An Equivalent Circuit Formulation of the Power Flow Problem with Current and Voltage State Variables

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Abstract— **Steady state analysis of power grids is typically performed using power flow analysis, where nonlinear balance equations of real and reactive power are solved to calculate the voltage magnitude, phase, and power at every bus. Transient analysis of the same power grids are performed using circuit simulation methods. We propose a novel approach to modeling the nonlinear steady state behavior of power grids in terms of equivalent circuits with currents and voltages as the state variables that is a step toward unifying transient and steady state models. A graph theoretic formulation approach is used to solve the circuits that enables incorporation of switch models for contingency analyses. Superior nonlinear steady state convergence is demonstrated by use of current as a state variable and application of circuit simulation methods. Furthermore, current and voltage state variables will offer greater compatibility with future smart grid components and monitors.**

Index Terms—**Power flow, power grid, smart grid**

I. Introduction

While the power flow method [1],[2] continues to be improved for optimization and real-time control modeling of the power grid, simulation of future smart-grid systems will rely on a combination of steady state and transient simulation methods under various operating conditions and scenarios that require multiple simulation engines. Power flow analysis is based on nonlinear balance equations of real and reactive power that are solved to calculate the voltage magnitude, phase, and power, while transient analyses follow similar methods used for circuit simulation. Unification of the models and methods, particularly for steady state and transient analyses, would provide a more robust simulation infrastructure for modeling and control.

Decades of research have advanced circuit simulation algorithms, namely those based on SPICE [3], which can be used for transient analysis of power grids. But these algorithms, based on solving for voltages and currents, have heretofore been incompatible with the power flow method and state variables used to evaluate the steady state behavior. In this paper we introduce a steady state simulation formulation that is based on solving for complex AC currents and voltages in Cartesian form as the state variables. An equivalent circuit of the power grid is formed and split into real and imaginary sub-circuits to facilitate the use of Newton-Raphson based solution of the nonlinear circuit equations. Modeling the grid in this manner enables decades of circuit simulation research to be applied to solve power flow problems. We discuss the benefits of the approach for sensitivity and N-1 analyses, modeling short and open circuits in the event of failures, and the ability to introduce other devices and models into this proposed simulation environment. The benefits of using current as a state variable for steady state analysis is shown to provide for a more robust convergence of the nonlinear equations as a function of the initial guesses for the variables.

II. Split Circuit Model

To begin, an equivalent circuit model of a power grid with currents and voltages as the state variables is constructed. Consider the three bus network shown in Figure 1(a). The buses and transmission lines can be replaced with circuit elements (voltage sources, impedances, etc.) as shown in Figure 1(b). The generator voltage is a complex function of the generator current; similarly, the load current is a complex function of the load voltage.

We would like to apply the Newton-Raphson (NR) algorithm to solve this nonlinear circuit, which involves taking a first-order Taylor expansion on the non-linear equations. However, a complex function with conjugate operator (e.g., generator voltage) is not an analytical function and, hence, not differentiable. In this case, NR cannot be directly applied to the circuit in Figure 1(b). A key insight is that the circuit can itself be split into two circuits: one real, and one imaginary (see Figure 2 and Table I). The two circuits are coupled by controlled sources. For example, consider the controlled source V_2 in the schematic. The voltage of this source is controlled by the current flowing through its counterpart in the imaginary circuit, V_5. By splitting the circuit, we are no longer solving complex functions; the actual generator voltage V_G, for example, is the sum of the voltages in the real circuit (V_{RG}) and imaginary circuit (V_{IG}) at the generator node ($V_G = V_{RG} + jV_{IG}$). NR is used to handle the quadratic non-linearities. The following sections describe how the circuit elements are derived.

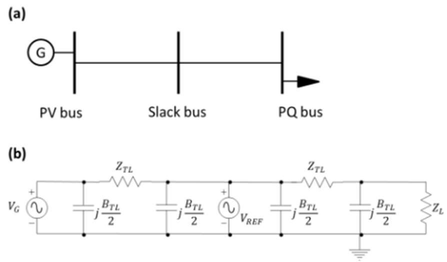

Figure 1 – (a) Three bus network with generator, slack bus, and load; (b) Equivalent circuit model of three bus network. $Z_{TL} = R_{TL} + X_{TL}$ is the impedance of the transmission line, B_{TL} is the shunt susceptance of the transmission line, and Z_L is the load impedance.

