Noise Macromodel for Radio Frequency Integrated Circuits[†]

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Abstract

Noise performance is a critical analog and RF circuit design constraint, and can impact the selection of the IC system-level architecture. It is therefore imperative that some model of the noise is represented at the highest levels of abstraction during the design process. In this paper we propose a noise macromodel for analog circuits and demonstrate it by way of implementation in a system level simulator based on MATLAB. We also explain our process of macromodel extraction via reformulation of frequency-domain noise analysis results, and the corresponding steps of model order reduction. The results demonstrate the efficacy of this macromodel for frequency domain system level simulation.

1.   Introduction

Noise performance in RF circuits links directly to system level specifications such as Signal-to-Noise-Ratio (SNR) and Bit-error-rate – which corresponds to the radius of service and data transfer rate for wireless communication applications. An efficient system level model and analysis that can include non-idealities such as noise can facilitate high-level architectural optimization and exploration that would otherwise be impossible. Although *noise figure* (NF) has been widely used by circuit designers for back-of-the-envelope calculations, it only specifies noise performance at a given frequency. In order to obtain noise characteristics in general requires analysis of the complete analog system. For this purpose we require a system-level noise macromodel with sufficient accuracy to capture noise behavior over a range of frequencies and in terms of key input design parameters.

Several advances have been made for modeling RF and analog circuit noise behavior, but in general they are not directly of the form required for system level simulation. The authors in [1] first introduced model order reduction techniques for noise modeling of linear time-invariant (LTI) systems. While these models are compact and suitable for system-level macromodeling, they are insufficient for RF circuits that are time-varying systems. Importantly, a mixer is known for its time-varying property since its function involves frequency translation. Moreover, low Noise Amplifiers (LNA) are usually treated as LTI systems, but when there is a large blocking signal in an adjacent channel, they too will behave as time-varying systems.

In [2], a behavioral noise model for mixers was proposed which focused on the time-varying transfer functions from each noise source to the output, without congregating all of the noise as input or output referred equivalent noise sources. For large RF circuits this can result in a macromodel that is overly large and complex for efficient system level simulation. Alternatively, reduced order modeling for time-varying systems were presented in [3], and the author suggested that techniques similar to those in [1] could ultimately be used to model noise in time-varying reduced order transfer functions.

In this paper we describe a variation on the noise macromodeling approach for time-varying systems that was outlined in [3], and demonstrate its implementation in a system level simulator. Our macromodel was simplified from that suggested in [3] by exploiting a symbolic representation of the noise power spectral density. This symbolic representation required fewer elements in the resultant noise analysis matrix than a corresponding numerical computation, and provided our macromodel with the relationship between the design parameters and the noise model. Our macromodel was further simplified by focusing on only the time-averaged power of the cyclostationary noise at the input and output ports of interest, thereby allowing us to model only the stationary noise component [5].

Using this simplification and reformulating the noise results into the form of a transfer function as suggested in [3], we obtained a closed form rational expression of the noise power spectral density (PSD). This expression, however, was extremely large since it increases proportionally with circuit size and number of harmonics.

Therefore, as a final step we applied model reduction via PRIMA [7] to produce a more compact noise macromodel. This noise macromodel is applied in frequency domain system level simulation that computes signal power spectral distribution at every node. The efficiency of this noise macromodel in the frequency domain simulation is apparent since the noise PSD is modeled directly.

We demonstrate the construction and application of our noise macromodels for two examples: an LNA with an adjacent channel blocking signal and a mixer with input port phase noise. The efficacy of these models is verified in a frequency domain system level simulation environment.

The remainder of this paper is organized as follows. In section 2 existing noise analysis methods are reviewed. In section 3 order-reduction-based noise macromodeling techniques for time-varying system are developed. Our examples are extracted and applied in system level simulator in sections 4 and 5, followed by our conclusions in section 6.

2. Review of circuit noise analysis

RF circuits produce noise at their outputs for which power varies significantly with time. If the power varies in a periodic fashion, the noise is said to be *cyclostaionary*. The time-averaged noise power represents the *stationary component* of cyclostationary noise. Traditional noise analysis algorithms that are available in tools such as Spice are not able to compute cyclostationary noise for RF circuits. We begin with a review of these noise analysis algorithms for time-invariant systems, then move on to describe more recent work for cyclostationary noise analysis for time-varying systems [4] [12]. We further describe extensions of cyclostationary noise analysis to account for input phase noise.

