Needles and Haystacks

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Outline

1 Quantum Chromodynamics and Strong Interaction
   - Basics of Quantum Chromodynamics
   - Jet Events
   - The Problem

2 Wavelet Analysis
   - Discrete Wavelet Transform
   - What Do Wavelets Look Like?
   - Wavelet Transform in Higher Dimensions

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The Strong Force

- The strong force is used to represent the interactions at the most basic level.
- It is a fundamental force mediated by gluons acting upon quarks, antiquarks and gluons themselves.
- It is detailed by the theory of Quantum Chromodynamics.
Particls Involved

- Quarks are one of the two basic constituents of matter. There are six different flavors of quarks — up, down, charm, strange, top and bottom. Antiquarks are the antiparticles of quarks.

- To uphold Pauli’s Exclusion Principle, there has to exist another internal quantum number for the quarks. This quantum number, given the whimsical name “color”, is the charge involved in the gauge theory of quantum chromodynamics (QCD).

- The gluon is the gauge boson of QCD. QCD being a non-Abelian gauge theory the force carrying particles are also colored — gluons come in eight colors.
Particles Involved (cont.)

- Mesons are particles made up of quark-antiquark pairs. They are bosons and come under the overall class of hadrons.

- Baryons are made up of three quarks. The baryons are fermions, and are also included under hadrons.

- Recent experimental evidence shows the existence of five-quark combinations called pentaquarks. It is classified as a baryon, although an “exotic” one.

- Besides charge and spin, two other quantum numbers are assigned to hadrons — Baryon Number and Strangeness. The conservation of baryon number is an important rule for interactions and decays of baryons, which no known interactions violate.
Quantum Chromodynamics displays two peculiar properties:

- **Confinement**, which means that the force between quarks does not diminish as they are separated. Because of this, it would take an infinite amount of energy to separate two quarks. They are forever bound into hadrons such as the proton and the neutron.

- **Asymptotic freedom**, which means that in very high-energy reactions, quarks and gluons interact very weakly.
Asymptotic Freedom

- Asymptotic Freedom is a direct consequence of *antiscreening*, an effect opposite of the screening found in Quantum Electrodynamics.
- As QCD is non-Abelian, the force carrying particles are themselves colored.
- The net effect of polarization of virtual gluons in the vacuum is not to screen the field, but to augment it.
- Getting closer to the quark diminishes the antiscreening effect of the surrounding virtual gluons, and this weakens the effective charge.
- This implies that within nucleons, quarks move mostly as free, non-interacting particles, which is termed “Asymptotic Freedom”.

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Confinement

- A consequence of antiscreening is that the color force experience by quarks remains constant regardless of their distance from each other (after a certain point).
- When two quarks become separated, as in particle accelerator collisions, at some point it becomes energetically more favorable for a new quark-antiquark pair to be created spontaneously in vacuum, than to allow the quarks to separate further.
- Quarks can never be separated, leading to “Confinement”. In such an interaction, instead of individual quarks, many color-neutral particles are detected.
- This process is called *hadronization*, and is one of the least understood processes in particle physics.
Jet Events

- The tight cone of particles created by the hadronization of a single quark is called a jet.
- These jets must be measured in a particle detector and studied in order to determine the properties of the original quark.
- Figure shows the simulation of a jet event.
Jet Event Depiction

**Figure:** Top quark and anti-top quark pair decaying into jets visible as collimated collections of particle tracks in CDF detector at Tevatron.
Simulation

- The properties of the quarks and other elementary particles can be accrued by studying the hadronization jets.
- These are simulated using PYTHIA, which is a program for the generation of high-energy physics events, including collisions at high energies between elementary particles.
- It contains theory and models for a number of physics aspects, including hard and soft interactions, parton distributions, initial- and final-state parton showers, multiple interactions, fragmentation and decay.
Each event generates particles in thousands, and hundreds of such events need to be analyzed. The volumes of data involved are huge.

These volumes will increase many times, when actual jet events are observed instead of simulations.

Analyzing these massive amounts of data, even for something as simple as figuring out direction of the jets, will take years for normal analysis.
This project attempts at using Multiresolution Analysis using Wavelets, to find the proverbial *needles in the haystack*.

The aim for the first stage was to figure out the appropriate angular resolution at which the analysis should be carried out to be optimal in accuracy and time-complexity.
What is A Wavelet Transform?