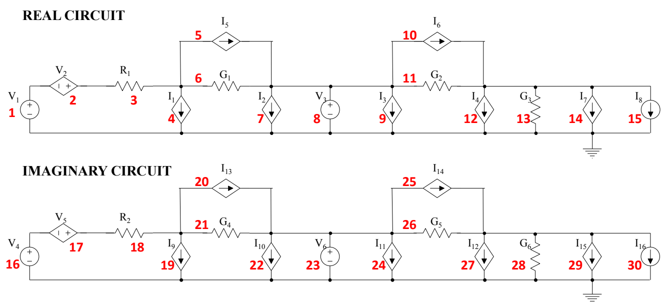

Figure 2 – Split circuit model of three bus system in Figure 1. Branch numbers are bolded and component values are given in Table I.

Table I – Component values/expressions for the circuit in Figure 2.

REAL CIRCUIT		IMAGINARY CIRCUIT	
Component	Value	Component	Value
V_1	$V_{RG}^k - \frac{\partial V_{RG}}{\partial I_{IG}}\vert_{I_{RG}^k,I_{IG}^k}(I_{IG}^k) - \frac{\partial V_{RG}}{\partial I_{RG}}\vert_{I_{RG}^k,I_{IG}^k}(I_{RG}^k)$	V_4	$V_{IG}^k - \frac{\partial V_{IG}}{\partial I_{RG}}\vert_{I_{RG}^k,I_{IG}^k}(I_{RG}^k) - \frac{\partial V_{IG}}{\partial I_{IG}}\vert_{I_{RG}^k,I_{IG}^k}(I_{IG}^k)$
V_2	$\frac{\partial V_{RG}}{\partial I_{IG}}\vert_{I_{RG}^k,I_{IG}^k}(I_{IG}^{k+1})$	V_5	$\frac{\partial V_{IG}}{\partial I_{RG}}\vert_{I_{RG}^k,I_{IG}^k}(I_{RG}^{k+1})$
V_3	$V_{REF}\cos\theta$	V_6	$V_{REF}\sin\theta$
R_1	$\frac{\partial V_{RG}}{\partial I_{RG}}\vert_{I_{RG}^k,I_{IG}^k}$	R_2	$\frac{\partial V_{IG}}{\partial I_{IG}}\vert_{I_{RG}^k,I_{IG}^k}$
I_1	$-\frac{B_{TL}}{2}(V_{19})$	I_9	$\frac{B_{TL}}{2}(V_4)$
I_2	$-\frac{B_{TL}}{2}(V_{22})$	I_{10}	$\frac{B_{TL}}{2}(V_7)$
I_3	$-\frac{B_{TL}}{2}(V_{24})$	I_{11}	$\frac{B_{TL}}{2}(V_9)$
I_4	$-\frac{B_{TL}}{2}(V_{27})$	I_{12}	$\frac{B_{TL}}{2}(V_{12})$
I_5	$\frac{X_{TL}}{R_{TL}^2 + X_{TL}^2}(V_{20})$	I_{13}	$-\frac{X_{TL}}{R_{TL}^2 + X_{TL}^2}(V_5)$
I_6	$\frac{X_{TL}}{R_{TL}^2 + X_{TL}^2}(V_{25})$	I_{14}	$-\frac{X_{TL}}{R_{TL}^2 + X_{TL}^2}(V_{10})$
I_7	$\frac{\partial I_{RL}}{\partial V_{IL}}\vert_{V_{RL}^k,V_{IL}^k}(V_{IL}^{k+1})$	I_{15}	$\frac{\partial I_{IL}}{\partial V_{RL}}\vert_{V_{RL}^k,V_{IL}^k}(V_{RL}^{k+1})$
I_8	$I_{RL}^k - \frac{\partial I_{RL}}{\partial V_{IL}}\vert_{V_{RL}^k,V_{IL}^k}(V_{IL}^k) - \frac{\partial I_{RL}}{\partial V_{RL}}\vert_{V_{RL}^k,V_{IL}^k}(V_{RL}^k)$	I_{16}	$I_{IL}^k - \frac{\partial I_{IL}}{\partial V_{RL}}\vert_{V_{RL}^k,V_{IL}^k}(V_{RL}^k) - \frac{\partial I_{IL}}{\partial V_{IL}}\vert_{V_{RL}^k,V_{IL}^k}(V_{IL}^k)$
G_1	$\frac{R_{TL}}{R_{TL}^2 + X_{TL}^2}$	G_4	$\frac{R_{TL}}{R_{TL}^2 + X_{TL}^2}$
G_2	$\frac{R_{TL}}{R_{TL}^2 + X_{TL}^2}$	G_5	$\frac{R_{TL}}{R_{TL}^2 + X_{TL}^2}$
G_3	$\frac{\partial I_{RL}}{\partial V_{RL}}\vert_{V_{RL}^k,V_{IL}^k}$	G_6	$\frac{\partial I_{IL}}{\partial V_{IL}}\vert_{V_{RL}^k,V_{IL}^k}$