The general form of any nonlinear circuit noise analysis can be expressed in the stochastic differential equation as following:

$$\frac{d}{dt}Q(X) + I(X) + b(t) + Au(t) = 0 \qquad (1)$$

where $u(t)$ denotes the stochastic noise signals. Differing from standard differential equations, now X is stochastic process that represents the time-domain circuit state variables, such as node voltage, etc. Vector $b(t)$ contains the large-signal excitations, and I and Q represent the "resistive" and "reactive" nonlinear elements of the circuit, respectively. The last term $Au(t)$ represents "small" perturbations to the system, e.g., from noise sources in devices or noise at input ports. A is an incidence matrix which describes how these noise sources are connected to the circuit.

When a nonlinear circuit operates under small signal conditions, the operating-point does not change the circuit can be analyzed as if it has only time-invariant (DC) excitations. The circuits are linearized about their DC operating point to construct an LTI model for noise analysis. From a Taylor expansion about the DC solution of (1) we obtain:

$$GX + C\frac{d}{dt}X + Au(t) = 0 \qquad (2)$$

Where G is admittance matrix and C is capacitance matrix.

The noise analysis problem, therefore, reduces to that of the propagation of some stochastic process through an LTI system. The general expression of the noise PSD at the output of the linear system $Syy(\omega)$ is well known as[1]:

$$S_{yy}(\omega) = H(\omega)S_{uu}(\omega)H^H(\omega) \qquad (3)$$

where $H(\omega)$ is the transfer function from each noise source to output and $H^H(\omega)$ denotes the conjugate transpose of $H(\omega)$. $Suu(\omega)$ is a diagonal matrix, each element denotes the PSD of a noise sources, and $Syy(\omega)$ is a scalar function of frequency. The many-to-one vector transfer function $H(\omega)$ of the linear system from the noise sources to an output port is given by:

$$H(\omega) = l^T(G + j\omega C)^{-1}A \qquad (4)$$

Where l denotes the incidence vector that corresponds to the output port of interest, A is same as denoted in (1). Combining (3) and (4) we obtain the expression for the noise PSD at the output of the LTI system:

$$S_{yy}(\omega) = l^T(G + j\omega C)^{-1}AS_{uu}(\omega)A^T(G + j\omega C)^{-H}l \qquad (5)$$

When a nonlinear circuit has a large input signal, however, it causes the operating points of the active devices to change with time. At steady state the nonlinear circuit can be linearized about its *time varying* operating-point to construct a linear time variant model for noise analysis [4].

Expanding (1) about its time-varying steady-state solution, we obtain:

$$J(t)X + C(t)\frac{d}{dt}X + Au(t) = 0 \qquad (6)$$

where

$$J(t) = G(t) + d/dt\,C(t)$$

The difference between (6) and (2) is that the capacitance matrix becomes a function of time and the *equivalent* admittance matrix $J(t)$ contains the time-varying admittance matrix and the time derivative of time-varying capacitance matrix. If the operating-point changes periodically, the system can be treated as a linear periodic time-varying (LPTV) system. Importantly, the bias dependent noise sources also become cyclostationary due to the time-varying operating-point. Equation (6) reflects the circuit response of cyclostationary noise sources

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propagating through the LPTV system. Similarly, we can derive the output noise and time-varying transfer function from many noise sources to output as described in following:

$$S_{yy}(\omega) = H_{FD}(\omega)S_{uu}(\omega)H_{FD}{}^{H}(\omega) \qquad (7)$$

$$H_{FD}(\omega) = D^{T}(J_{FD} + j\omega C_{FD})^{-1}A_{FD} \qquad (8)$$

Here $H_{FD}(\omega)$ is frequency domain harmonic transfer function, A_{FD}, J_{FD} and C_{FD} are frequency domain representation of noise source incidence matrix and time-varying equivalent admittance and capacitance matrices, and D denotes the incidence matrix that corresponds to the output port of interest. The detailed derivation of above equations is described in [4]. Combining (7) and (8) we have a structurally similar expression as in (9) for the total output noise of time-varying system. This similarity will be exploited in later section when extracting the cyclostationary noise macromodel.

