Electrical & Computer ENGINEERING

From Finance to Flip Flops:

Using the Mathematics of Money and Risk to Understand the Statistics of Nanoscale Circuits

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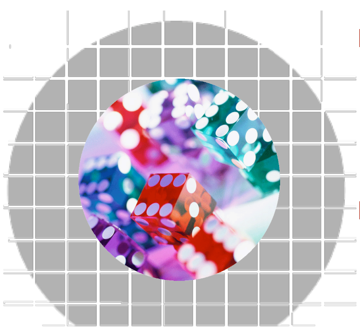
About This Talk



- Statistics for nanoscale ckts
  - The new challenge
- Monte Carlo analysis
  - How we do statistical analysis
- Mathematics of money+risk
  - Surprising source for very sophisticated Monte Carlo tools

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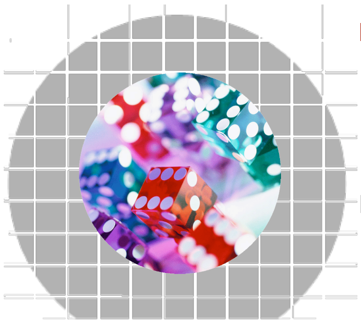

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  - **The new challenge**
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  - **How we do statistical analysis**
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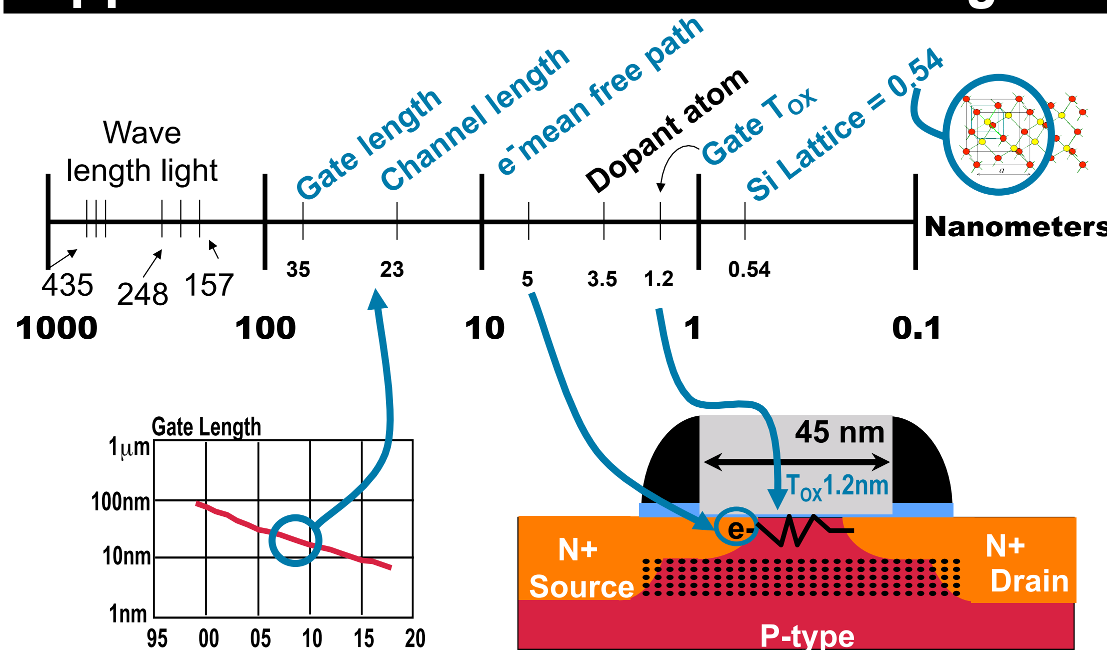
  - **Surprising source for most sophisticated Monte Carlo tools**

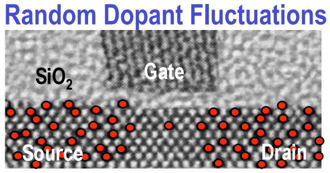
Approach *Atomic* Scale → *Challenges*



[Source: Scott Thompson, U Florida]

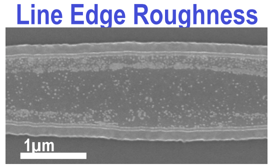
One Challenge: *Statistical Variation*

Random Dopant Fluctuations



A. Brown *et al.*, *IEEE Trans. Nanotechnology*, p. 195, 2002

Line Edge Roughness



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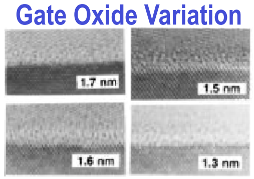
Gate Oxide Variation



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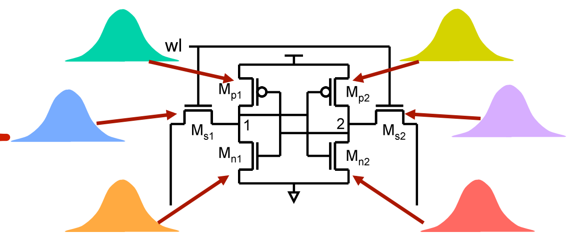
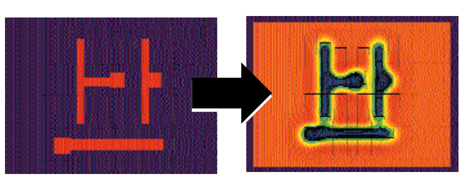
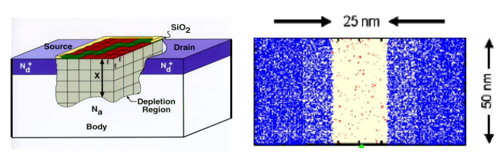


- **At nanoscale, nothing is *deterministic* anymore**
- **Everything is *statistical***

Statistical Variability: Two Flavors

- **Systematic variation**
  - Ex: Lithography
  - Optics, chemistry to print small mask shapes
- **Random variation**
  - Ex: Dopant fluctuation
  - How many individual dopant atoms; where?





