

# Thermal Modeling for Energy-Efficient Smart Building With Advanced Overfitting Mitigation Technique

(invited)

Wandi Liu\*, Hai Wang\*, Hengyang Zhao<sup>†</sup>, Shujuan Wang<sup>†</sup>,  
Haibao Chen<sup>¶</sup>, Yuzhuo Fu<sup>¶</sup>, Jian Ma\*, Xin Li<sup>‡</sup>, Sheldon X.-D. Tan<sup>†</sup>

\* School of Microelectronics and Solid-State Electronics,

University of Electronic Science and Technology of China, Chengdu 610054 China

<sup>†</sup> Dept. Electrical and Computer Engineering, University of California, Riverside, CA 92521

<sup>‡</sup> Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA 15213

<sup>¶</sup> Department of Micro/Nano-electronics, Shanghai Jiao Tong University, Shanghai, 200240 China

**Abstract**—Building energy accounts large amount of the total energy consumption, and smart building energy control leads to high energy efficiency and significant energy savings. A compact and accurate building thermal model is important for designing the efficient energy control system. In this paper, we propose an accurate thermal behavior modeling technique for general and complicated buildings. This new modeling technique builds compact thermal model by system identification using temperature and power data obtained from EnergyPlus software, which can provide realistic temperature, weather and power data for buildings. In order to make the best use of data from EnergyPlus and avoid the overfitting problem associated with the system identification method, a cross-validation technique is employed to generate multiple thermal models to find the optimal model order. The final model is then generated by performing a regular system identification using the previously selected order. Experimental results from a case study of a 5-zone building have shown that the proposed method is able to find the optimal model order, and the building models built by the proposed method can achieve 1-3% average errors and less than 10-18% maximum errors for the estimation of zone temperatures for about a one year period.

## I. INTRODUCTION

It is estimated that building section accounts for about 40% energy consumption in United States. The building section is also responsible for 70% of electricity use. About 50% of the energy consumed in buildings are directly related to space heating, cooling and ventilation [1]. As a result, it is important to control the heating, ventilation and air conditioning (HVAC) system in a more energy-efficient way.

Smart buildings today have sophisticated and distributed control systems as part of a Building Automation System (BAS). The task of a BAS is to maintain building climate within a specified range, control the lighting based on the occupancy schedule, and monitor the system performance and failures. To achieve these tasks, a BAS needs complicated and advanced control techniques such as model predictive control (MPC) method to minimize energy consumption of the HVAC system and satisfy the temperature and input constraints [12].

However, the MPC based control could have high computational requirements depending on the number of control variables, the time horizon for the optimization, the discretization

for the control decisions and model complexity. One critical issue in effective MPC control is to have compact state space thermal model of large buildings.

Building performance simulation has been well studied in the past [11]. Simulation programs such as EnergyPlus [3] from U.S. Department of Energy, and TRNSYS [2], which are both based on the first principle of dynamic heat transfer and air flow have been developed. However, for practical buildings, those simulators have very high computational cost due to their detailed modeling of the underlying physics and interactions among the components of buildings, which makes them less suitable for MPC based thermal control for smart buildings.

To mitigate this problem, compact thermal modeling methods for buildings have been proposed recently [4], [6], [9]. Existing approaches include the reduced order modeling method using the classic truncated balanced realization method [9], aggregation-based reduction approach, which performs localized reduction (aggregation) so that some network properties can be preserved [4], and the ad-hoc model reduction method, which extracts the basic linear dynamics of thermal behaviors of a building from the EnergyPlus program [6]. All those existing compact modeling approaches can be viewed as the white-box models, which start with accurate and detailed models from the first principle and then perform approximation to obtain compact models via model order reduction, aggregation or ad-hoc dynamics extraction. But those methods suffer from several problems. First, those methods need to know the detailed structures or equations of the thermal systems to start with, which need steep learning curves to learn and extract. For commercial building simulation tools, this will become impossible. Second, they may suffer significant accuracy loss during the reduction process as many assumptions were made such as linear system models [6], ignorance of cooling and exhaust loads [4].