$$S_{yy}(\omega) = D^{T}(J_{FD} + j\omega C_{FD})^{-1}A_{FD}S_{uu}(\omega)A_{FD}{}^{T}(G_{FD} + j\omega C_{FD})^{-H}D \qquad (9)$$

When the large deterministic signal that causes the system to vary with time is not purely sinusoidal due to the presence of input phase noise, the noise analysis technique in [5] extends the work in [4] to take this effect into account:

$$G(t)X + C(t)\frac{d}{dt}X + Au(t) - \sqrt{c}C(t)\dot{X}_{s}(t)u_{0}(t) = 0 \qquad (10)$$

The time domain representation of phase noise is the signal period with increasing variance, which is reflected in the above equation by c to denote the rate of variance increase. $\dot{X}_{s}(t)$ is the time derivative of steady-state response. Equation (10) suggested that the contribution of input phase noise to the wide-band amplitude noise could be treated as an additional white noise source modulated by $-\sqrt{c}C(t)\dot{X}_{s}(t)$. Hence we can modify the existing cyclostationary noise analysis technique described early to accommodate this effect.

The frequency domain cyclostationary noise analysis discussed so far computes all of the harmonic PSD, which produces the frequency spectrum along with information about the correlations in the noise between sidebands. It has been proven that we only need to consider the stationary component of cyclostationary noise in non-autonomous circuits[5]. This leads to our noise macromodeling approach that focuses on the stationary component of cyclostationary noise.

3. Noise macromodel for RF circuits

When we build a noise macromodel for RF circuits it is often desirable to represent all of the noise contributions by some equivalent noise sources at the inputs or outputs.

Unlike the internal noise source, such as thermal noise and flicker noise, which has relatively simple frequency dependence, the power spectral density of the input or output referred equivalent noise sources (even the stationary component) usually has complicated frequency-dependence. This is because the nonlinear circuit contains energy storage elements, such as capacitors and inductors, which shape the PSD of the noise signal by its transfer characteristics. In order to reflect this frequency dependence accurately at the macromodel level, extensive computations of the original system are required, thus defeating the purpose of macromodeling. Therefore, to find approximate, yet computationally inexpensive forms of this frequency dependent noise is of great value. To achieve this goal, as suggested in [3], the noise PSD of a time-varying system can be reformulated because (9) is structurally similar to (5) – which has been effectively used for modeling noise of time-invariant system [1]. We will begin with reformulation of noise analysis results to obtain a closed form rational expression of the cyclostationary noise PSD, then simplify this result into our macromodel.

If we denote $s=j\omega$, the output noise representation in (9) can be reformulated into the following form when only white noise sources are considered:

$$S_{yy}(s) = \begin{bmatrix} D^{T} & 0 \end{bmatrix} \left(\begin{bmatrix} 0 & J_{FD}^{T} \\ J_{FD} & -A_{FD}S_{uu}A_{FD}{}^{T} \end{bmatrix} + s\begin{bmatrix} 0 & -C_{FD} \\ C_{FD} & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} D \\ 0 \end{bmatrix}$$

This is because white noise sources Suu are not a function of frequency and can be absorbed into the frequency independent equivalent admittance matrix. The extension to non-white noise sources such as flicker noise can be achieved by expressing $Suu(\omega)$ as a matrix polynomial as adopted in [1]. The methodology of reformulation for non-white noise sources is similar to that for white noise except that the former results in larger equivalent admittance and capacitance matrices. For notational simplicity in this paper we restrict our discussion to white noise sources in the following sections.

The reformulated form above has the same information as contained in (9) except that it is in the form of transfer function. The impact of phase noise is regarded as an additional white noise source that it is represented by the $-A_{FD}S_{uu}A_{FD}{}^{T}$ term. If we denote:

$$\tilde{D} = \begin{bmatrix} D \\ 0 \end{bmatrix}, \quad \tilde{J}_{FD} = \begin{bmatrix} 0 & J_{FD}^{T} \\ J_{FD} & -A_{FD}S_{uu}A_{FD}{}^{T} \end{bmatrix}, \quad \tilde{C}_{FD} = \begin{bmatrix} 0 & -C_{FD} \\ C_{FD} & 0 \end{bmatrix}.$$

we arrive at the following form that is suitable for model order reduction:

$$S_{yy}(s) = \tilde{D}^{T}\left[\tilde{J}_{FD} + s\tilde{C}_{FD}\right]^{-1}\tilde{D} \qquad (11)$$

