Probabilistic Modeling of Dependencies During Switching Activity Analysis

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***Abstract*—This paper addresses, from a probabilistic point of view, the issue of switching activity estimation in combinational circuits under the zero-delay model. As the main theoretical contribution, we extend the previous work done on switching activity estimation to explicitly account for complex spatiotemporal correlations which occur at the primary inputs when the target circuit receives data from real applications. More precisely, using lag-one Markov chains, two new concepts—conditional independence and signal isotropy—are brought into attention and based on them, sufficient conditions for exact analysis of complex dependencies are given. From a practical point of view, it is shown that the relative error in calculating the switching activity of a logic gate using only pairwise probabilities can be upper-bounded. It is proved that the conditional independence problem is NP-complete and thus, relying on the concept of signal isotropy, approximate techniques with bounded error are proposed for estimating the switching activity. Evaluations of the model and a comparative analysis on benchmark circuits show that node-by-node switching activities are strongly pattern dependent and therefore, accounting for spatiotemporal dependencies is mandatory if accuracy is a major concern.**

***Index Terms*— Markov chains, power estimation, signal isotropy, spatiotemporal correlations, switching activity analysis.**

I. Introduction

CAD tools play a significant role in the efficient design of the high-performance digital systems. In the past, time, area, and testability were the main concerns of the CAD community during the optimization phase. With the growing need for low-power electronic circuits and systems, power analysis and low-power synthesis have also become primary concerns for the CAD community.

To calculate the average power consumption in a gate-level implementation of a CMOS circuit, one can use the well-known formula $P_{\rm avg} = [(f_{\rm clk}/2)V_{dd}^2]\ \Sigma_x\ C(x)sw(x)$ where $f_{\rm clk}$ is the clock cycle frequency, V_{dd} is the supply voltage, $C(x)$ and $sw(x)$ represent the load capacitance and the switching activity, respectively, at the output of any gate x in the circuit [1]. As we can see, the average switching activity per node (gate) is a key parameter that needs to be correctly determined because charging and discharging different load capacitances is by far the most important source of energy dissipation in digital CMOS circuits.

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Power estimation techniques must be fast and accurate to be applicable in practice. Not surprisingly, these two requirements interact and at some point conflict with one another. Existing techniques for power estimation at gate and circuit-level can be divided in two classes: *dynamic* and *static* [1]. Dynamic techniques explicitly simulate the circuit under a "typical" input stream. Because their results depend on the simulated sequence, the required number of simulated vectors is usually high. These techniques can provide a high level of accuracy, but the run time is very high. A few years ago, the static techniques came into the picture and demonstrated their usefulness by providing sufficient accuracy with low computational overhead. These techniques rely on probabilistic information about the input stream (e.g., switching activity of the input signals, temporal correlations, etc.) to estimate the internal switching activity of the target circuit. From the very beginning, the major concern in probabilistic power estimation approaches was switching activity estimation because accounting for all dependencies which relate to the sequence and the circuit under consideration is by no means a trivial task.

Common digital circuits exhibit many dependencies; the most known one is the dependency due to reconvergent fanout among different signal lines, but even structurally independent lines may have dependencies (induced by the sequence of inputs applied to the circuit) which cannot be neglected. To date, only some dependencies have been considered and even then, only heuristics have been proposed. This is a consequence of the difficulty in managing complex data dependencies at acceptable levels of computational work. In addition to the dependencies described above (called also *spatial correlations*), another type of correlations, namely *temporal correlations*, may appear in digital circuits.

Let us consider a simple case to illustrate these issues. The circuit in Fig. 1 is fed successively by three input sequences, S_1, S_2, and S_3; S_1 is an exhaustive pseudorandom sequence, S_2 is also an exhaustive sequence but it is generated by a 3-bit counter, and S_3 is obtained from a "faulty" 3-bit counter.