In this work, we look at the more powerful top-down black-box based behavioral modeling technique to address this issue. The idea is to build mathematical behavioral models from the input and output traces of the dynamic thermal systems via robust numerical approaches. Existing works consist of the matrix pencil method [7], [10] and the subspace identification method [5], [13]. The major advantage of such pure behavioral modeling methods are their flexibility and simplicity as no physical restrictions and assumptions are made or required for the models. They are also very accurate as the training

This research was supported in part by National Natural Science Foundation of China grant under No. 61404024, in part by the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry, in part by the Open Foundation of State Key Laboratory of Electronic Thin Films and Integrated Devices (KFJJ201409). The work was also funded in part by a 985 research fund from Shanghai Jiao Tong University.

is based on accurate data from physical measurement or detailed numerical simulation. The new approach is based on the recently proposed subspace identification method to obtain the control-friendly state space models for building thermal behaviors [8]. The training data is obtained from the EnergyPlus software package [3], which is a suite of algorithms that calculate the energy required to operate a building and its resulting thermal behavior based on numerous considerations ranging from the specifics of the structure, ambient temperature, heating, ventilation, and air-conditioning (HVAC) inputs, power consumption of CPUs, lighting, number of occupants in a zone, resulting temperature traces of zones, and other factors that we are interested in. We also study the relationship between model order and model prediction accuracy, and use cross-validation technique to find the optimal order to avoid both the overfitting and underfitting problems. Experimental results from a case study of a 5-zone building show that the building models built by the proposed method can achieve 1-3% average RMS errors and 10-18% maximum errors for the estimation of zone temperatures for about a one year period.

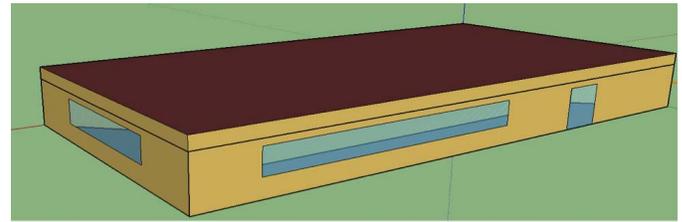
## II. REVIEW OF ENERGYPLUS FOR ENERGY SIMULATION OF BUILDING

In this section, we review the EnergyPlus software program, which provides accurate input and output traces from buildings for the new thermal modeling algorithm.

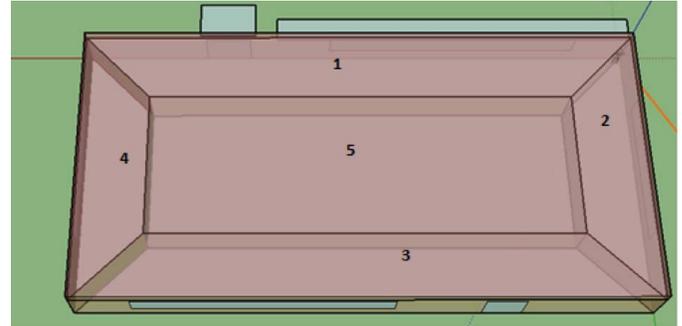
The EnergyPlus software package [3] is a suite of algorithms that calculate the energy required to operate a building and its resulting thermal behavior based on numerous considerations ranging from the specifics of the structure, to heat sources and sinks within the building, and weather. EnergyPlus consists of an integrated solution manager which manages the calculation of the heat balance of various surfaces in the building, the heat balance of the air, and the heat balance on the mechanical systems. The solution to each of these three elements are calculated separately and communicated to each other using the manager at each time step. Due to its modularity, it is easy to establish links to other programming links.

An input data file (IDF) and weather file are needed for the EnergyPlus simulation. The IDF includes all the information of the building such as size, structure, position and the HVAC subsystem etc. The IDF editor in EnergyPlus can be used to change parameters of the building, the schedule of the HVAC subsystem and also the output information. The selected output information will be generated in the spreadsheet file after running the simulation.