- ***Not* really random**
  - Physics is understood, expensive to compute
- **Really (*really*) random**
  - Physics is fundamentally random for these effects

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End Result for Us Design/CAD Folks

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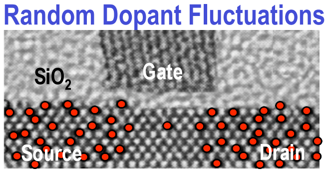
The Challenge: *Statistical Variation*

Random Dopant Fluctuations



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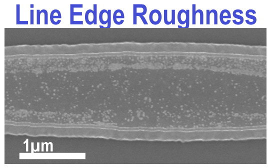

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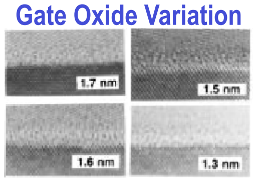
Gate Oxide Variation



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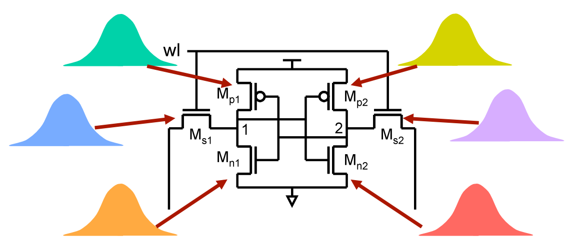
- **Everything is *statistical***
  - How do we analyze designs?



- **Old:** Delay = T
  Find T
- **New:** Prob[Delay < T] = 0.9
  Find T

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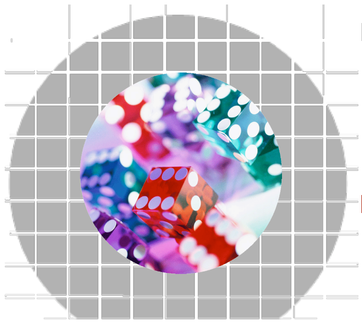
About This Talk



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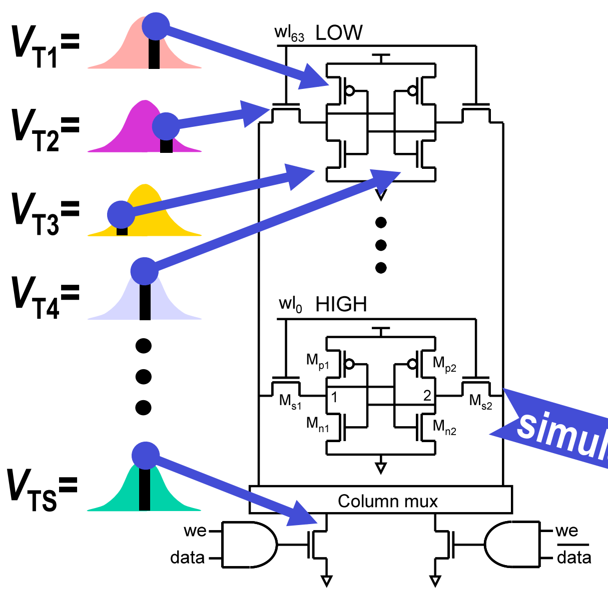
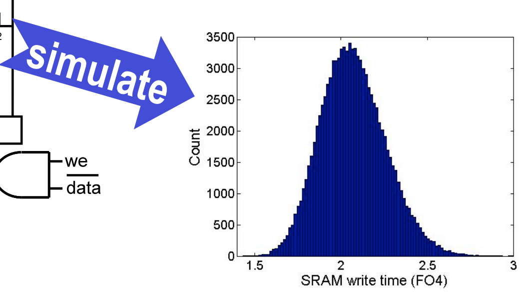
To Evaluate Circuit Impact: *Monte Carlo*



- **Sample each statistical variable**
- **Parameterize one circuit, simulate it**
- **Repeat--n samples**



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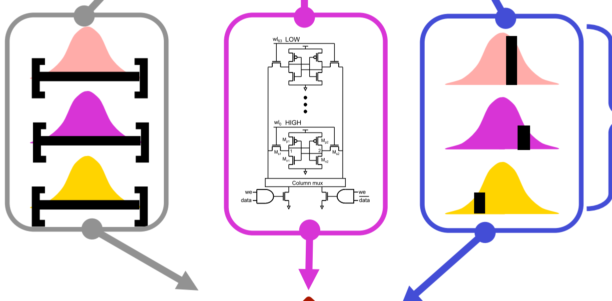
Monte Carlo Math: Just A Big Integral

$$\text{Est} = \int \cdots \iint f_{ckt}(\mathbf{x})\, p(\mathbf{x})\, d\mathbf{x}$$

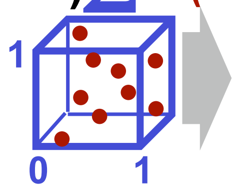
Stat space



s statistical vars

s-dimensional prob

$$\approx (1/n) \sum \hat{F}(\mathbf{u})$$



Can transform to sample uniformly from s-dim unit cube

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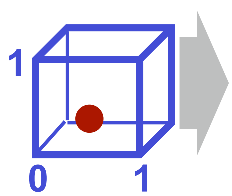
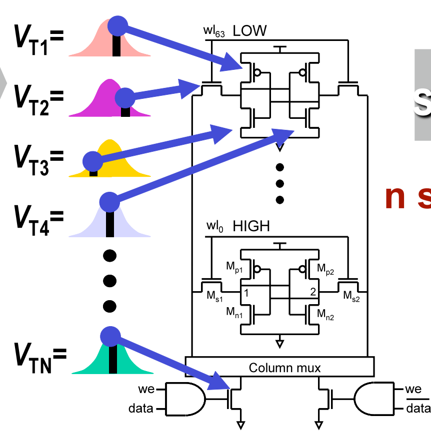
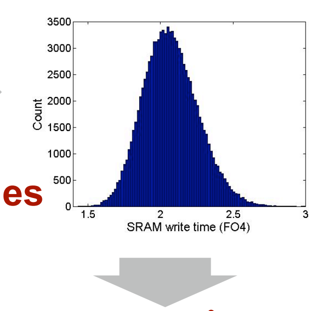
Evaluate Circuit Impact: Monte Carlo



Uniform random s-dim sample



n samples



$$\approx (1/n) \sum \hat{F}(\mathbf{u})$$

- **PRO:** Accurate, flexible, general
- **CON:** Slow, slow, s l o w…

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Why is Monte Carlo Painful?

- **High-dim problems:** ***s is big (100-1000)***
- **Profoundly nonlinear:** ***Nanoscale physics***
- **Accuracy matters:** ***~1-5% error***
- **Speed matters:** ***Many samples***
- **Samples expensive:** ***Simulate each circuit***

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Question: Who *Else* Has This Problem?