All three sequences have the same signal probability on lines x, y, and c, but are otherwise very different. There are two other measures which differentiate these sequences, namely *transition* and *conditional* probabilities. More intuitively, these sequences exercise the circuit such that the number of transitions on each internal signal line (and hence the total number of transitions) is quite different once we feed S_1, S_2, or S_3. To accurately compute the number of transitions, we should undoubtedly account for the influence of the reconvergent fanout (e.g., in Fig. 1, a and b cannot

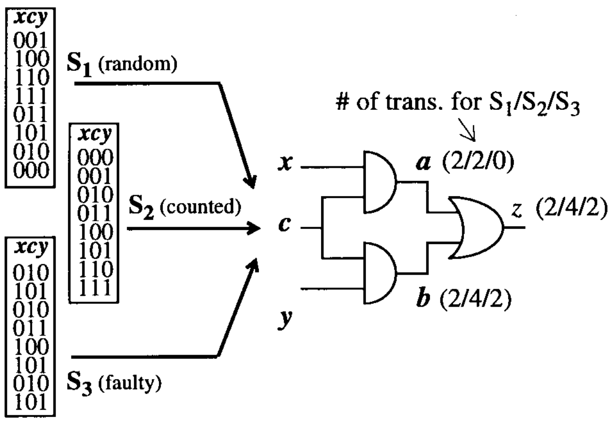


Fig. 1. The effect of spatiotemporal correlations.

be considered independent signal lines). This problem can be solved (albeit for small circuits only) by expressing the function of each signal line in terms of the circuit inputs, but even then, neglecting the correlations among circuit inputs can lead to incorrect results. As we see, assuming input independence for sequences S_2 and S_3 is unrealistic because the patterns in each of them are temporally correlated (e.g., each pattern in sequence S_2 is obtained from the previous one by adding a binary 1). Even more than this, transitions such as $0 \rightarrow 1$ or $1 \rightarrow 0$ on apparently independent signal lines (e.g., x and c in Fig. 1) are correlated and a detailed analysis on these input streams can reveal a strong spatial relationship. Consequently, to accurately compute the switching activity, one has to account for both spatial and temporal dependencies starting from primary inputs and continuing throughout the circuit.

Addressing these issues, this paper proposes a new analytical model which accounts for spatiotemporal correlations under the zero-delay model. Its mathematical foundation consists of using lag-one Markov chains to capture different kinds of dependencies in combinational circuits [2]. Temporal correlations for any signal x are considered through a Markov chain with only two states whereas spatial correlations for any pair of signals (x, y) are modeled by a four-state Markov chain. The basic assumptions used throughout the paper are:

- the target circuit is combinational and the logic value of any signal line x can only be 0 or 1;
- under the zero-delay model, any signal line x can switch at most once within each time step.

Under these hypotheses, we present theoretical and practical evidences showing that *conditional independence* is a concept powerful enough to overcome difficulties arising from the presence of structural dependencies and external input dependencies [3]. More precisely, based on conditional independence and *signal isotropy*, we give a formal proof showing that the statistics taken for pairwise correlated signals are sufficient to characterize larger sets of dependent signals.

A detailed analysis presented here illustrates the importance of being accurate node-by-node (not only for the total power consumption) and identifies potential drawbacks in the previous approaches when patterns feeding the inputs become highly correlated. To support the potential impact of this research, experimental results are presented for common benchmark circuits.

The paper is organized as follows. First, we review the prior work relevant to our research. In Section III, we introduce the analytical model for switching activity estimation which accounts for spatiotemporal correlations. In Section IV, we present global and incremental propagation mechanisms for transition probabilities and transition coefficients calculation; we also discuss the complexity of the proposed propagation mechanisms. In Section V, we improve the results in Section IV by providing an enhanced propagation mechanism based on conditional independence and signal isotropy. In Section VI, we report our results on benchmark circuits. Finally, we conclude by summarizing our main contribution.