Fig. 1 shows the side view and the top view of an office building with 5 zones and HVAC modeled in the EnergyPlus. The heat sources for this building can be HVAC, light, occupants, electric equipment, air filtration, etc. The zone temperature is also affected by the weather (ambient temperature). The zone temperature is controlled by the HVAC system with coil and fan as shown in Fig. 2. The air of a zone is cooled or heated in the coil and goes back to the zone. The coil itself is a heat exchanger. Besides the air loop, the coil has a water loop coming from the boiler or chiller, to control the temperature of the air. Fig. 3 shows the simulated temperature changes over 500 hours in the 5 zones from this building from EnergyPlus.



(a) side view



(b) top view

Fig. 1: The 5-zone office building.

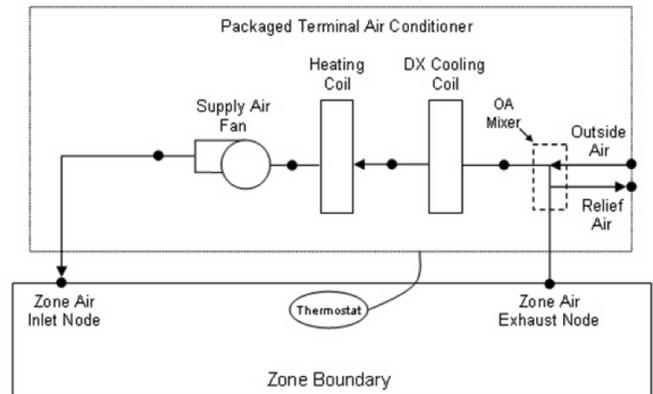


Fig. 2: A HVAC in a zone with coil and fan system

## III. REVIEW OF SUBSPACE METHODS FOR SYSTEM IDENTIFICATION

Given input  $u(t)$  and output  $y(t)$ , subspace identification method identifies the state matrices  $A$ ,  $B$ ,  $C$ , and  $D$  of (1).

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \quad (1)$$

where  $A \in \mathbb{R}^{l \times l}$  is a stable matrix,  $l$  is the number of states.  $B \in \mathbb{R}^{l \times k}$ ,  $C \in \mathbb{R}^{q \times l}$ , and  $D \in \mathbb{R}^{q \times k}$ .

The subspace identification basically tries to first identify the system states (Kalman states), then the state matrices will be obtained by the least square based optimization method [8]. There are several implementations such as the Ho-Kalman' method, the MOSEP method and the N4SID (Numerical algorithms for Subspace System Identification) method [13]. In this paper, we apply the widely used N4SID method for this system identification problem.

