Coping with Heterogeneity and Privacy in Communication-Efficient Federated Optimization

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FedVision@CVPR Jun. 2022

Acknowledgements





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Empirical Risk Minimization (ERM)

Given a set of data \mathcal{M} ,

$$\mathsf{minimize}_{\boldsymbol{x}} \quad f(\boldsymbol{x}) = \frac{1}{N} \sum_{\boldsymbol{z} \in \mathcal{M}} \ell(\boldsymbol{x}; \boldsymbol{z})$$

Here, N = number of total samples.

- convex: least squares, logistic regression
- non-convex: PCA, training neural networks (focus of this talk)



Distributed ERM

Distributed/Federated learning: due to privacy and scalability, data are distributed at multiple locations / workers / agents.

Let $\mathcal{M} = \cup_i \mathcal{M}_i$ be a data partition with equal splitting:

$$f(\boldsymbol{x}) := \frac{1}{n} \sum_{i=1}^{n} f_i(\boldsymbol{x}), \quad \text{where} \quad f_i(\boldsymbol{x}) := \frac{1}{(N/n)} \sum_{\boldsymbol{z} \in \mathcal{M}_i} \ell(\boldsymbol{x}; \boldsymbol{z}).$$



Challenges in federated/decentralized learning

- Communication efficiency: limited bandwidth, stragglers, ...
- Heterogeneity: non-iid data across the agents
- Privacy: does not come for free without sharing data



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We will focus on communication compression methods.



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PS coordinates *global* information sharing





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Network/decentralized model

agents share *local* information over a graph topology





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Coping with heterogeneity





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Coping with heterogeneity

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Somewhat surprisingly, direct compression doesn't work!

A counter-example

Consider n = 3 and let $f_i(x) = (\boldsymbol{a}_i^{\top} \boldsymbol{x})^2 + \frac{1}{2} \|\boldsymbol{x}\|^2$, where $\boldsymbol{a}_1 = (-4, 3, 3)^{\top}$, $\boldsymbol{a}_2 = (3, -4, 3)^{\top}$ and $\boldsymbol{a}_3 = (3, 3, -4)^{\top}$.



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, and the compressor be top₁,
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- The next iteration

$$x^{1} = x^{0} - \eta \frac{1}{3} \sum_{i=1}^{3} C(\nabla f_{i}(x^{0})) = (1 + 5\eta)x^{0}$$

and then $\boldsymbol{x}^t = (1+5\eta)^t \boldsymbol{x}^0$ diverges exponentially.

A better scheme: shift compression

(Stich et al., 2018; Richtárik et al., 2021)

• Model update:

$$\boldsymbol{x}^{t+1} = \boldsymbol{x}^t - \frac{\eta}{n} \sum_{i=1}^n \boldsymbol{g}_i^t$$

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• Update g_i^t with a shift compression:

$$\boldsymbol{g}_{i}^{t+1} = \boldsymbol{g}_{i}^{t} + \underbrace{\mathcal{C}(\nabla f_{i}(\boldsymbol{x}^{t+1}) - \boldsymbol{g}_{i}^{t})}_{\text{difference compression}}$$

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We'll consider algorithms using shift compression!

BEER: Fast Decentralized Nonconvex Optimization with Communication Compression



Haoyu Zhao Princeton

Boyue Li CMU





Peter Richtarik KAUST



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- slow convergence rates (need more communication rounds) and
- Incompatible with heterogeneity: bounded gradient or dissimilarity

$$\mathbb{E}_{\xi_i \sim \mathcal{D}_i} \|\nabla f(\boldsymbol{x}; \xi_i)\| \le G^2 \quad \text{or} \quad \mathbb{E}_i \|\nabla f_i(\boldsymbol{x}) - \nabla f(\boldsymbol{x})\| \le G^2$$



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Can we converge at the rate $O\left(\frac{1}{s}\right)$ under arbitrary heterogeneity?

Yes, by using gradient tracking!

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At optimal point \boldsymbol{x}^{\star} : $\nabla f(\boldsymbol{x}^{\star}) = \boldsymbol{0}$, but $\nabla f_i(\boldsymbol{x}^{\star}) \neq \boldsymbol{0}$

How do we fix this?