II. Prior Work

Most of the existing work in pseudorandom testing and power estimation relies on probabilistic methods and signal probability calculations. One of the earliest works in computing the signal probabilities in combinational circuits is presented in [4]. While the algorithm is simple and general, its worse case time complexity is exponential. For tree circuits which consist of simple gates, the exact signal probabilities can be computed during a single post-order traversal of the network [5]. An algorithm, known as the *cutting algorithm*, which computes lower and upper bounds on the signal probability of reconvergent nodes is presented in [6]. The algorithm runs in polynomial time in the size of the circuit. Ercolani *et al.* present in [7] a procedure for propagating the signal probabilities from the circuit inputs toward the circuit outputs using only pairwise correlations between circuit lines and ignoring higher order correlations. The signal probability of a product term is estimated by breaking down the implicant into a tree of 2-input AND gates, computing the correlation coefficients of the internal nodes and then the signal probability at the output. Similarly, the signal probability of a sum term is estimated by breaking down the implicate into a tree of 2-input OR gates.

People working in power estimation have also considered the issue of signal probability estimation. An exact procedure based on ordered binary-decision diagrams (OBDD's) [8] which is linear in the size of the corresponding function graph (the size of the graph, of course, may be exponential in the number of circuit inputs) can be found in [9]. Using an event-driven simulation-like technique, the authors describe a mechanism for propagating a set of probability waveforms throughout the circuit. Unfortunately, this approach does not take into account the correlations that might appear due to reconvergent fanout among the internal nodes of the circuit. The authors in [10] use symbolic simulation to produce exact boolean conditions for switching at a particular node of the circuit. However, this approach is expensive in terms of computational cost (time and space requirements) and ignores the correlations at the primary inputs.

Recently, a few approaches which account for correlations have been proposed. Using an event-driven probabilistic simulation technique, Tsui *et al.* account in [11] only for first-order

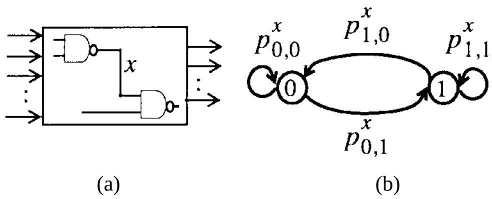


Fig. 2. A lag-one Markov chain describing temporal effects on line x.

spatial correlations among probabilistic waveforms. Kapoor in [12] suggests an approximate technique to deal with structural dependencies, but on average the accuracy of the approach is modest. In [13], Schneider *et al.* rely on lag-one Markov chains and account for temporal correlations; unfortunately, they assume independent transition probabilities among the primary inputs and use global OBDD's to evaluate switching activity (thus limiting the size of the circuits they can process).

In what follows, we introduce a new model that improves the state-of-the-art by taking into account spatiotemporal correlations at the primary inputs of the target circuit and by providing a general model for handling them inside the circuit.

III. An Analytical Model for Dependencies

We adopt the conventional probability model which consists of the triplet (Ω, Σ, p) where Ω represents the sample space, Σ denotes the class of events of interest, and p is the probability measure associated to Σ.

A. Temporal Correlations

Let us consider first a combinational logic module fed by the input vectors $V_1, V_2, \cdots, V_n$ [Fig. 2(a)]. While the input vectors $V_1, V_2, \cdots, V_n$ are applied to the primary inputs of the circuit, at time steps 1, 2, $\cdots, n$, the logic value of any internal line x may be 0 or 1. Hence, under the zero-delay model, x may switch at most once during each clock cycle. Let x_n be a random variable which describes the state of line x at any time n. If $\{x_n\}_{n\geq 1}$ is modeled as a lag-one Markov chain [Fig. 2(b)], then its behavior, over the state set $\Omega = \{0, 1\}$ can be described through the transition matrix Q [14]

$$Q = \begin{bmatrix} p^x_{0,0} & p^x_{0,1} \\ p^x_{1,0} & p^x_{1,1} \end{bmatrix}. \quad (1)$$

Every entry $p^x_{i,j}$ in the Q matrix represents the conditional probability of signal line x and may be viewed as the one-step transition probability to state j at step n from state i at step $n-1$.