A. Generator Model

For the generator, we need to express the real and imaginary voltage as a function of the real and imaginary current (I_{RG} and I_{IG}, respectively). To do this, we solve the following equations simultaneously, which relate the aforementioned variables to the provided/controlled values of real power (P_G) and voltage magnitude ($|V_G|$):

$$P_G = V_{RG}I_{RG} + V_{IG}I_{IG} \tag{1}$$
$$|V_G|^2 = V_{RG}^2 + V_{IG}^2 \tag{2}$$

The resulting expressions for voltage are:

$$V_{RG} = \frac{P_G I_{RG} \pm I_{IG}\sqrt{V_G^2(I_{RG}^2 + I_{IG}^2) - P_G^2}}{I_{RG}^2 + I_{IG}^2} \tag{3}$$
$$V_{IG} = \frac{P_G I_{IG} \pm I_{RG}\sqrt{V_G^2(I_{RG}^2 + I_{IG}^2) - P_G^2}}{I_{RG}^2 + I_{IG}^2} \tag{4}$$

The correct root to choose for each expression depends on the sign of the reactive power (Q) for the generator; for negative values of Q (generator supplying reactive power), the positive root is selected for V_{RG} and the negative root for V_{IG}. For positive Q, the negative root is chosen for V_{RG} and the positive root for V_{IG}.

The equations are linearized via a first-order Taylor expansion; for example, the expansion of the real voltage at the $(k+1)^{\text{th}}$ iteration is:

$$V_{RG}^{k+1} = \frac{\partial V_{RG}}{\partial I_{RG}}\Big|_{I_{RG}^k,I_{IG}^k}(I_{RG}^{k+1}) + \frac{\partial V_{RG}}{\partial I_{IG}}\Big|_{I_{RG}^k,I_{IG}^k}(I_{IG}^{k+1}) + V_{RG}^k - \frac{\partial V_{RG}}{\partial I_{IG}}\Big|_{I_{RG}^k,I_{IG}^k}(I_{IG}^k) - \frac{\partial V_{RG}}{\partial I_{RG}}\Big|_{I_{RG}^k,I_{IG}^k}(I_{RG}^k) \tag{5}$$

The first term represents a non-linear resistor (R_1 in Figure 2 and Table I), where the voltage across it is proportional to the current through it; the second term represents a dependent voltage source (V_2 in Figure 2 and Table I), where the controlling variable is the imaginary generator current flowing in the opposite sub-circuit; the final terms represent an independent voltage source (V_1 in Figure 2 and Table I) based on values from the previous iteration. Symmetric elements in the imaginary sub-circuit (R_2, V_5, V_4) can be derived in an identical manner.

B. Load Model

Real and imaginary load current (I_{RL} and I_{IL}, respectively) as a function of real and imaginary load voltage (V_{RL} and V_{IL}, respectively) are derived by solving the following equations simultaneously:

$$P_L = V_{RL}I_{RL} + V_{IL}I_{IL} \tag{6}$$
$$Q_L = -V_{RL}I_{IL} + V_{IL}I_{RL} \tag{7}$$

yielding the following expressions:

$$I_{RL} = \frac{P_L V_{RL} + Q_L V_{IL}}{V_{RL}^2 + V_{IL}^2} \tag{8}$$

$$I_{IL} = \frac{P_L V_{IL} - Q_L V_{RL}}{V_{RL}^2 + V_{IL}^2} \tag{9}$$

Linearizing the load via Taylor expansion results in three elements in parallel in both circuits: a conductance, a voltage-controlled current source, and an independent current source. The values are given in Table I and the models in Figure 2. G_3, I_7, and I_8 are the load elements in the real circuit, and G_6, I_{15}, and I_{16} are the load elements in the imaginary circuit.