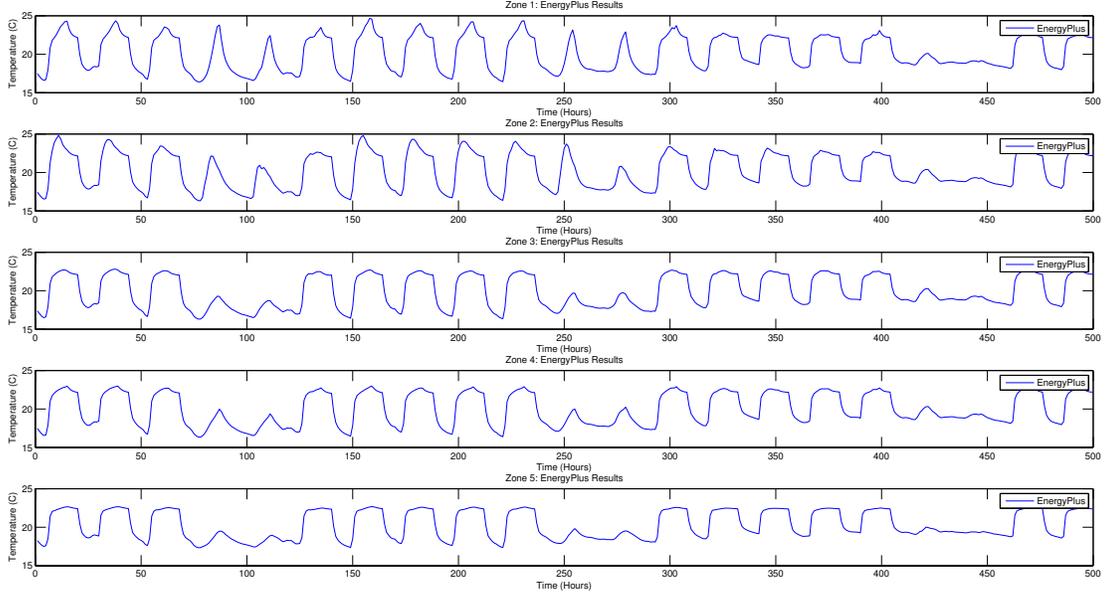


Fig. 3: The temperature changes of 5 zones in over 500 hours

#### A. Method flow of N4SID

Defining the input Hankel matrix of an  $l$ -order system as

$$U_{a|b} := \begin{bmatrix} u(a) & u(a+1) & \cdots & u(a+N-1) \\ u(a+1) & u(a+2) & \cdots & u(a+N) \\ \vdots & \vdots & \ddots & \vdots \\ u(b) & u(b+1) & \cdots & u(b+N-1) \end{bmatrix} \quad (2)$$

$\in \mathbb{R}^{(b-a+1)p \times N}$ ,

and the *output Hankel matrix*  $Y_{a|b}$  is defined accordingly. The *state sequence* is defined according to a given number  $a$  and the arbitrary number  $N$  as

$$X(a) := [x(a), x(a+1), \dots, x(a+N-1)] \in \mathbb{R}^{l \times N}. \quad (3)$$

Based on the previous definition, the *past* input, output Hankel matrices and state sequence is defined as

$$U_p := U_{0|k-1}, \quad Y_p := Y_{0|k-1}, \quad X_p := X(0), \quad (4)$$

and the *future* input, output Hankel matrices and state sequence as

$$U_f := U_{k|2k-1}, \quad Y_f := Y_{k|2k-1}, \quad X_f := X(k), \quad (5)$$

The *past* data matrix and *future* data matrix now is defined as

$$W_p := \begin{bmatrix} U_p \\ Y_p \end{bmatrix}, \quad W_f := \begin{bmatrix} U_f \\ Y_f \end{bmatrix}. \quad (6)$$

The number  $k$  and  $N$ , which determine the row and column size of the input and output Hankel matrices, are determined by the user according to the number of input samples available along the time axis. Also,  $k > l$  should be satisfied given that  $l$  is the order of the system.

Additionally, the extended observability matrix  $\mathcal{O}_k$  is defined as follows

$$\mathcal{O}_k := \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{k-1} \end{bmatrix} \in \mathbb{R}^{kq \times l}, \quad (7)$$

where  $q$  is the number of output ports.

In N4SID algorithm, an important property which can be proved is

$$\mathcal{P}_{U_f}(Y_f, W_p) = \mathcal{O}_k X_f, \quad (8)$$

where  $\mathcal{P}_B(A, C)$  represents an oblique projection of the row space of  $A$  onto the row space of  $C$  along row space of  $B$  [8], [13], [14].

By applying Singular Value Decomposition (SVD) on the left hand side of (8), there is

$$\mathcal{P}_{U_f}(Y_f, W_p) = [U_1 \ U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 \Sigma_1 V_1^T. \quad (9)$$

From (8) and (9), the extended observability matrix  $\mathcal{O}_k$  and the future state sequence  $X_f$  are readily identified as

$$\mathcal{O}_k = U_1 \Sigma_1^{1/2}, \quad (10)$$

$$X_f = \Sigma_1^{1/2} V_1^T. \quad (11)$$

Now the state sequence  $X_f = X(k) = [x(k), x(k+1), \dots, x(k+N-1)]$  is identified, we can proceed to determine the system matrices  $A, B, C, D$ . Specifically, define the following

$(N - 1)$ -column matrices (compared to the previously defined  $N$ -column matrices) as

$$\bar{X}_{k+1} := [x(k+1), x(k+2), \dots, x(k+N-1)], \quad (12)$$

$$\bar{X}_k := [x(k), x(k+1), \dots, x(k+N-2)], \quad (13)$$

$$\bar{U}_{k|k} := [u(k), u(k+1), \dots, u(k+N-2)], \quad (14)$$

$$\bar{Y}_{k|k} := [y(k), y(k+1), \dots, y(k+N-2)]. \quad (15)$$

All the four matrices are known or identified already. Then, the system matrices are solved as a least square problem directly from

$$\begin{bmatrix} \bar{X}_{k+1} \\ \bar{Y}_{k|k} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \bar{X}_k \\ \bar{U}_{k|k} \end{bmatrix}. \quad (16)$$

#### IV. THE PROPOSED NEW COMPACT MODELING ALGORITHM FOR BUILDING

In this section, we present the modeling scheme by using the subspace identification method to build the compact thermal model for the building.