$$S_{yy}(\omega) = \begin{bmatrix} & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & S_{yy-2}(-\omega+2\omega_0) & S_{yy-1}(-\omega+2\omega_0) & S_{yy0}(-\omega+2\omega_0) & S_{yy1}(-\omega+2\omega_0) & S_{yy2}(-\omega+2\omega_0) & \cdots \\ \cdots & S_{yy-2}(-\omega+\omega_0) & S_{yy-1}(-\omega+\omega_0) & S_{yy0}(-\omega+\omega_0) & S_{yy1}(-\omega+\omega_0) & S_{yy2}(-\omega+\omega_0) & \cdots \\ \cdots & S_{yy-2}(-\omega) & S_{yy-1}(-\omega) & S_{yy0}(-\omega) & S_{yy1}(-\omega) & S_{yy2}(-\omega) & \cdots \\ \cdots & S_{yy-2}(-\omega-\omega_0) & S_{yy-1}(-\omega-\omega_0) & S_{yy0}(-\omega-\omega_0) & S_{yy1}(-\omega-\omega_0) & S_{yy2}(-\omega-\omega_0) & \cdots \\ \cdots & S_{yy-2}(-\omega-2\omega_0) & S_{yy-1}(-\omega-2\omega_0) & S_{yy0}(-\omega-2\omega_0) & S_{yy1}(-\omega-2\omega_0) & S_{yy2}(-\omega-2\omega_0) & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \quad (12)$$

With the reformulated rational expression in (11), further simplification can be derived. Writing the noise analysis results explicitly as in (12), we see that $S_{yy}(\omega)$ is an $n \times n$ matrix where n is the number of harmonic PSDs we considered. Each element $S_{yyi}(\omega)$ of this matrix denotes the ith harmonic PSD at frequency ω. A close observation reveals that each column of the $S_{yy}(\omega)$ matrix represents the same harmonic PSD at different frequencies. For numercial simulation it is beneficial to obtain multiple responses, but for the symbolic representation of noise harmonic PSD it would be redundant to compute all the elements in $S_{yy}(\omega)$. Instead, we need only to compute one element in each column to represent the corresponding harmonic PSD. Moreover, since we are only interested in the closed form of stationary noise PSD, only one element in the entire $S_{yy}(\omega)$ matrix needs to be computed. Instead of computing the $n \times n$ elements of the entire matrix, one element will suffice to represent the frequency dependence of the stationary noise PSD. Following these observations, we simplify (11) into:

$$\tilde{S}_{yy}(s) = \tilde{l}^T \left[\tilde{J}_{FD} + s\tilde{C}_{FD} \right]^{-1} \tilde{l} \quad (13)$$

where l is only the center column of D matrix. The results will be a scalar function of frequency.

After we derive the simplified reformulation of the noise PSD in the structure of a rational function, model order reduction is employed. This is imperative because the above expression is extremely large and increases with circuit size and number of harmonics of interest. With model order reduction we produce an approximation to match the first few moments of the rational function. These approximations have been well established for LTI systems via direct moment matching such as AWE [10], and implicit methods such as PVL [9] and PRIMA [7]. The author in [3] proposed a Time-varying Pade' (TVP) approximation to perform order reduction for linear time-varying systems. It essentially converts the time-varying system to an equivalent LTI system of much larger size. In this paper, the PRIMA algorithm is employed to implement the model order reduction step.

To apply PRIMA as described in [7], we first construct an equivalent circuit equation:

$$\begin{cases} \tilde{C}_{FD} \dfrac{dx}{dt} + \tilde{J}_{FD} x = \tilde{l} u_p \\ i_p = \tilde{l}^T x \end{cases} \quad (14)$$

This circuit equation denotes the same transfer characteristic as in (11), with the objective of calculating a subspace X, or *Krylov subspace*, of matrix $A \equiv -\tilde{J}_{FD}^{-1}\tilde{C}_{FD}$, such that:

$$X^T X = I_q, X^T A X = H_q \quad (15)$$

where Hq is a block Hessenberg matrix. The rank of X denoted as q is much smaller than the rank of the original matrix A. A reduced-order-model can be generated as follows:

$$\hat{J} = X^T \tilde{J}_{FD} X \quad \hat{C} = X^T \tilde{C}_{FD} X \quad \hat{l} = X^T \tilde{l} \quad (16)$$