C. Slack Bus Model

The slack bus is the simplest bus type to model. In the real circuit, it appears as an independent voltage source of value $V_{REF}\cos\theta$, and in the imaginary circuit it appears as a voltage source of value $V_{REF}\sin\theta$. When the phase θ is 0° the imaginary component appears as a short to ground.

D. Transmission Line Model

A transmission line is represented in the equivalent circuit as a pi-model as shown in Figure 1(b). To split this model into the real and imaginary sub-circuits, we begin with

$$\begin{aligned}\tilde{I} = \tilde{Y}\tilde{V} = (Y_R + jY_I)(V_R + jV_I) \\ = (Y_R V_R - Y_I V_I) \\ + j(Y_I V_R + Y_R V_I)\end{aligned} \tag{10}$$

The admittance of the shunt elements in the pi-model is purely imaginary ($Y_{shunt} = j\frac{B}{2}$), but the admittance of the branch connecting them has both real and imaginary components ($Y_{branch} = \frac{1}{R+jX} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$). Plugging these expressions into equation (10) yields:

$$I_{R,branch} = \frac{R}{R^2+X^2}V_{R,branch} + \frac{X}{R^2+X^2}V_{I,branch} \tag{11}$$

$$I_{I,branch} = \frac{R}{R^2+X^2}V_{R,branch} - \frac{X}{R^2+X^2}V_{I,branch} \tag{12}$$

$$I_{R,shunt} = -\frac{B}{2}V_{I,shunt} \tag{13}$$

$$I_{I,shunt} = \frac{B}{2}V_{R,shunt} \tag{14}$$

An element where the current through it is proportional to the voltage across it is represented as a conductance (G_1, G_2 in the real circuit, G_4, G_5 in the imaginary circuit). An element where the current through it is proportional to the voltage across its companion element in the opposite sub-circuit is represented as a voltage-controlled current source (I_5, I_6, in the real circuit, which are dependent on the voltages across I_{13}, I_{14} in the imaginary circuit, and vice versa).

E. Transformer Model

Although a transformer is not present in the simple 3-bus example of Figure 1, they appear as branch elements connecting buses in nearly every network of reasonable size. To derive their split circuit equivalent model, we begin by relating the primary and secondary voltages ($\tilde{V}_1$ and $\tilde{V}_2$) through the turns ratio n and the phase angle θ (which is only non-zero for phase shifters):

$$\frac{\tilde{V}_1}{\tilde{V}_2} = ne^{j\theta} \rightarrow \frac{V_{1R}+jV_{1I}}{V_{2R}+jV_{2I}} = n\cos\theta + jn\sin\theta \tag{15}$$

If we solve for the real and imaginary components of V_2, we obtain the following expressions for secondary side elements:

$$V_{2R} = \frac{V_{1R}\cos\theta}{n} + \frac{V_{1I}\sin\theta}{n} \tag{16}$$

$$V_{2I} = \frac{V_{1I}\cos\theta}{n} - \frac{V_{1R}\sin\theta}{n} \tag{17}$$

The first term of equation (16) represents a voltage-controlled voltage source, where the controlling voltage is the primary side voltage in the real circuit. The second term is a voltage-controlled voltage source, but here the controlling voltage is the primary side voltage in the imaginary circuit. The same types of elements are found in equation (17).