Computational finance

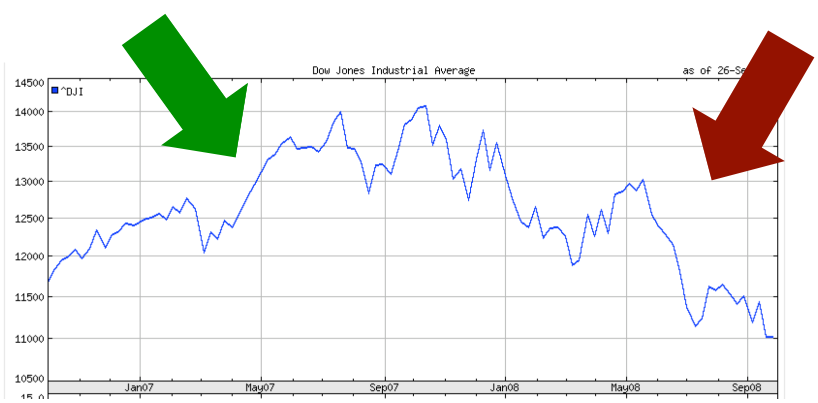
- **Valuing complex financial instruments, derivatives**
- **High-dimensional, nonlinear, statistical integrals**
- ***Speed+accuracy* matters here, e.g., ~real-time decision-making**

A Brief Aside: About "Finance…"

- These recruiting signs common in my building at CMU…
- …*last year*

A Brief Aside: In Defense of Finance…

- 12-18 months ago…
  - *"Wow, analyzing yield is like pricing a bond? Cool!"*
- 12-18 days ago…
  - *"Wow, you're using the same stuff that killed Wall Street?!"*



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A Brief Aside: In Defense of Finance


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The New York Times
Tuesday, September 23, 2008

Technology

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September 18, 2008, 7:52 AM

How Wall Street Lied to Its Computers

By SAUL HANSELL

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Before I started covering the Internet in 1997, I spent 13 years covering trading and finance. I covered my share of trading disasters from junk bonds, mortgage securities and the financial blank canvas known as derivatives. And I got to know bunch of quantitative analysts ("quants"): mathematicians, computer scientists and economists who were working on Wall Street to develop the art and science of risk management.

(Credit: Fred R. Conrad/The New York Times)

"The people who ran the financial firms **chose** to program their risk-management systems with **overly optimistic assumptions** and to feed them **oversimplified data**. …

… Wall Street executives had **lots of incentives to make sure their risk systems didn't see much risk**."

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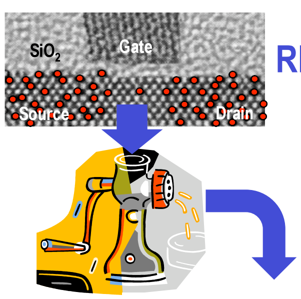
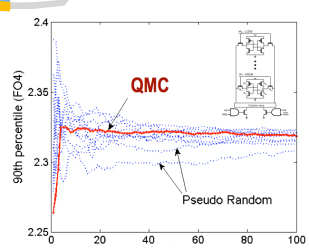
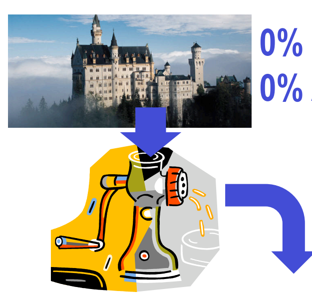
From Finance to Physics…

- Moral of story: If you start with ***honest physics*** as your input, you can get great results…



RDF





0% down!
0% APR!


Tuesday, September 23, 2008

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REVIEW & OUTLOOK | SEPTEMBER 23, 2008

The End of Wall Street

Article

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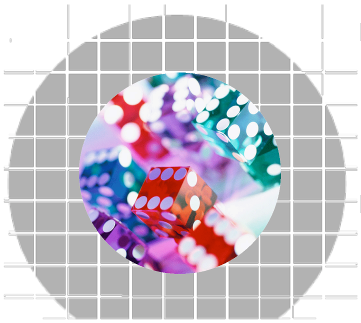
"The world had changed," said the Morgan Stanley spokesperson yesterday, and you can mark that down as the understatement of the year.

About This Talk



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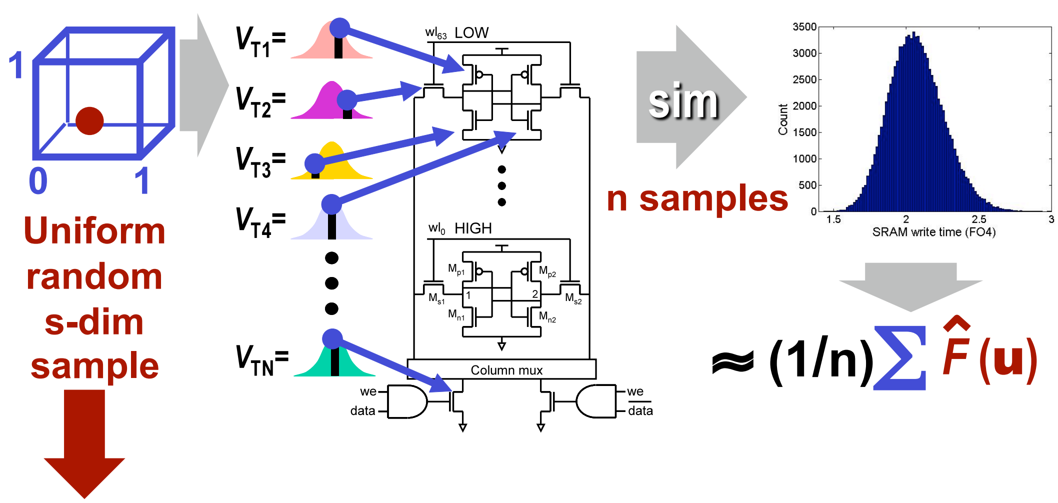
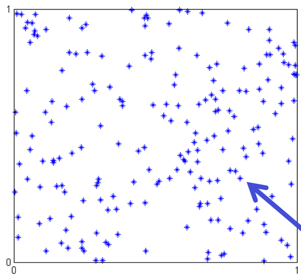
Monte Carlo Revisited: Uniform Sampling



- **Start from *uniform* random sampling in $[0,1]^s$**
- **"Unit cube" thing seems like a *minor* detail**
- **... but it turns out to be *crucial***