DGD with gradient tracking

Use dynamic average consensus (Zhu and Martinez, 2010) to track the global gradient s_i^t :

$$\begin{split} \boldsymbol{x}_{i}^{t} = & \underbrace{\sum_{j} w_{ij} \boldsymbol{x}_{j}^{t-1}}_{\text{mixing}} - \eta \boldsymbol{s}_{i}^{t} \\ \boldsymbol{s}_{i}^{t} = & \underbrace{\sum_{j} w_{ij} \boldsymbol{s}_{i}^{t-1}}_{\text{mixing}} + \underbrace{\nabla f_{i}(\boldsymbol{x}_{i}^{t}) - \nabla f_{i}(\boldsymbol{x}_{i}^{t-1})}_{\text{gradient tracking}} \end{split}$$

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- EXTRA (Shi, Ling, Wu and Yin, 2015); NEXT (Di Lorenzo and Scutari, 2016); NIDS (Li, Shi, Yan, 2017); ADD-OPT (Xi, Xin, and Khan, 2017); DIGING (Nedic, Olshevsky, and Shi, 2017); DGD (Qu and Li, 2018);
- many, many more...

 $X = [x_1, x_2, \cdots, x_n]$: local models. $\nabla F(X) = [\nabla f_1(x_1), \nabla f_2(x_2), \cdots, \nabla f_n(x_n)]$: local gradients.

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$$\boldsymbol{X}^{t+1} = \boldsymbol{X}^t + \gamma \underbrace{\boldsymbol{H}^t(\boldsymbol{W} - \boldsymbol{I})}_{\text{mixing}} - \eta \underbrace{\boldsymbol{V}^t}_{\text{gradient}}$$

where H^t is the accumulated compressed surrogate of X^t , and V^t is the global gradient estimates across the agents.

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• Both H^t and G^t are updated using shift compression.

Theoretical convergence of BEER

Theorem (Zhao et al., 2022)

To achieve $\mathbb{E} \| \nabla f(\boldsymbol{x}^{\text{output}}) \|^2 \leq \varepsilon$, BEER requires at most

$$O\left(\frac{1}{\rho^3 \alpha \varepsilon}\right)$$

communication rounds, without the bounded gradient assumption. Here, α is the compression ratio, β is the spectral gap of the network.

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BEER vs CHOCO-SGD



Figure: Training gradient norm and testing accuracy against communication rounds for classification on the *unshuffled* MNIST dataset using a simple neural network. Both BEER and CHOCO-SGD employ the biased $gsgd_b$ compression with b = 20.

SoteriaFL: A Unified Framework for Private FL with Communication Compression



Zhize Li CMU

Haoyu Zhao Princeton



Boyue Li CMU

Motivation: a unified framework?

- **Privacy:** need to preserve the privacy of local data
- Communication: shift compression with many options, e.g. sparsification or quantization
- Computation: stochastic local gradient estimators with many options, e.g. SGD, SVRG or SAGA



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Can we develop a unified framework for private FL with compression, with a characterization of the privacy-utility-communication trade-off?

SoteriaFL: a unified framework for compressed private FL



Highlights of SoteriaFL:

- Flexible local gradient estimators
- Protect local data privacy
- State-of-the-art shift compression scheme
- Privacy-utility-communication trade-offs

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At each client:



Privacy-utility-communication trade-off



Under (ϵ, δ) local differential privacy:

• Utility/accuracy: $\frac{\sqrt{\alpha \log(1/\delta)}}{\epsilon}$ • Communication: $\frac{\epsilon}{\sqrt{\alpha^3 \log(1/\delta)}}$

Summary



Provably efficient communication-compressed FL algorithms for heterogeneous and private data!

Future work:

privacy-preserving decentralized algorithms under data heterogeneity.

Thank you!

- BEER: Fast O(1/T) Rate for Decentralized Nonconvex Optimization with Communication Compression
 H. Zhao, B. Li, Z. Li, P. Richtarik, and Y. Chi, arXiv:2201.13320.
- SoteriaFL: A Unified Framework for Private Federated Learning with Communication Compression
 Li, H. Zhao, B. Li, and Y. Chi, arXiv today.