We can also express primary and secondary side currents ($\tilde{I}_1$ and $\tilde{I}_2$) in terms of the turns ratio:

$$\frac{\tilde{I}_2}{\tilde{I}_1} = -ne^{j\theta} \rightarrow \frac{I_{2R}+jI_{2I}}{I_{1R}+jI_{1I}} = -(n\cos\theta + jn\sin\theta) \tag{18}$$

From equation (18), we can derive expressions for the primary side current:

$$I_{1R} = -\frac{I_{2R}\cos\theta}{n} + \frac{I_{2I}\sin\theta}{n} \tag{19}$$

$$I_{1I} = -\frac{I_{2I}\cos\theta}{n} - \frac{I_{2R}\sin\theta}{n} \tag{20}$$

The first term of equation (19) represents a current-controlled current source, where the controlling current is the current which flows through the secondary side in the real circuit. The second term represents a current-controlled current source, but here the controlling current is the current which flows through the secondary side in the imaginary circuit. The same types of elements are found in equation (20).

The final term to model is the leakage impedance, $Z_{TR} = R_{TR} + jX_{TR}$,. This is handled the same way as the branch impedance found in transmission lines. A similar analysis results in the following equations:

$$I_{R,TR} = \frac{R_{TR}}{R_{TR}^2+X_{TR}^2}V_{R,TR} + \frac{X_{TR}}{R_{TR}^2+X_{TR}^2}V_{I,TR} \tag{21}$$

$$I_{I,TR} = \frac{R_{TR}}{R_{TR}^2+X_{TR}^2}V_{R,TR} - \frac{X_{TR}}{R_{TR}^2+X_{TR}^2}V_{I,TR} \tag{22}$$

The first term of equation (21) is a conductance and the second term is a voltage-controlled current source; likewise for equation (22). A full equivalent circuit model for the transformer is shown in Figure 3.

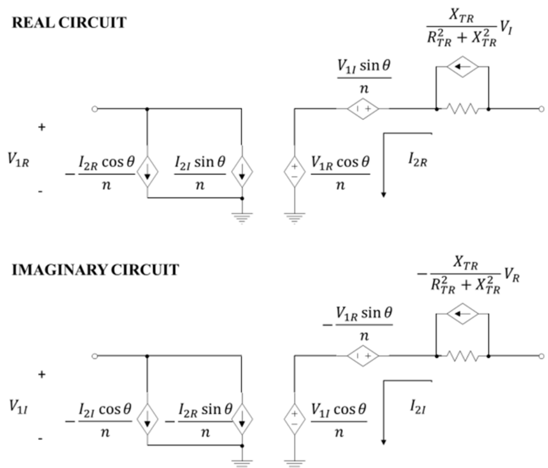

Figure 3 – Real and imaginary circuit models for a transformer.

F. Shunt Model

Shunt elements at buses are modeled similarly to the transmission line shunts. We begin from equation (10), and note that $Y_{sh} = G_{sh} + jB_{sh}$. Usually the shunts are purely susceptive. The following expressions are obtained:

$$I_{R,shunt} = G_{sh}V_R - B_{sh}V_I \tag{23}$$
$$I_{I,shunt} = B_{sh}V_R + G_{sh}V_I \tag{24}$$

The first term of (23) represents a conductance to ground and the second term represents a controlled current source to ground, where the controlling voltage is the voltage across the conductance element in the imaginary circuit. Likewise, the first term of (24) represents a conductance to ground and the second term represents a controlled current source to ground, where the controlling voltage is the voltage across the conductance element in the real circuit.

III. Tree-Link Analysis and Formulation of Equations

A graph theory-based method known as tree-link analysis (TLA) is used to solve the circuit for voltages and currents [4]-[6]. A directed graph of the circuit is constructed, and a spanning tree is found that touches all nodes and forms no loops. Branches not included in the tree form a set of links, or co-tree. A system of equations is formulated such that the solutions are the tree branch voltages and the link currents. Inclusion of an element in the tree is based on priority ordering, with greatest preference given to voltage sources, capacitors, and small-valued resistors, for which solving for the voltage is most efficient. Current sources, inductors, and large resistors are included in the links, as it is more efficient to solve for their currents. The TLA formulation yields provably optimal matrix conditioning [6].