Definition 1 (Conditional Probabilities): We define the *conditional probabilities* of any signal line x as

$$\begin{aligned} p^x_{i,j} &= p[(x_n = j) | (x_{n-1} = i)] \\ &= \frac{p[(x_n = j) \cap (x_{n-1} = i)]}{p(x_{n-1} = i)} \quad \forall i, j = 0, 1. \end{aligned} \quad (2)$$

We note that Q is a *stochastic* matrix, that is, every row adds to unity

$$p^x_{0,0} + p^x_{0,1} = 1 \quad p^x_{1,0} + p^x_{1,1} = 1. \quad (3)$$

A lag-one Markov chain has the property that one-step transition probabilities do not depend on the "history," i.e, they are the same irrespective of the number of previous steps. The process $\{x_n\}_{n\geq 1}$ is *homogeneous* and *stationary*: indeed, because any combinational circuit is a memoryless device (ignoring the floating nodes inside complex gates) having a homogeneous and stationary distribution at the primary inputs is a sufficient condition for homogeneity and stationarity to hold throughout the circuit [14]. Because the process $\{x_n\}_{n\geq 1}$ is homogeneous and stationary, $\mathcal{P}$, the probability distribution of the chain, may be expressed as

$$\mathcal{P} = \mathcal{P}\mathcal{Q} \quad (4)$$

where Q is the transition matrix of the chain.

Proposition 1[1]*:* The signal probabilities may be expressed in terms of conditional probabilities as follows:

$$p(x=0) = \frac{p^x_{1,0}}{p^x_{1,0} + p^x_{0,1}} \quad p(x=1) = \frac{p^x_{0,1}}{p^x_{1,0} + p^x_{0,1}}. \quad (5)$$

Definition 2 (Transition Probabilities): We define the *transition probabilities* of any signal line x as

$$p(x_{i\to j}) = p[(x_n = j) \cap (x_{n-1} = i)] \quad \forall i, j = 0, 1. \quad (6)$$

Signal, conditional, and transition probabilities associated with any signal line x are not independent measures. The following two propositions describe quantitatively the relationship between them.

Proposition 2: Transition probabilities may be expressed in terms of conditional probabilities as

$$\begin{aligned} p(x_{0\to 0}) &= \frac{p^x_{1,0} p^x_{0,0}}{p^x_{1,0} + p^x_{0,1}} \quad p(x_{0\to 1}) = \frac{p^x_{1,0} p^x_{0,1}}{p^x_{1,0} + p^x_{0,1}} \\ p(x_{1\to 0}) &= \frac{p^x_{1,0} p^x_{0,1}}{p^x_{1,0} + p^x_{0,1}} \quad p(x_{1\to 1}) = \frac{p^x_{1,1} p^x_{0,1}}{p^x_{1,0} + p^x_{0,1}}. \end{aligned} \quad (7)$$

□

Proposition 3: Conditional probabilities may be expressed in terms of transition probabilities as

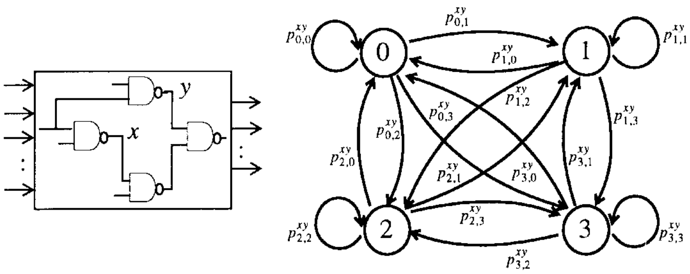
$$\begin{aligned} p^x_{0,0} &= \frac{p(x_{0\to 0})}{p(x_{0\to 0}) + p(x_{0\to 1})} \quad p^x_{0,1} = \frac{p(x_{0\to 1})}{p(x_{0\to 0}) + p(x_{0\to 1})} \\ p^x_{1,0} &= \frac{p(x_{1\to 0})}{p(x_{1\to 0}) + p(x_{1\to 1})} \quad p^x_{1,1} = \frac{p(x_{1\to 1})}{p(x_{1\to 0}) + p(x_{1\to 1})}. \end{aligned} \quad (8)$$