Monte Carlo: Uniform Sampling



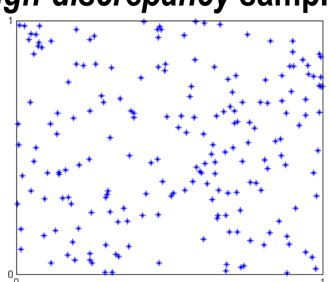
2-D example: unit cube is $[0,1]^2$

Independent, uniform random samples **(x,y)** in 2-D cube

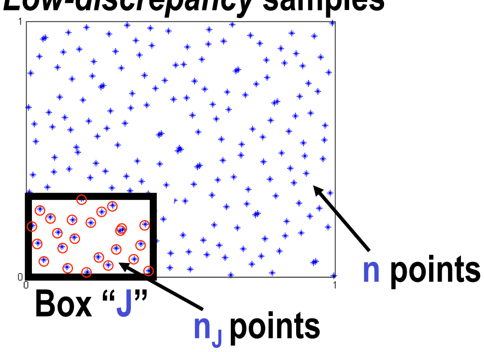
- **Classical Monte Carlo sampling**
  - Uses uniform pseudo-random pts (i.e, **rand( )**)
  - Surprise: *Not* very uniform
  - See clumps, holes in 2D example
  - Turns out this is inefficient

Uniformity Matters: Discrepancy

High-discrepancy **samples**



Low-discrepancy **samples**



Mathematically: the **discrepancy** is a measure of "uniformity"

$$D_n^* = \sup_J \left| \frac{n_J}{n} - Vol(J) \right|$$

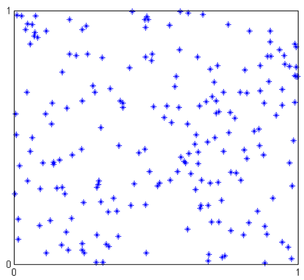
Fraction of points in J

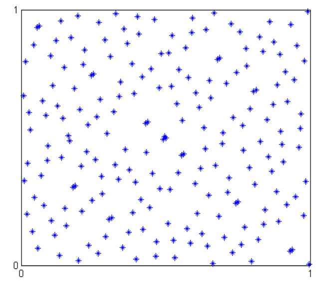
Fraction of volume occupied by J

How well does sampled n_J / n approximate *relative volume* of box?

For *low-discrepancy* sequences, answer is: *always very well.*

Big Idea: Quasi Monte Carlo (QMC)





- Classical Monte Carlo
  - Uniform pseudo-random pts
  - Problem: *not* very uniform
- Error for n samples

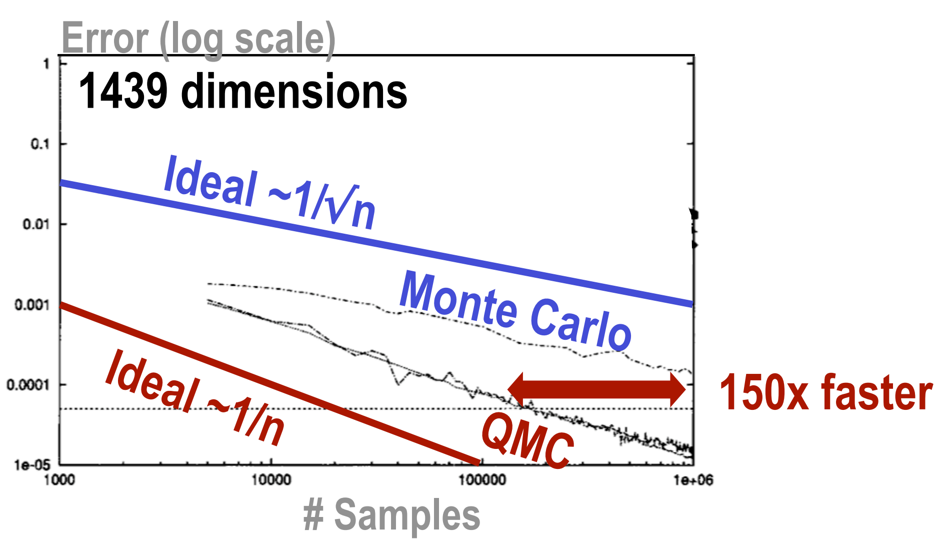
  $O(1 / \sqrt{n})$

- Quasi Monte Carlo
  - "Low-discrepancy" seq's
  - *Deterministic* samples
- Error for n samples

  $O(1 / n)$

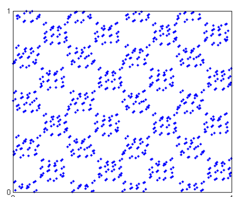
Computational Finance Example

- Eval 5-year discount price for a bond
  - From [Ninomiya,Tezuka, App Math Finance 1996]

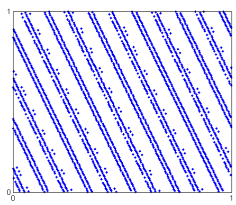


New Problem: Pattern Artifacts

- **Problem: Low Discrep Seq's show *patterns* in high dim's**
  - Need too many points for good uniformity

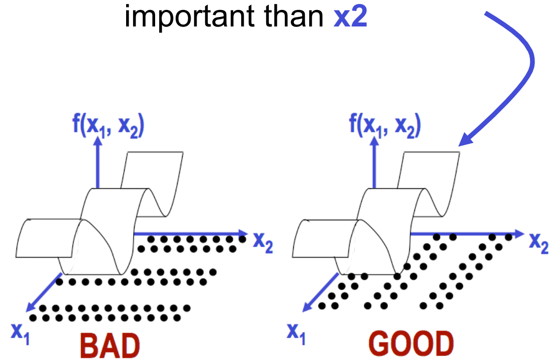


2 dims from 500-D Sobol' pts



2 dims from 500-D Faure pts

- **Solution: Since *earlier* dimensions *less* affected…**
  - Calculate ***statistical sensitivity*** of all vars
  - Put sensitive vars first
  - Ex: in **f(x1,x2)** if **x1** more important than **x2**