The macromodel of the stationary component of the cyclostationary noise, therefore, can be expressed as:

$$\hat{S}_{yy}(s) = \hat{l}^T \left[\hat{J}_{FD} + s\hat{C}_{FD} \right]^{-1} \hat{l} \quad (17)$$

This algorithm requires only matrix-vector products. Even when $\tilde{C}_{FD}$ and $\tilde{J}_{FD}$ matrices are large, dense or difficult to factor, exploiting structure and using iterative linear algebra techniques can make these computations scale almost linearly with problem size. When these fast techniques are applied, the complexity grows approximately linearly with circuit size and number of harmonics, thus making it useful for large problems.

The order of approximation, denoted as q in equation (15), represents the size of the resultant system after model order reduction. Usually approximations that are adequate for system level analysis are obtained with fairly low orders of approximation, as demonstrated in following section.

4. Examples: noise macromodel for LNA and Mixer

We implemented our noise macromodeling algorithm in Matlab using a prototype frequency domain harmonic balance simulator to compute the steady-sate response and time-varying admittance and capacitance matrices. The simplified reformulation for the noise PSD is calculated, and then PRIMA is applied to generate the

reduced order noise macromodel. We present two examples to demonstrate the approach.
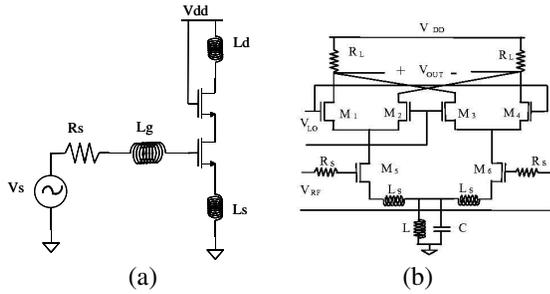
Fig. 1 schematic of LNA and Mixer

4.1 LNA with single tone blocking signal in adjacent channel

The first example is an LNA with blocking signal. The schematic of a single ended LNA is shown in Fig.1 (a). As specified in most wireless communication blocking characteristics, a relatively large blocking signal at an adjacent channel will appear at the input of LNA. The large blocking signal will modulate the operating-point of LNA periodically. This modulation can be interpreted as multiplication in time domain or convolution in frequency domain, thereby causing noise to mix up and down. This phenomenon is often referred to as *noise folding*, which is depicted graphically in Fig. 2.
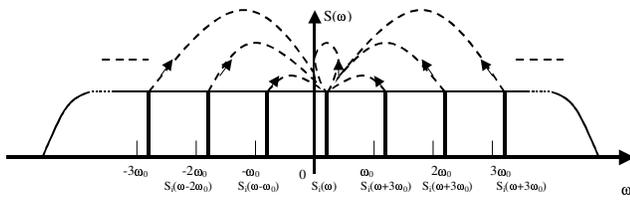
Fig. 2 Noise folding effect

The simulation of a simple LNA reveals that, when the input signal power is –102dBm with a –38dBm
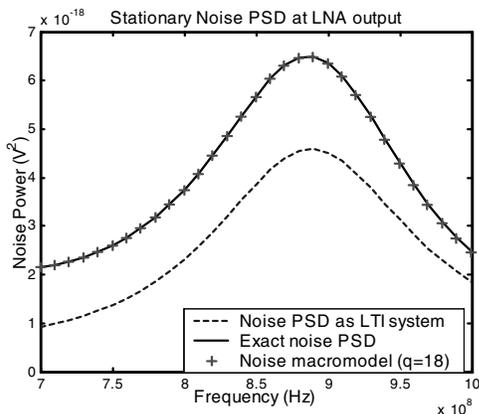
Fig. 3 LNA noise macromodel

blocking signal, which is specified in the GSM blocking characteristic, the output noise power spectral density increases by about 30% compared with no blocking signal. The macromodel of the noise PSD will capture this noise folding effect as shown in Fig. 3. It requires a macromodel of order 18 to accurately represent this frequency dependency. The maximum error between macromodel and exact results, which is obtained using full-fledged cyclostationary noise analysis, is less than 0.01%.