- A wavelet is a kind of mathematical function used to divide a given function into different frequency components and study each component with a resolution that matches its scale.
- A wavelet transform is the representation of a function by wavelets.
- The wavelets are scaled and translated copies (known as “daughter wavelets”) of a finite-length or fast-decaying oscillating waveform (known as the “mother wavelet”).
Wavelet transforms have advantages over traditional Fourier transforms for

- representing functions that have discontinuities and sharp peaks
- for accurately deconstructing and reconstructing finite, non-periodic and/or non-stationary signals.
In continuous wavelet transforms, a given signal of finite energy is projected on a continuous family of frequency bands.

It is computationally impossible to analyze a signal using all wavelet coefficients.

It is sufficient to pick a discrete subset of the upper halfplane to be able to reconstruct a signal from the corresponding wavelet coefficients.

One such system is the affine system for some real parameters $a > 1, b > 0$.

$$\psi_{m,n}(t) = a^{-m/2} \psi \left( a^{-m} t - nb \right).$$
Wavelet Transform by Hardware

- The signal is passed through a quadrature mirror filter.
- The filter outputs are then downsampled by 2.
- This decomposition is repeated, the approximation coefficients decomposed and down-sampled again.

**Figure:** Level 3 filter bank
The Discrete Wavelet Transform (DWT) as an algorithm consists of applying a wavelet coefficient matrix hierarchically, first to the full data, then to the smooth part of half the length, then the smooth-smooth part of quarter the length, and so on.

The Haar wavelet can be considered as replacing every two consecutive data values by their average and difference (neglecting a numerical factor), and then rearranging the resulting data to have all the averages first and the differences later.

To invert the transform, the process is reversed, with the inverse of the matrix being used.
What Do Wavelets Look Like?

Table: DAUB1 (Haar), DAUB4 and DAUB20 wavelets.
Wavelet Transform in Higher Dimensions

- A wavelet transform of a $d$-dimensional array is obtained by transforming the array sequentially on its first index for all values of its other indices, then on its second, and so on.
- Each transformation corresponds to a multiplication by an orthogonal matrix, hence by matrix associativity, the result is independent of the order in which the indices are transformed.
- The levels of analysis and the indices can be interchanged without affecting the input, i.e., one particular index can be analyzed to the highest level before starting on the next one.
2D Wavelet Transform

Figure: Two dimensional wavelet transform
The first step of the analysis is to choose the optimum resolution at which to search for the jets. So, we have to carry out two preliminary steps:

1. Convert the data to spherical \((r, \theta, \phi)\) co-ordinates
2. Bin the data into appropriately sized parts and get a count of the number of particles per unit solid angle in each part, like a two dimensional histogram.
The initial resolution of the histogram is arbitrarily taken to be 1024 in both $\theta$ and $\phi$.

This does not divide the space into areas subtending equal solid angles.

While creating the histogram, the actual number of particles in that area has to be divided by the solid angle it subtends at the center.

If the bin is between $\theta_1$ and $\theta_2$ and has a width of $\Delta \phi$, the solid angle subtended is

$$\Delta \Omega = (\cos \theta_1 - \cos \theta_2) \cdot \Delta \phi.$$  \hspace{1cm} (2)
The optimum resolution can be taken to be the one where the histogram shows the most uneven distribution, or the most entropy.

First we carry out a two-dimensional wavelet transform on the histogram, down to the lowest level.

This will give us, for a resolution of 1024 in each direction, 1 average component ($LL$), and 9 different levels with three detail components ($LH$, $HL$ and $HH$) each.

The energy is defined as the average of the squares of all the detail components.

The energy is calculated for each component, and the level with the maximum energy is outputted for each event.
96% of the events showed a maximum energy at level 2, i.e., at a resolution of 4 in each direction, which implies that an optimal resolution of 4 should be used at each step of the analysis.

Once the area with the maximum particles is identified, the next step can again be undertaken with a further resolution of 4, in that area.

This recursion would have to be implemented at least 4 times, to get the position of the jets accurate to degrees.
**Results**

**Figure:** Optimal Resolutions in 100 Events.
The fact that a reasonable resolution was arrived at in almost all of the analyzed events, indicates that the analysis is on the right track. Stage two will involve using the results of the optimum resolution to perform the actual analysis of the data. The final aim is to extract as many properties of the jets as will be useful in studying the properties of the elementary particles. The second stage will also involve improving the code written for the first stage, maybe by using Fast Wavelet Transform (FWT) instead of DWT.
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