Putting It All Together

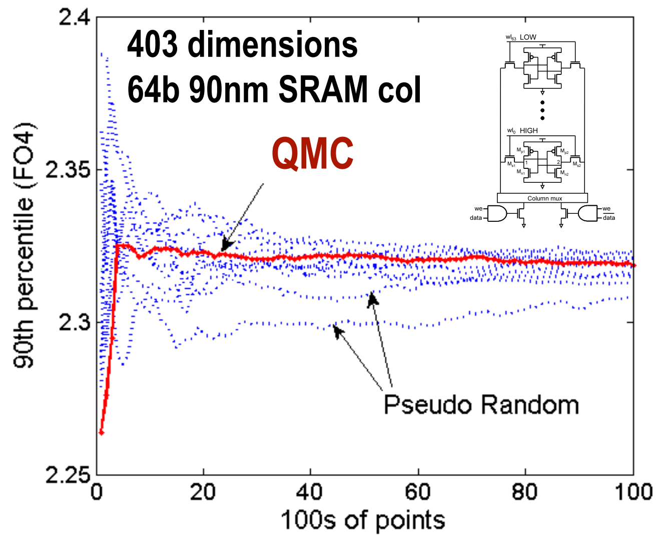
QMC steps

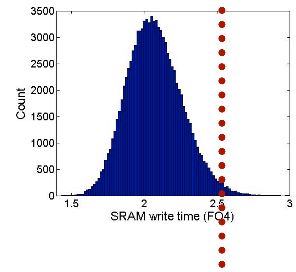
1. **Run** small, initial Monte Carlo
2. **Compute** sensitivity R_i for every statistical param x_i and output y
3. **Sort** x_i with decreasing sensitivity R_i for QMC sampling
4. **Foreach(** sample point **X=(x1, x2, …)** **)** from low discrepancy seq
   a. **Parameterize** circuit with **X**
   b. **Run** SPICE on this circuit

- **Technical details**
  - For multiple outputs, **add** the correlations for each input before sorting the x_i
  - We use **Sobol'** LDS points for our experiments

Does QMC Work for Circuits?

- **Yes!**
  - See: [Singhee, Rutenbar, ISQED 2007]
  - Example: Complete SRAM column @ 90nm

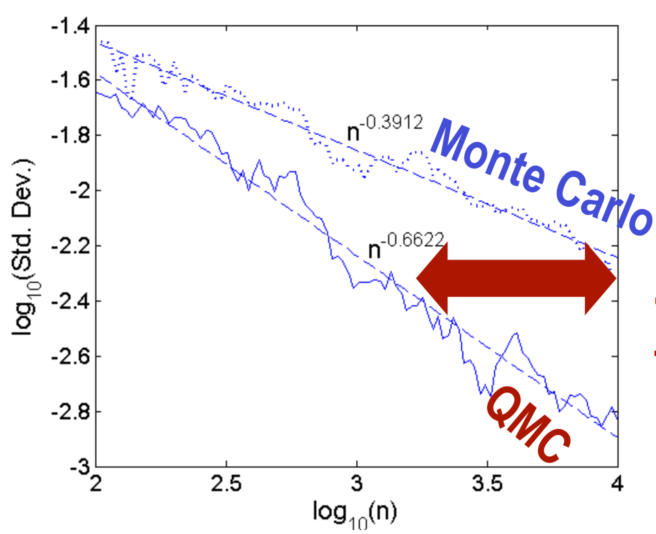




Pr(write < t_W) =0.9

Very Promising Speedups

- **Same 403-dimensional, 64b SRAM column**



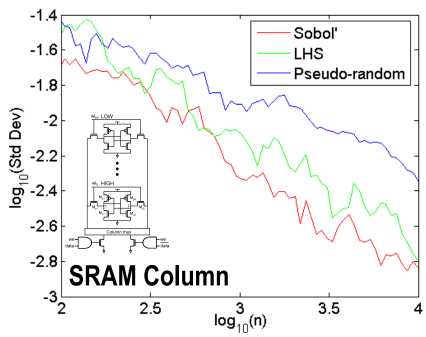
~9x faster for 1% error

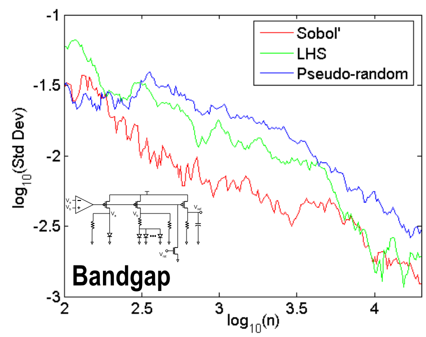
[Singhee, Rutenbar, ISQED 2007]

Isn't This Just Latin Hypercube Sampling?

- **No**
  - LHS sample set actually a *randomized* low-discrep seq
  - Considered "advanced" in EDA, *but inferior* to QMC
  - (Nobody prices bonds with LHS – it's all QMC)

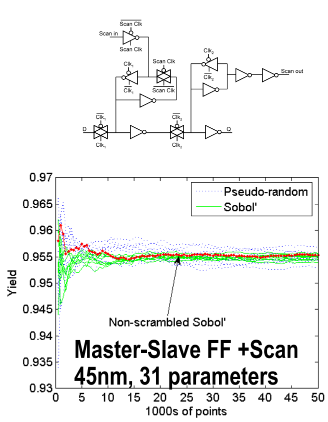
Plots: Error (est. variance across 10 runs) vs #samples *n*

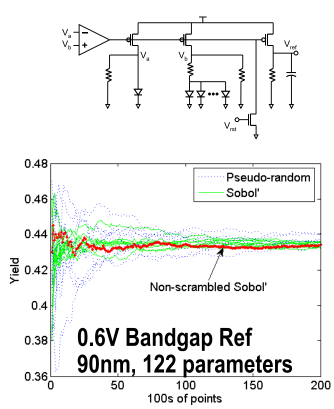


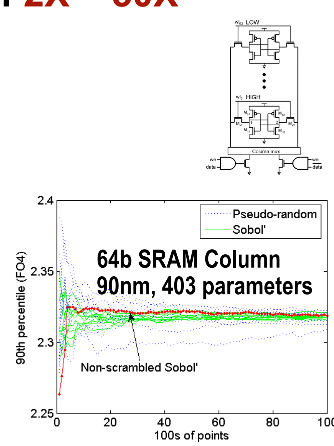


Good Behavior on Other Circuits

- **MC vs QMC variance, with #samples, *n***
  - 10 MC runs to compute MC variance
  - 1 QMC run + 9 *scrambled* QMC runs for variance
  - **General result: See speedups of 2X – 50X**