To build the compact thermal models for the building, we know that a zone temperature is affected by many factors. First of all, the zone temperatures are controlled by HVAC systems inside the building, as well as other factors such as lighting and number of occupants. Besides that, the ambient temperature or weather, which varies from day to night, and summer to winter, also has a huge impact on the zone temperatures. In this work, we treat all the HVAC power inputs, ambient temperature and other factors such as occupant factor, electrical equipment and zone infiltration (as a heat loss factor), as inputs to the system and individual zone temperatures as their corresponding outputs.

For subspace identification method, one critical issue is in finding the best order to avoid the overfitting and underfitting problems [8]. If we select larger model order than the actual order needed for the system we are modeling, we may model many noises instead of the true system dynamics (its poles/zeros), this is called overfitting. On the other hand, if we use less order than the true orders of the system, we may run into an underfitting problem.

In order to find the optimal order to avoid both overfitting and underfitting problems, one straightforward way is through performing a linear sweep by varying the order number and record the simulation's validation phase absolute RMS error. Then, the order with the smallest error can be chosen as the desired order to avoid both overfitting and underfitting problems. However, performing such an order selection has two major drawbacks. First, we cannot use all samples to do the training in the process of finding the optimal order. That is because we have to reserve part of the sample for validation in order to compute the error. Second, for each order, we only obtain one identified model, with only one corresponding error. Such identified model has uncertainties to be possibly more accurate or less accurate than "normal" (here "normal" accuracy indicates the average error from models built from different data samples, with the same order), by "better" or "worse" than average set of training samples used. Such two problems may lead to chosen order which is only suboptimal.

In order to mitigate the mentioned problems, we use cross-validation technique in the order selection process. As discussed before, in traditional methods, part of the data need to be used in the validation process in order to get the error

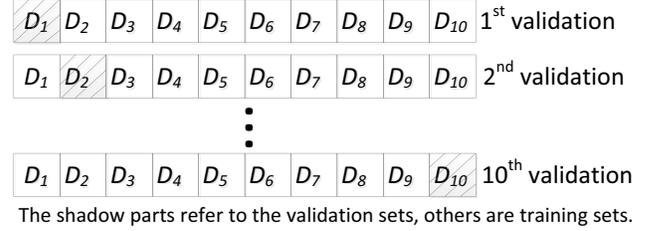


Fig. 4: The algorithm processing chart of 10-fold cross-validation.

information. Then, such data cannot be used in training, and is considered to be "wasted", because their information is not used to build the model for order selection. Cross-validation solves such problem by dividing the data samples into  $k$  parts. Then, for each potential model order, it will perform training and validation process for  $k$  rounds. For the  $i$ -th round, the  $i$ -th part of the data is taken for validation, and the remaining data are taken for training. As a result, all data will be used for  $k - 1$  times for training in the total  $k$  rounds, and no data is wasted. After  $k$  rounds of training and validation, the average error is computed using all  $k$  errors from  $k$  rounds of validation process for each potential model order. Then, the optimal order is selected as the order with the smallest average error. Such average error scheme further relieved the uncertainty problem in traditional order selection scheme. When we choose  $k$  partitions in the data set and perform  $k$  rounds of training and validation, we call such method as  $k$ -fold cross-validation. Choosing different values of  $k$  affects efficiency and accuracy of cross-validation: larger value of  $k$  leads to less uncertainty but slower order selection speed. The most common choice of  $k$  is 10, and one simple illustration of 10-fold cross-validation is shown in Fig. 4. After the optimal order is selected, all data (all  $k$  parts) are used for the final training process and generate the final model with the previously determined optimal order.