Figure 4 shows the graph representation of the three-bus split circuit, where each node of the graph represents a node in the circuit and each branch represents a two-terminal element in the circuit. The direction given to each branch represents the current flow through the element and is based on the associated reference direction of that element; for example, the current through a voltage source is defined as positive when directed from the positive to the negative terminal of the source. An incidence matrix ($\boldsymbol{A}$) of size (#nodes×#branches) is used to represent the graph, such that:

$\boldsymbol{A}(i,j) = 1$ if branch j is directed away from node i
$\boldsymbol{A}(i,j) = -1$ if branch j is directed to node i
$\boldsymbol{A}(i,j) = 0$ if branch j does not touch node i

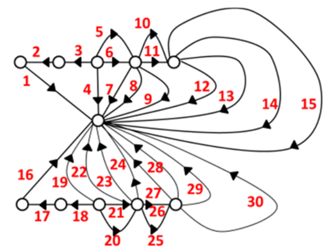

Figure 4 – Directed graph representation of circuit shown in Figure 2.

After the elements are ordered into a tree and co-tree, $\boldsymbol{A}$ is row-reduced to form the cutset matrix $\boldsymbol{F}$, where each row of $\boldsymbol{F}$ represents a fundamental cutset. The tree branch currents ($\boldsymbol{i_t}$) can be expressed in terms of the link currents ($\boldsymbol{i_l}$) via $\boldsymbol{F}$:

$$\boldsymbol{i_t} = -\boldsymbol{F}\boldsymbol{i_l} \tag{25}$$

This is also a statement of Kirchoff's current law (KCL).

To determine the tree branch voltages ($\boldsymbol{v_t}$) we solve:

$$[\boldsymbol{1} - \boldsymbol{\alpha}\boldsymbol{F^T} + \boldsymbol{R}\boldsymbol{F}(\boldsymbol{1}+\boldsymbol{\beta}\boldsymbol{F})^{-1}\boldsymbol{G}\boldsymbol{F^T}]\boldsymbol{v_t} = \boldsymbol{V_t} - \boldsymbol{R}\boldsymbol{F}(\boldsymbol{1}+\boldsymbol{\beta}\boldsymbol{F})^{-1}\boldsymbol{I_l} \tag{26}$$

where

$$\boldsymbol{v_t} = \boldsymbol{R}\boldsymbol{i_t} + \boldsymbol{V_t} + \boldsymbol{\alpha}\boldsymbol{v_l} \tag{27}$$
$$\boldsymbol{i_l} = \boldsymbol{G}\boldsymbol{v_l} + \boldsymbol{I_l} + \boldsymbol{\beta}\boldsymbol{i_t} \tag{28}$$

$\boldsymbol{R}$ is a matrix of resistances used to calculate the voltage across a tree branch from the current flowing through the branch; it is a statement of Ohm's law. $\boldsymbol{V_t}$ is a vector of independent voltage sources that appear on tree branches. $\boldsymbol{\alpha}$ is a matrix where the only non-zero entries represent dependent voltage sources in the tree that are controlled by link voltages. $\boldsymbol{G}$, $\boldsymbol{I_l}$, and $\boldsymbol{\beta}$ have similar meanings; in this case, $\boldsymbol{G}$ is a matrix of conductances, $\boldsymbol{I_l}$ a vector of independent current sources, and $\boldsymbol{\beta}$ a matrix of tree branch current-controlled current sources that appear in the co-tree. Tree branch currents and link voltages can be back-calculated from these equations if necessary.

IV. Results and Discussion

A prototype solver was implemented in MATLAB. The program reads in power flow case files in the standard IEEE CDF format and replaces each bus, line, and transformer with their equivalent circuit models from Section II. A graph and spanning tree are built from this circuit and the TLA equations are formulated. These equations are solved on every Newton-

Raphson iteration. If the result of an iteration would yield an infeasible solution on the next iteration (e.g., a complex number for a tree branch voltage in the real circuit, which can occur if the discriminant of equation (3) or (4) becomes negative), no values are updated and the iteration is repeated with damping. The sign of the change in value remains the same, but the magnitude of the change is reduced. If the solution is acceptable, the last remaining step is to calculate the reactive power of the generator buses and the voltage magnitude and phase of the load buses. This is trivial, because the solution of the tree/link equations of the split circuit yields all values for real and imaginary voltages and currents at every bus.

The proposed implementation successfully simulates the 14-, 30-, 57-, 118-, and 300-bus IEEE test cases. Iteration counts are given in Table II. The large number of iterations required, especially for the bigger test systems, is primarily due to damping. Reducing the iteration count with a better damping mechanism is a direction for future research.