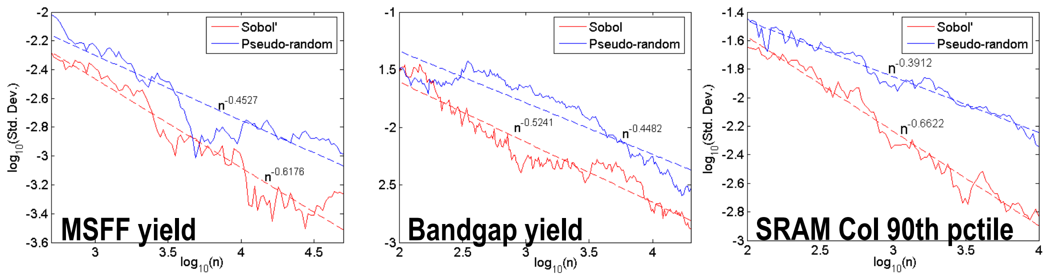






MC Variance vs. QMC Variance

- **Plot: Est. variance (10 runs) vs #samples *n***
  - Fit lines in log-log plot to estim convergence rate $O(n^{-\alpha})$.



- **Results**
  - QMC always shows **lower variance** than MC
  - QMC always shows **faster convergence** than MC.

Are We Done Yet? (Nope…)

- **Lots of ideas to exploit in this space**

Money…

…Risk

Another Aside: In Defense of Finance

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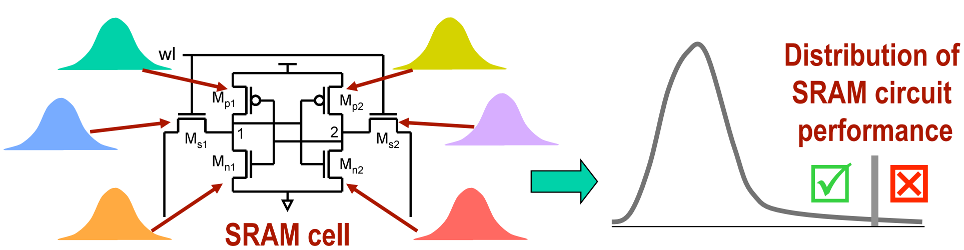
(Credit: Fred R. Conrad/The New York Times)

"In fact, most Wall Street computer models radically underestimated the risk of the complex mortgage securities … partly because the level of financial distress is 'the equivalent of the 100-year flood'…"

Related Problem: "Rare Event" Statistics

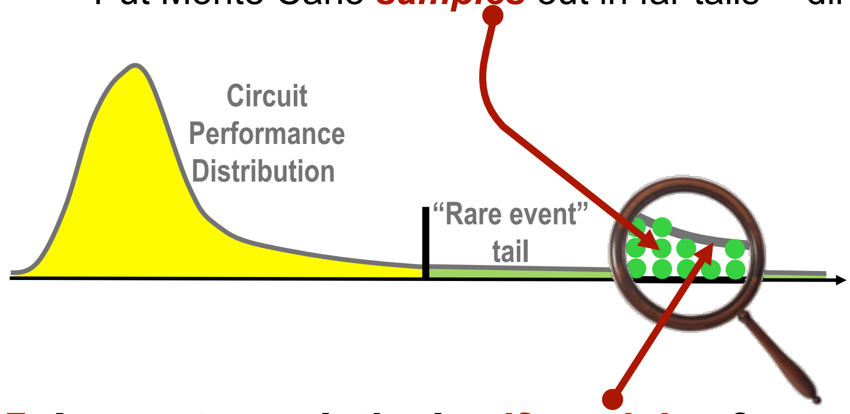
- **SRAM reliability is all about *far tails* of stats**
  - **Why?** High replication (~10^8 bits) of core circuits
  - 3σ doesn't cut it for 100M cells; need **6σ, 7σ, 8σ**…



- **Problem: *Intractable* Monte Carlo runs**
  - 1M Monte Carlo sims predicts (unreliably) to **~4.5σ**

What Do We Need To Solve This…?

- **Ultra fast *sampling* of rare events**
  - Put Monte Carlo ***samples*** out in far tails -- directly



- **Accurate analytical *pdf models* of rare tails**
  - Using these samples, model lets us predict ***farther***

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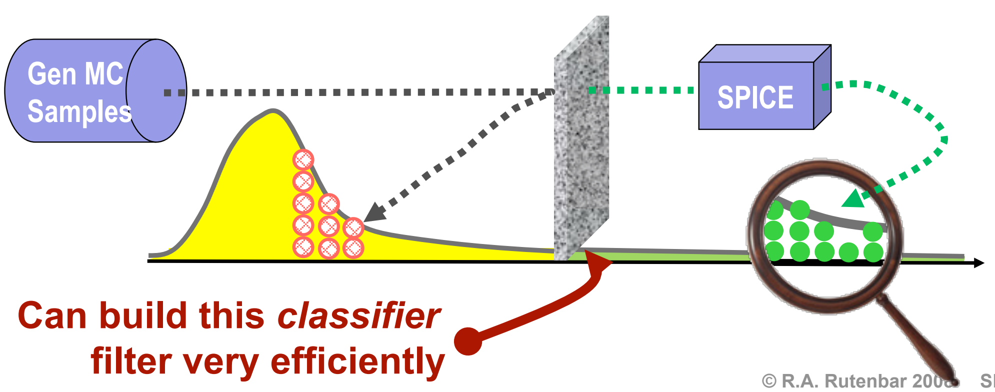
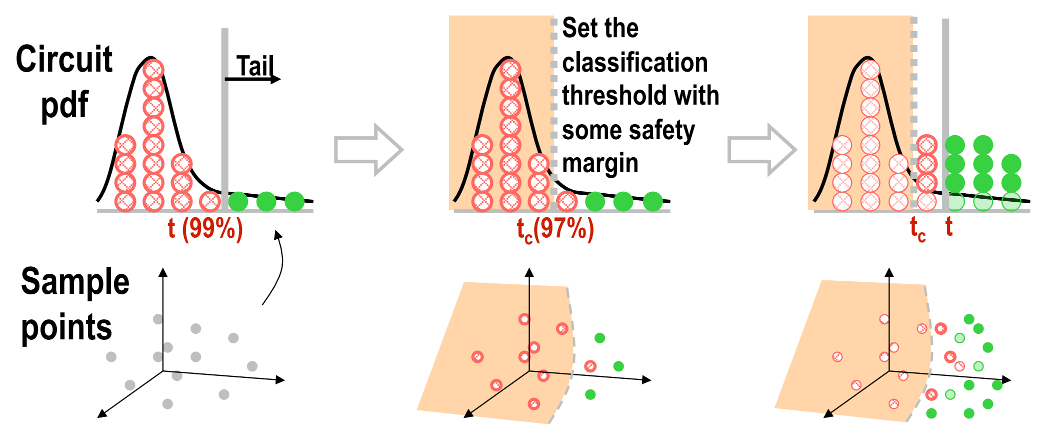
Efficiently Sampling *Just* the Tail