#### V. NUMERICAL RESULTS AND DISCUSSIONS

The proposed method has been implemented in MATLAB with its System Identification Toolbox.

In the sequel, the relative error is calculated by  $Error = \frac{|s-e|}{e}$  where  $s$  is the estimated result from model and  $e$  is the EnergyPlus data. Then relative root mean square (RMS) error is calculated. And the maximum relative error is determined by taking the largest value of the relative error vector.

First, for the office building with 5 zones shown in Fig. 1, we treat HVAC data and these extra factors as system inputs and the 5 individual zone temperatures as outputs, and collected the input-output samples for one year (one sample per hour for 8760 hours in one year). From all these data, 7760 samples of 5 zones is used for training and validation to select the proper order for the model, while another set of 1000 samples is reserved as the testing set to evaluate the accuracy of the final model.

Then, we use 10-fold cross-validation method on the 7760 samples to find the optimal order. The average error of 10 validation errors (by 10 training and validation rounds) for each order is recorded, and shown in Fig. 5. The optimal order selected by cross-validation is 57, as can be also seen

TABLE I: The validation errors for the 57-order model selected by the cross-validation method and 65-order model selected by the traditional method.

		Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
Order=57	absolute RMS error	0.7788	0.7190	0.4961	0.4925	0.3620
	Relative RMS error	0.0326	0.0322	0.0236	0.0231	0.0163
	Absolute maximum error	4.9824	3.8895	2.1354	2.2024	1.5029
	Relative maximum error	0.1794	0.1506	0.1122	0.1023	0.0671
Order=65	absolute RMS error	0.7970	0.7418	0.5118	0.5321	0.3794
	Relative RMS error	0.0332	0.0330	0.0245	0.0248	0.0170
	Absolute maximum error	5.1348	4.0042	2.0801	2.5728	1.5115
	Relative maximum error	0.1849	0.1430	0.1090	0.1196	0.0688

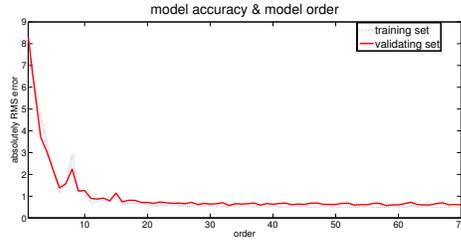


Fig. 5: The RMS error vs. model order using cross-validation method. The “training set” line means accuracy tested on the training data set, and the “validating set” line means accuracy tested using the validation data set.

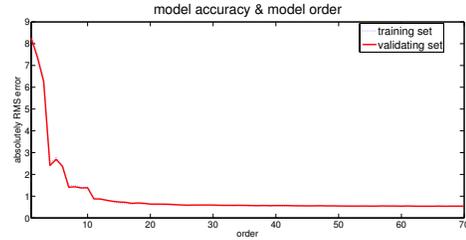
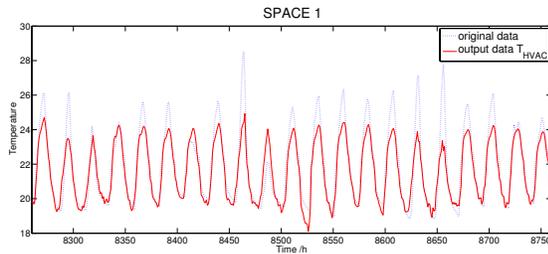
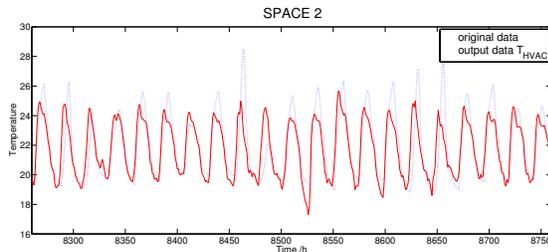


Fig. 7: The RMS error vs. model order using traditional method. The “training set” line means accuracy tested on the training data set, and the “validating set” line means accuracy tested using the validation data set.