The An ill-conditioned 11-bus system from the literature [7] was also tested. The solution converges in nine iterations from a flat start with all load voltages initially set to 1+j0. The Newton-Raphson residue at each iteration for this test case is shown in Figure 5.

Table II – Iteration count for IEEE standard test cases.

Case	Iteration Count
14-bus	29
30-bus	30
57-bus	30
118-bus	98
300-bus	265

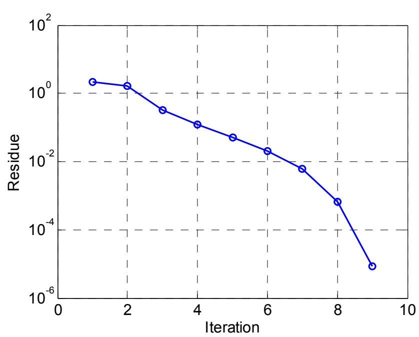

Figure 5 – Newton-Raphson residue decreases with iteration number for ill-conditioned 11-bus system.

Traditional power flow methods are known to be sensitive to the choice of the initial guess of the solution. Consider the simple two-bus system in Figure 6 with a slack bus and a load. The system has two solutions for the load voltage, one high (stable) and one low (unstable). An initial guess of the load voltage (including both real and imaginary components) is required to run a power flow simulation. This guess can be represented as a point on a complex plane, with the y-axis representing the imaginary component of the voltage and the x-axis representing the real component. Ten thousand choices of initial guess were supplied to MATPOWER, of which 6785 led to convergence to the low voltage solution (red points in Figure 7) and 49 led to non-convergence in 1000 iterations (black points in Figure 7).

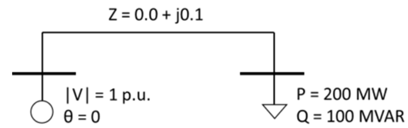

Figure 6 – Sample two-bus system.

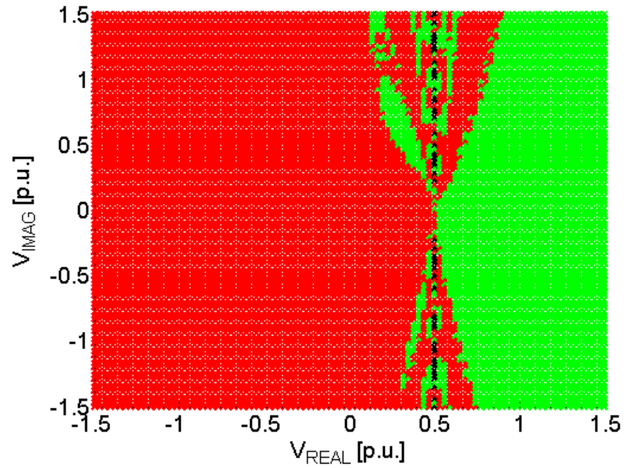

Figure 7 – Nearly 70% of initial seeds lead to non-convergence (black points) or convergence to a low voltage solution (red points) in a power flow simulation of a two bus system.

The same experiment was run using the current/voltage TLA simulator. Figure 8 shows that this method is significantly less sensitive to the initial guess than traditional power flow, with only 131 out of the 10000 seeds leading to convergence to the low voltage solution. However, the equivalent circuit formulation allows for borrowing techniques from circuit simulators to further improve the convergence properties. A common method to find the DC solution to a non-linear circuit is power stepping, where the supply voltage (VDD) is scaled down and stepped back up to its original value in increments [8]. This technique is similar to representing "turning on the circuit" in simulation, and can equally be applied to "turning on the grid." All generation and loads are scaled back by a factor of 0.001. The solution to this scaled system is then used as the initial guess for the next iteration where the loads are increased, and this is repeated until all loads are returned to their initial values. Figure 9 shows that any choice of initial guess leads to convergence to the correct solution when this approach is applied. This is because unstable solutions only occur when the current is large enough such that a small voltage magnitude can still yield the constant P and Q demanded by the load. When P and Q are scaled down as they are here, low values for the current are found even for low voltage initial guesses. This allows the simulation to avoid being steered toward high current solutions that lead to low bus voltages.