- **Note: *Generating* MC samples is cheap,**
  ***Simulating* these samples is costly**
- **Idea:**
  1. **Generate** regular MC samples…
  2. …but ***block*** points that are "very probably" ***not*** in tail
  3. **Simulate** the rest – i.e., the points we do not block



We Call the Idea: *Statistical Blockade*



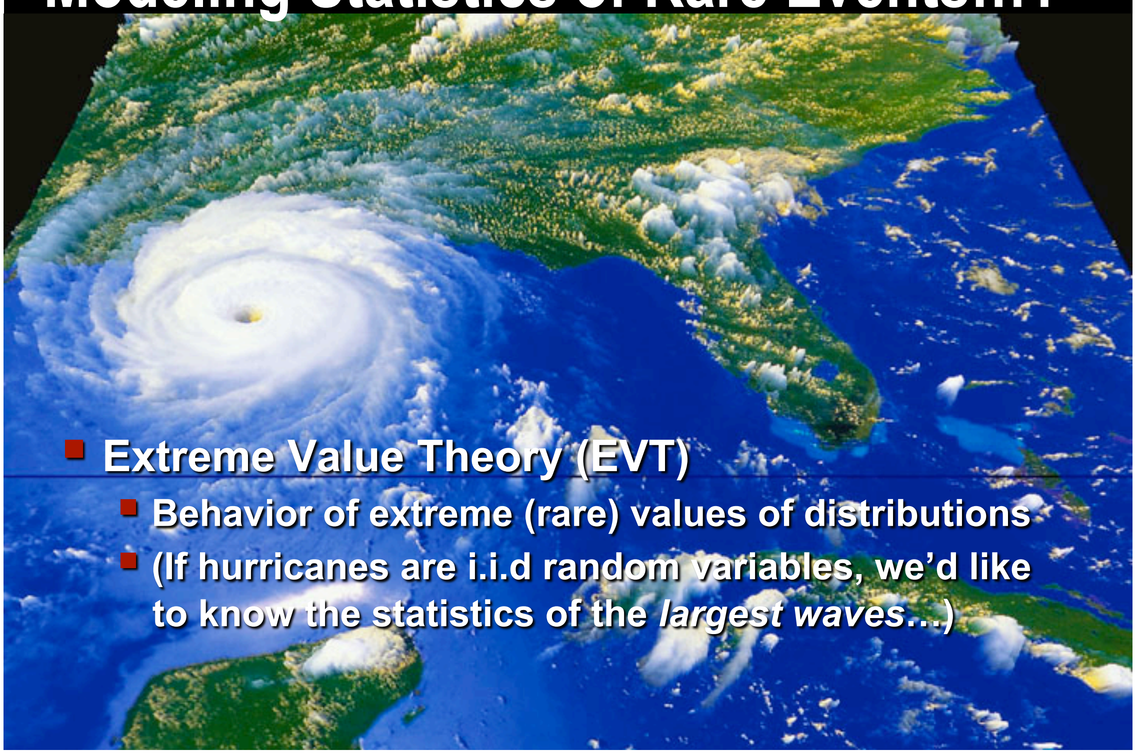
Simulate starting set
(few points, *fast*)

Build *classifier* (*fast*)
(uses ideas from data-mining; we use a *Support Vector Machine*)

Generate MC samples (*fast*)
Classify sample points (*fast*)
Block nontail points (*fast*)
Simulate the rest (*slow*)

[Singhee, Rutenbar DATE 2007]

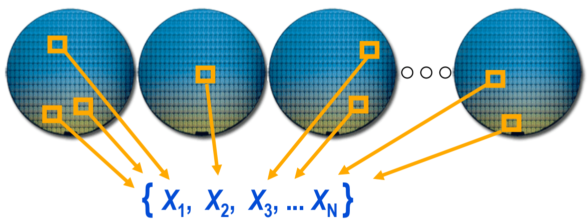
Modeling Statistics of Rare Events…?



- Extreme Value Theory (EVT)
  - Behavior of extreme (rare) values of distributions
  - (If hurricanes are i.i.d random variables, we'd like to know the statistics of the *largest waves*…)

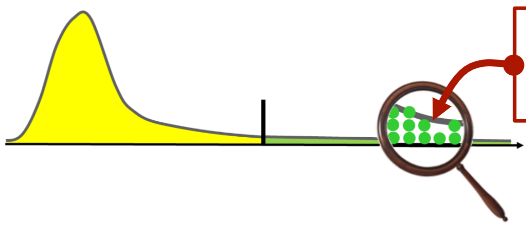
EVT: Modeling the PDF in the Tail

- **Recall Central Limit Theorem:** Σ(i.i.d. samples) → Gaussian
  - **Question**: Is there a similar result for these tails of “extreme” results ...?
  - **Answer**: YES – Extreme Value Theory (EVT)



On each of *N* wafers, identify cells slower than threshold t. What is their distrib? EVT tells us!