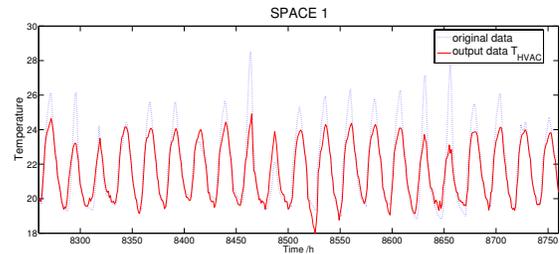


(a) Temperature results of zone one.

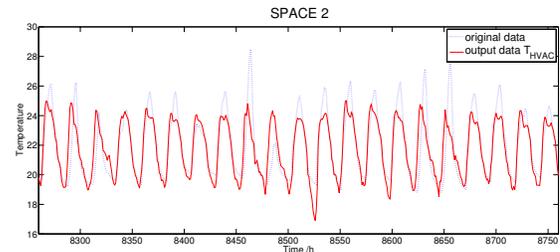


(b) Temperature results of zone two.

Fig. 6: Accuracy test of the 57th-order model. Only results of the first two zones are shown. “original data” means the temperature data obtained from EnergyPlus, and serves as samples. “output data” denotes the temperature data calculated by the 57th-order model.



(a) Temperature results of zone one.



(b) Temperature results of zone two.

Fig. 8: Accuracy test of the 65th-order model. Only results of the first two zones are shown. “original data” means the temperature data obtained from EnergyPlus, and serves as samples. “output data” denotes the temperature data calculated by the 65th-order model.

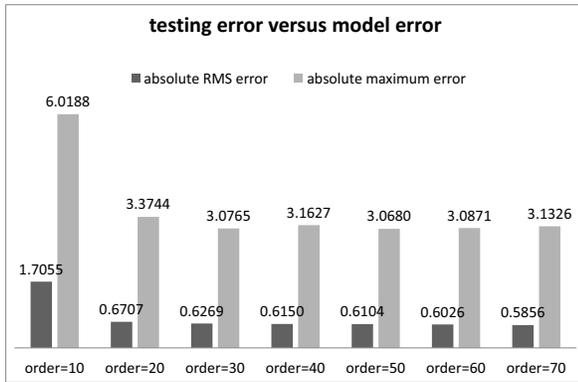


Fig. 9: The testing error vs. model order. The errors are generated using the reserved 1000 testing data set.

in the figure. Then, we use all the 7760 samples for subspace identification training process, and generate the final model with the order of 57. In order to test the accuracy of the final model, we use the reserved 1000 samples from the testing set to evaluate the error of the final model. Fig. 6 shows part of the testing results of the final model. Due to the page limitation, only results of two zones are shown, and the other three zones have similar results.

In order to make comparison, we take a traditional experiment, that is, treating the last 776 samples from the 7760 samples as validation set, and taking the other samples as training set. Then we record the errors of models with different orders, and get the curve as shown in Fig. 7. Please note that this time, each error is generated by one model only, comparing with the average error by multiple models in the cross-validation case. This time, model with order of 65 has the minimum validation error, and 65 is chosen as the final order. Then, we train the final 65-order model using all data as training data. The accuracy test of the final model is also performed on the reserved 1000 testing samples. Fig. 8 shows the testing result of the 65-order final model, which does not show significant difference comparing with the previous 57-order model case, meaning both of them are accurate enough as the thermal behavior model of the building.

In order to study the results numerically, we computed the RMS errors and maximum errors of different zones for both 57-order case found by cross-validation and 65-order case by traditional method. The results are given in Table I. Through the table, a conclusion can be drawn that cross-validation yields more accurate results, even with a lower order model, which means the order selected by cross-validation is better than that selected by traditional method.

Finally, in order to study the testing results of models with different orders, we computed the testing errors of 7 models, with orders ranging from 10 to 70. All the 7 models are trained by using all 7760 samples, and tested on the 1000 testing data. Fig. 9 presents histogram of the testing RMS and maximum errors of different orders. The testing error has a fast decrease from order 10 to order 20, revealing underfitting for low order models. The error becomes steady starting from the order of

30 to 70, with a slightly larger maximum error at 70, showing that there is no need to choose a higher order.

