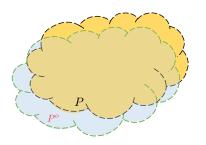
$$\mathcal{U}^{\sigma}(\underline{P}^{o}) = \{P : \rho(P, \underline{P}^{o}) \leq \sigma\}$$





Uncertainty set of the nominal transition kernel P^o :

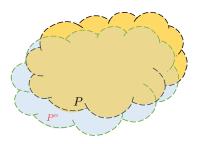
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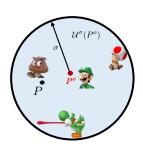




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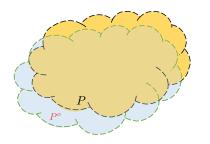
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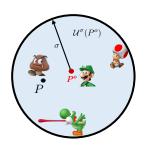




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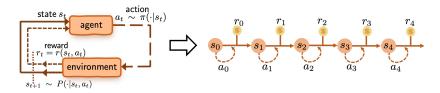
$$\mathcal{U}^{\sigma}(\underline{P}^{o}) = \{P : \rho(P, \underline{P}^{o}) \leq \sigma\}$$





• Examples of ρ : f-divergence (TV, χ^2 , KL...)

Robust value/Q function



Robust value/Q function of policy π :

$$\forall s \in \mathcal{S}: \qquad V^{\pi,\sigma}(s) := \inf_{P \in \mathcal{U}^{\sigma}(P^{o})} \mathbb{E}_{\pi,P} \left[\sum_{t=0}^{\infty} \gamma^{t} r_{t} \mid s_{0} = s \right]$$

$$\forall (s,a) \in \mathcal{S} \times \mathcal{A}: \quad Q^{\pi,\sigma}(s,a) := \inf_{P \in \mathcal{U}^{\sigma}(P^{o})} \mathbb{E}_{\pi,P} \left[\sum_{t=0}^{\infty} \gamma^{t} r_{t} \mid s_{0} = s, a_{0} = a \right]$$

Measures the worst-case performance of the policy in the uncertainty set.

Distributionally robust MDP

Robust MDP

Find the policy π^{\star} that maximizes $V^{\pi,\sigma}$

(Iyengar. '05, Nilim and El Ghaoui. '05)

Distributionally robust MDP

Robust MDP

Find the policy π^* that maximizes $V^{\pi,\sigma}$

(Iyengar. '05, Nilim and El Ghaoui. '05)

Robust Bellman's optimality equation: the optimal robust policy π^\star and optimal robust value $V^{\star,\sigma}:=V^{\pi^\star,\sigma}$ satisfy

$$\begin{split} Q^{\star,\sigma}(s,a) &= r(s,a) + \gamma \inf_{P_{s,a} \in \mathcal{U}^{\sigma}\left(P_{s,a}^{o}\right)} \left\langle P_{s,a}, V^{\star,\sigma} \right\rangle, \\ V^{\star,\sigma}(s) &= \max_{a} \, Q^{\star,\sigma}(s,a) \end{split}$$

Distributionally robust MDP

Robust MDP

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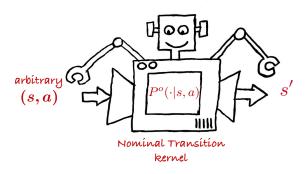
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Distributionally robust value iteration (DRVI):

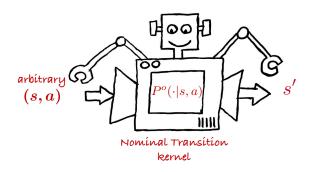
$$Q(s,a) \leftarrow r(s,a) + \gamma \inf_{P_{s,a} \in \mathcal{U}^{\sigma}\left(P_{s,a}^{o}\right)} \langle P_{s,a}, V \rangle,$$

where $V(s) = \max_a Q(s, a)$.

Learning distributionally robust MDPs



Learning distributionally robust MDPs

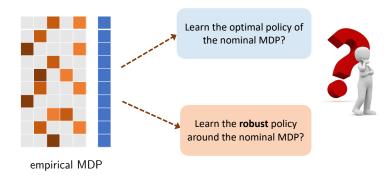


Goal of robust RL: given $\mathcal{D} := \{(s_i, a_i, s_i')\}_{i=1}^N$ from the *nominal* environment P^0 , find an ϵ -optimal robust policy $\widehat{\pi}$ obeying

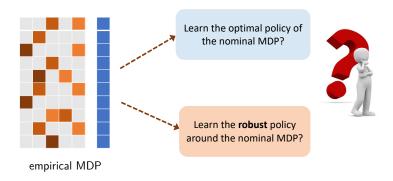
$$V^{\star,\sigma} - V^{\widehat{\pi},\sigma} \leq \epsilon$$

— in a sample-efficient manner

A curious question

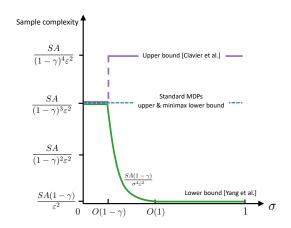


A curious question



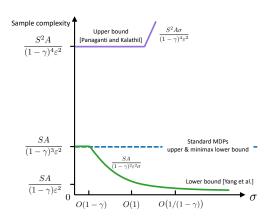
Robustness-statistical trade-off? Is there a statistical premium that one needs to pay in quest of additional robustness?