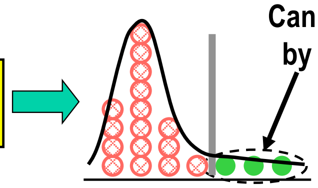
- **EVT gives simple analytical form for *conditional tail distrib***



$$G_{a,k}(x) = \begin{cases} 1-\left(1+\dfrac{kx}{a}\right)^{1/k} & , k \neq 0 \\ 1-e^{-x/a} & , k = 0 \end{cases}$$

EVT Says: GPD Form Fits Tail Stats

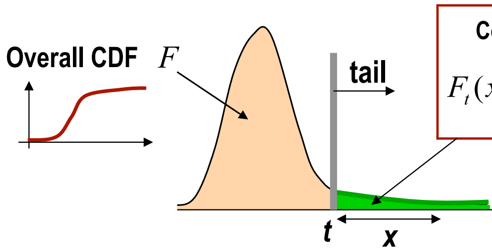
Run some sampling (Monte Carlo)



Can fit a GPD model to these tail points by fitting just 2 parameters

$$G_{a,k}(x) = \begin{cases} 1-\left(1+\dfrac{kx}{a}\right)^{1/k} & , k \neq 0 \\ 1-e^{-x/a} & , k = 0 \end{cases}$$

- **Aside: we actually fit the *conditional* CDF of the tail**
  - Ex: Prob[SRAM write-time > 2 FO4 delay] – the bad values

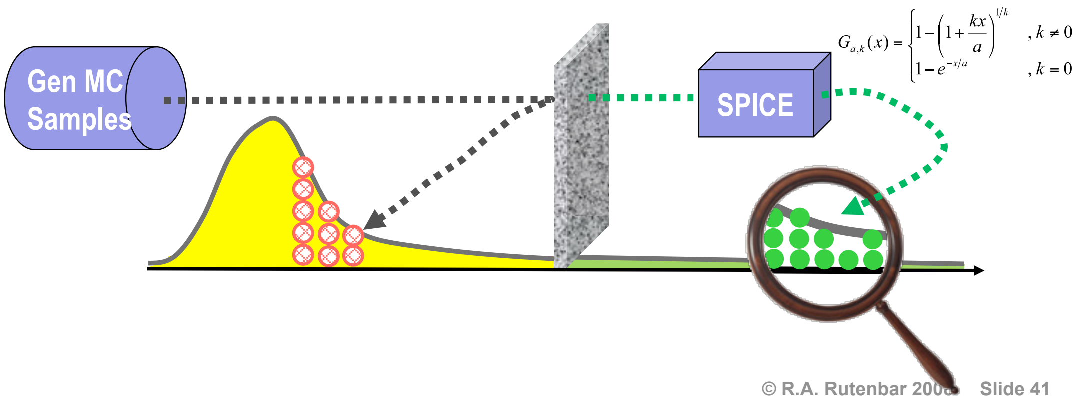


Conditional tail CDF

$$F_t(x) = \frac{F(x+t)-F(t)}{1-F(t)}$$

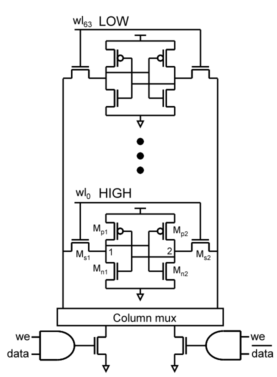
Put It All Together: Statistical Blockade

- **Statistical Blockade**
  1. **Generate** small set of regular MC samples
  2. **Build** classifier (the wall)
  3. **Generate** many MC samples, use classifier to ***block*** nontail pts
  4. **Simulate** the rest – i.e., the points we do not block
  5. **Use** EVT-GPD theory to build **analytical model** of tail stats

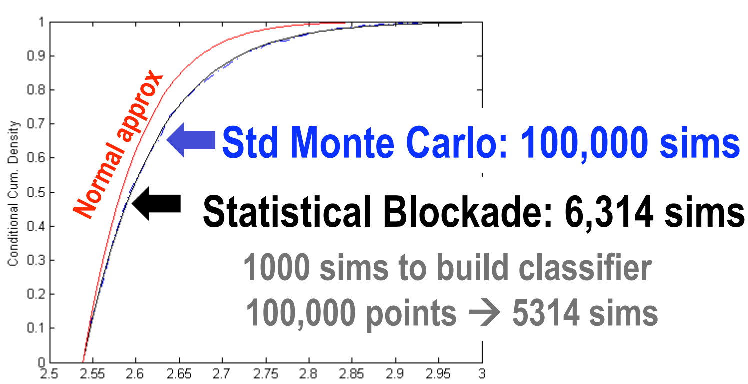


$$G_{a,k}(x) = \begin{cases} 1-\left(1+\frac{kx}{a}\right)^{1/k} & ,k \neq 0 \\ 1-e^{-x/a} & ,k = 0 \end{cases}$$

Result: Complete 64b SRAM Column



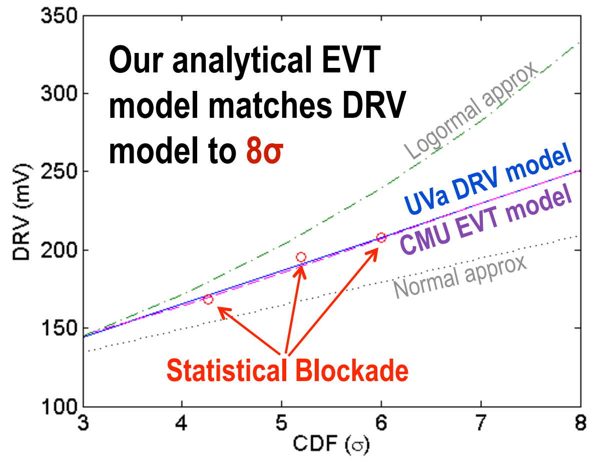
- **90nm 64b SRAM column with write driver and column mux**
  - **~ 400 devices; model Write-time CDF**
  - **Speedup: ~16X**



Result: Validating Model Out to 8σ

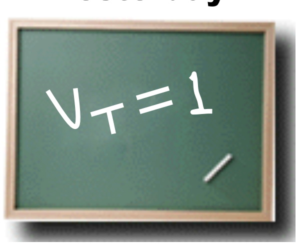
- **Recently validated novel analytical DRV model**
  - Model of Data Retention Voltage, [Calhoun et al. UVa, ESSCIRC'07]
  - Validated to **6σ**, via **billion element** Monte Carlo run…
  - …but only did 41,721 SPICE sims – **recursive** extension of Blockade
  - **Speedup ~23,000X**

[Singhee at al, 2008 Conf on VLSI Design]

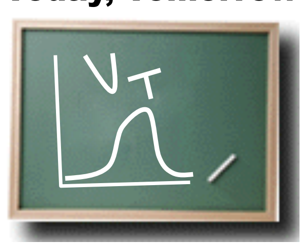


Summarizing

Yesterday



Today, Tomorrow



- **At nanoscale, nothing is deterministic…**
- **Brute-force Monte Carlo hurts (a lot)**
- **We can do *much better with smart methods***
  - (Many of which involve $$$ + risk…)
  - CMU results: **10x – 10,000x speedups**

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Thank You!

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