The same experiment was tried on all IEEE test systems, with all loads assumed to begin with the same voltage magnitude and angle. Fewer than 20 of the 10000 initial guesses cause convergence to a low-voltage solution in each case without power stepping. When power stepping is applied, 100% of initial guesses lead to the correct solution.

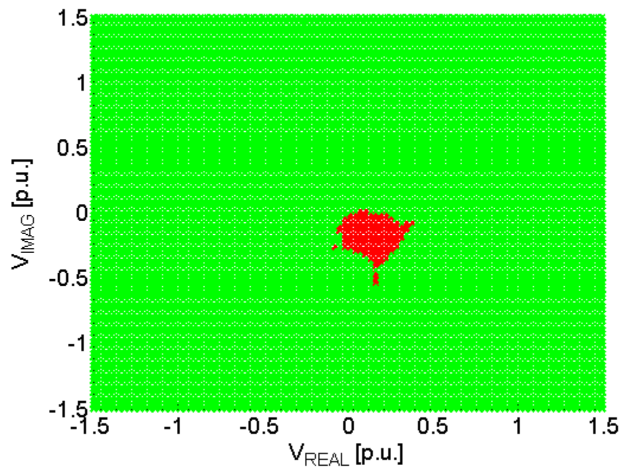

Figure 8 – Only 1.3% of initial seeds lead to convergence to a low voltage solution in the current/voltage TLA approach.

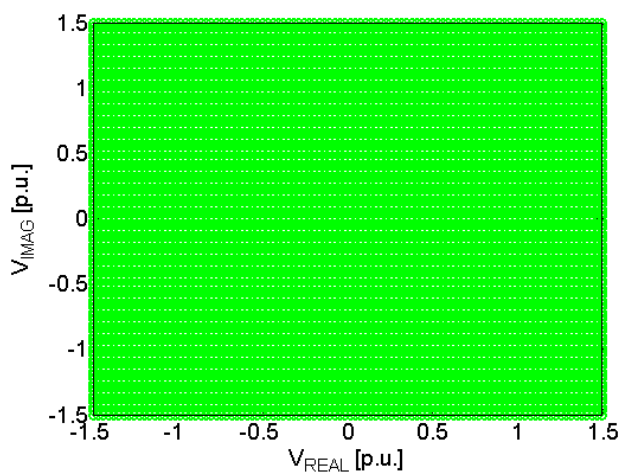

Figure 9 – 100% of initial seeds ultimately yield the high voltage solution in the TLA simulator when power stepping is applied.

It is worth noting that this technique does nothing to improve the convergence properties of traditional power flow methods. This is because scaling P and Q does not affect the Jacobian, nor does it help force the current to be small (due to the lack of a current variable). Applying this technique to traditional power flow results in a plot similar to Figure 7. The only difference is that every initial guess converges to one of the two solutions, but nearly 70% of those initial guesses cause convergence to the low voltage solution.

V. Conclusion and Future Work

We have introduced an equivalent circuit formulation for the steady state analysis of power grids in terms of current and voltage state variables. By splitting the circuit into real and imaginary sub-circuits we can accommodate the use of Newton-Raphson to iteratively compute the nonlinear solution. Initial results demonstrate that this formulation can solve power flow problems with improved robustness as a function of the initial guess. The use of an equivalent circuit for the steady state analysis is a first step toward unification of steady state and transient models, and further enables the use of circuit simulation methods to be applied to power grid problems.

With tree-link circuit analysis, for example, it is trivial to model both short- and open-circuit elements. This could enable more efficient contingency analysis, where it may be necessary to simulate a short or open between any two nodes in the event of a catastrophe. Replacing lines with shorts or opens does not require the entire problem to be reformulated; only local changes to the tree are required. Sensitivity analysis methods that are borrowed from the circuit simulation community [4],[9]-[10] could also be applied to the equivalent circuit models for the power grid to perform optimal power flow analysis. And most importantly, since any element that can be expressed in terms of voltages and currents (e.g. converters, solar cells, or high voltage DC components) can be incorporated into the equivalent circuit, this circuit-based unification of models could enable powerful new capabilities for modeling and monitoring future smart grids.

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