## VI. CONCLUSION

In this article, we have proposed a new compact building thermal modeling method for control-oriented building performance analysis, which is critical for energy-efficient smart building control. The approach is based on the recently proposed subspace identification method to obtain the state space models for building thermal behaviors. The input power traces and output zone temperatures were obtained from the EnergyPlus program, which can accept various transient inputs such as ambient temperature, heating, ventilation, and air-conditioning (HVAC) inputs, power consumption of CPUs, lighting and number of occupants in a zone and resulting temperature traces of zones. We also studied the relationship between model order and model prediction accuracy, and used cross-validation technique to avoid both the overfitting and underfitting problems. Experimental results from a case study of a 5-zone building have shown that the proposed method is able to find the optimal model order, and the building models built by the proposed method can achieve 1-3% average errors and less than 10-18% maximum errors for the estimation of zone temperatures for about a one year period.

## REFERENCES

- [1] Building Energy Data Book of DOE. [Online]. Available: <http://buildingsdatabook.eren.doe.gov>
- [2] TRNSYS – A transient systems simulation program. [Online]. Available: <http://www.trnsys.com/>
- [3] D. B. Crawley, L. K. Lawrie, F. C. Winkelmann, W. F. Buhl, Y. J. Huang, C. O. Pedersen, R. K. Strand, R. J. Liesen, D. E. Fisher, M. J. Witte, and J. Glazer, "EnergyPlus: creating a new-generation building energy simulation program," *Energy and Buildings*, vol. 33, no. 4, pp. 319–331, Apr. 2001.
- [4] K. Deng, P. Barooah, P. G. Mehta, and S. P. Meyn, "Building Thermal Model Reduction via Aggregation of States," in *American Control Conference, June 30-July 02, 2010, Baltimore, MD*, Baltimore, MD, Jun. 2010, pp. 5118–5123.
- [5] T. Eguia, S. X.-D. Tan, R. Shen, D. Li, E. H. Pacheco, M. Tirumala, and L. Wang, "General parameterized thermal modeling for high-performance microprocessor design," *IEEE Trans. on Very Large Scale Integration (VLSI) Systems*, 2011.
- [6] B. Eisenhower and I. Mezic, "Extracting Dynamic Information from Whole-building Energy Models," in *Proc. of Conference on Dynamics for Design (DFD 2012)*, Chicago, IL, Apr. 2012, pp. 3–10.
- [7] Y. Hua and T. Sarkar, "Generalized pencil of function method for extracting poles of an em system from its transient responses," *IEEE Trans. on Antennas and Propagation*, vol. 37, no. 2, pp. 229–234, Feb. 1989.
- [8] T. Katayama, *Subspace Methods for System Identification*. Springer, 2005.
- [9] D. Kim and J. E. Braun, "Reduced-Order Building Modeling For Application to Model-Based Predictive Control," in *5th National Conference of Innational Building Performance Simulation Association (IBPSA-USA)*, Aug. 2012, pp. 554–561.
- [10] D. Li, S. X.-D. Tan, E. H. Pacheco, and M. Tirumala, "Parameterized architecture-level dynamic thermal models for multicore microprocessors," *ACM Trans. Des. Autom. Electron. Syst.*, vol. 15, no. 2, pp. 1–22, 2010.
- [11] A. Malkawi and G. Augenbroe, Eds., *Advanced Building Simulation*. Spon Press, Jun. 2004.
- [12] M. Massoumy, Q. Zhu, C. Li, F. Meggers, and A. Sangiovanni-Vincentelli, "Co-design of Control Algorithm and Embedded Platform for Building HVAC Systems," in *International Conference on Cyber Physical Systems (ICCPS)*, Philadelphia, PA, Apr. 2013, pp. 61–70.
- [13] P. V. Overschee and B. D. Moor, "N4SID: Subspace algorithms for the identification of combined deterministic-stochastic systems," *Automatica*, vol. 30, no. 1, pp. 75–93, 1994.
- [14] —, *Subspace Identification for Linear Systems, Theory - Implementation - Applications*. Kluwer Academic Publishers, 2006.