Prior art: TV uncertainty



- Large gaps between existing upper and lower bounds
- Unclear benchmarking with standard MDP

Prior art: χ^2 uncertainty



- · Large gaps between existing upper and lower bounds
- Unclear benchmarking with standard MDP

Our theorem under TV uncertainty

Theorem (Shi et al., 2023)

Assume the uncertainty set is measured via the TV distance with radius $\sigma \in [0,1)$. For sufficiently small $\epsilon > 0$, DRVI outputs a policy $\widehat{\pi}$ that satisfies $V^{\star,\sigma} - V^{\widehat{\pi},\sigma} \leq \epsilon$ with sample complexity at most

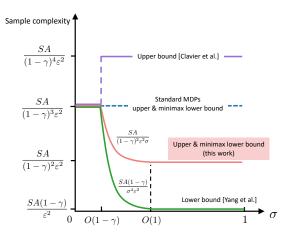
$$\widetilde{O}\left(\frac{SA}{(1-\gamma)^2 \max\{1-\gamma,\sigma\}\epsilon^2}\right)$$

ignoring logarithmic factors. In addition, no algorithm can succeed if the sample size is below

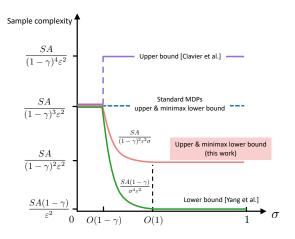
$$\widetilde{\Omega}\left(\frac{SA}{(1-\gamma)^2\max\{1-\gamma,\sigma\}\epsilon^2}\right).$$

 Establish the minimax optimality of DRVI for RMDP under the TV uncertainty set over the full range of σ.

When the uncertainty set is TV



When the uncertainty set is TV



RMDPs are easier to learn than standard MDPs.

Our theorem under χ^2 uncertainty

Theorem (Upper bound, Shi et al., 2023)

Assume the uncertainty set is measured via the χ^2 divergence with radius $\sigma \in [0,\infty)$. For sufficiently small $\epsilon>0$, DRVI outputs a policy $\widehat{\pi}$ that satisfies $V^{\star,\sigma}-V^{\widehat{\pi},\sigma} \leq \epsilon$ with sample complexity at most

$$\widetilde{O}\left(\frac{SA(1+\sigma)}{(1-\gamma)^4\epsilon^2}\right)$$

ignoring logarithmic factors.

Our theorem under χ^2 uncertainty

Theorem (Upper bound, Shi et al., 2023)

Assume the uncertainty set is measured via the χ^2 divergence with radius $\sigma \in [0,\infty)$. For sufficiently small $\epsilon>0$, DRVI outputs a policy $\widehat{\pi}$ that satisfies $V^{\star,\sigma}-V^{\widehat{\pi},\sigma} \leq \epsilon$ with sample complexity at most

$$\widetilde{O}\left(\frac{SA(1+\sigma)}{(1-\gamma)^4\epsilon^2}\right)$$

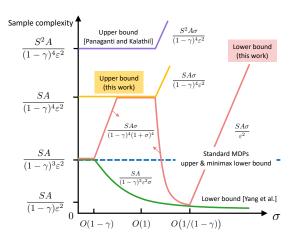
ignoring logarithmic factors.

Theorem (Lower bound, Shi et al., 2023)

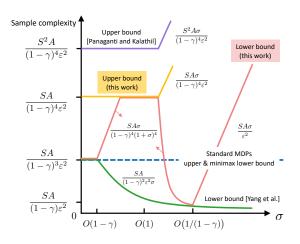
In addition, no algorithm succeeds when the sample size is below

$$\left\{ \begin{array}{ll} \widetilde{\Omega}\left(\frac{SA}{(1-\gamma)^3\epsilon^2}\right) & \text{if } \sigma \lesssim 1-\gamma \\ \widetilde{\Omega}\left(\frac{\sigma SA}{\min\{1,(1-\gamma)^4(1+\sigma)^4\}\epsilon^2}\right) & \text{otherwise} \end{array} \right.$$

When the uncertainty set is χ^2 divergence

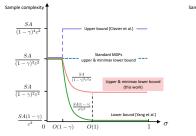


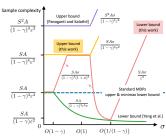
When the uncertainty set is χ^2 divergence



RMDPs can be harder to learn than standard MDPs.

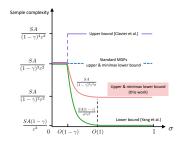
Summary

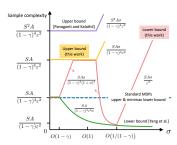




The price of robustness varies: choice of the uncertainty set matters.

Summary





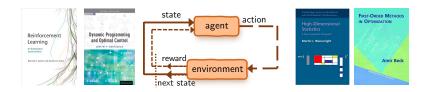
The price of robustness varies: choice of the uncertainty set matters.

Ongoing and future work:

- Other choices of uncertainty sets: KL divergence.
- Function approximation.



Concluding remarks



Understanding non-asymptotic performances of RL algorithms sheds light to their empirical successes (and failures)!

Thanks!

- The Blessing of Heterogeneity in Federated Q-Learning: Linear Speedup and Beyond, arXiv: 2305.10697. Short version at ICML 2023.
- The Curious Price of Distributional Robustness in Reinforcement Learning with a Generative Model, arXiv:2305.16589.





https://users.ece.cmu.edu/~yuejiec/