4.2 Mixer with phase noise at one input port

A double balance mixer, shown in Fig.1 (b) is used as second example. It is analyzed first using a pure sinusoidal signal for the local oscillator (LO) signal. Then phase noise of –90dBc/Hz at 600Khz offset is added into the LO port to model the phase noise that is present in an actual system. About 10% of the noise power increase can be observed at the intermediate frequency (IF). Fig. 4 shows that the noise macromodel with order of 20 captures this impact of input phase noise together with noise folding effect. The error between macromodel and analysis results is also less than 0.01%.
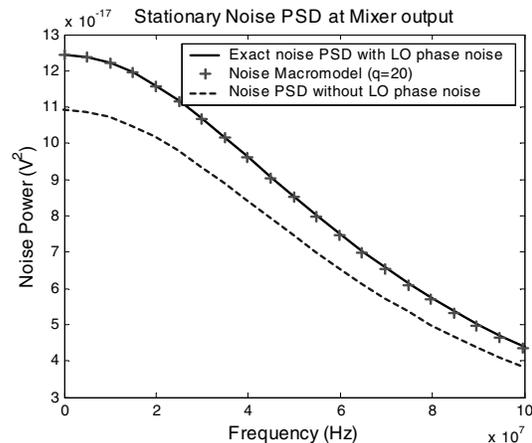
Fig. 4 Mixer noise macromodel

5. System level simulation results

We incorporated our noise macromodels in a prototype frequency domain system level simulation platform that provides a high-level analysis and optimization tool for design of RF circuits. The interface signals in this system level simulation environment are represented as stochastic processes in terms of the PSD distribution. This representation makes it easy to incorporate the noise impact into the system level simulation since the noise PSD is macromodeled directly. The algorithm of computing the signal PSD propagating through time-varying system is revealed through the following equation:

$$S_o(\omega) = \sum_{n=-\infty}^{\infty} |H_{-i}(\omega + n\omega_o)|^2 S_i(\omega + n\omega_o)$$

Where S_i and S_0 denote the input and out noise PSD, $H_{-i}(\omega + n\omega_0)$ represents the transfer function from input frequency of $\omega + n\omega_0$ to ω. This means that noise at the output for a particular frequency ω has contributions from the sources at frequency $\omega + n\omega_0$, where n is an integer and ω_0 is the fundamental frequency of the periodicity of the time-varying system.

Using the algorithm described above, we build a system level simulation platform for a GSM system. The block diagram is shown in Fig. 5.
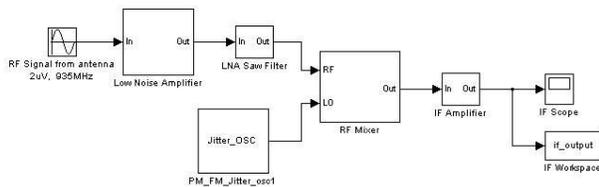
Fig.5 Block diagram of GSM system level simulation

The noise PSD distribution in each block is macromodeled and the impact to output is essentially to compute noise signals propagating through time-varying system. In this example, the LNA noise PSD is first macromodeled and then modulated by the linear time-varying transfer function of mixer. The noise generated by mixer itself is added at the output. The total output noise PSD of mixer is shown in Fig. 6. Therefore, the noise macromodel is seamlessly incorporated into the frequency domain system level simulation.
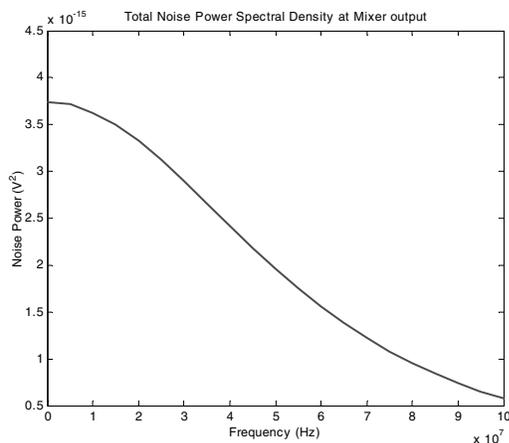
Fig. 6 Noise simulation results of GSM system

6.   Conclusions

A noise macromodel for radio frequency integrated circuits is extracted by simplified reformulation of frequency-domain noise analysis results and subsequent model order reduction via PRIMA. System-level simulation in terms of signal PSD is performed to incorporate the developed noise model. The efficiency of this macromodel is demonstrated in frequency domain system level simulation.

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