□

Example 1: Suppose that the signal line x takes the following successive values: "*aababaaabb*," where $a, b \in \{0, 1\}$. Then we have: $p(x = a) = 6/10, p(x = b) = 4/10, p^x_{a,a} = 3/6, p^x_{a,b} = 3/6, p^x_{b,b} = 1/4, p^x_{b,a} = 3/4, p(x_{a\to b}) = p(x = a)p^x_{a,b} = 3/10$, and $p(x_{b\to a}) = 3/10$.

As we can see, we need less information to compute the signal probabilities, but the ability to derive anything else is severely limited. On the other hand, once we get either conditional or transition probabilities, we have all we need to characterize that particular signal.

[1] Proofs are available from http://atrak.usc.edu/~radu/tech/tech.html.



(a) (b)

Fig. 3. A lag-one Markov chain describing spatial correlations between lines x and y.

Definition 3 (Switching Activity): For any signal line x the switching activity is defined as

$$sw(x) = p(x_{0\to 1}) + p(x_{1\to 0}) = 2\frac{p^x_{1,0}p^x_{0,1}}{p^x_{1,0}+p^x_{0,1}}. \tag{9}$$

As we can see, the switching activity depends only on the statistics for two consecutive time steps and thus, using lag-one Markov chains is sufficient for estimating switching activity.

Note: We should point out that (9) reduces to the well-known formula $sw(x) = 2p(x = 1)[1 - p(x = 1)]$ *only if* the events are *temporally uncorrelated*. As long as we deal with temporally correlated signals, the exact relation (9) should be used. For instance, in Example 1, $sw(x) = p(x_{0\to 1}) + p(x_{1\to 0}) = 3/10 + 3/10 \neq 2\times(6/10)\times(4/10)$.

B. Spatial Correlations

These correlations have two important sources:

- *structural dependencies* due to reconvergent fanout in the circuit;
- *input dependencies* that is, spatial and/or temporal correlations among the input signals which result from the actual input sequence applied to the target circuit.

Referring to the combinational module in Fig. 3(a), lines x and y are obviously correlated due to the reconvergent fanout; on the other hand, even independent signal lines like the primary inputs of this module may also become correlated due to a particular input sequence (as is the case for sequences S_2 and S_3 in Fig. 1 when structurally independent lines x and c become correlated).

To take into account the exact correlations is practically impossible even for small circuits. To make the problem more tractable, we allow only *pairwise correlated signals*, which is undoubtedly an approximation, but provides good results in practice. Consequently, we consider the correlations for all 16 possible transitions of a pair of signals (x, y) and model them by a lag-one Markov chain with four states [denoted by 0, 1, 2, and 3 which stand for the encoding 00, 01, 10, and 11 of (x, y) in Fig. 3(b)].

Definition 4 (Pairwise Conditional Probabilities): We define the *conditional probabilities* of a pair of signals (x, y) as:

$$p^{xy}_{a,b} = p[(x_n = k \cap y_n = l)|(x_{n-1} = i \cap y_{n-1} = j)] \tag{10}$$

where $a, b = 0, 1, 2, 3, a$ being encoded as ij and b as kl. Relation (10) basically describes the probability that the pair of signals (x, y) goes from state ij at time $n-1$ to state kl at time step n.

Ercolani *et al.* consider in [7] structural dependencies between any two signals in a circuit, through the *signal correlation coefficients (SC's)*

$$SC^{xy}_{ij} = \frac{p(x = i \cap y = j)}{p(x = i)p(y = j)} \tag{11}$$

where $i, j = 0, 1$. Assuming that higher order correlations of two signals to a third one can be neglected, the following approximation is used:

$$SC^{xyz}_{ijk} = \frac{p(x = i \cap y = j \cap z = k)}{p(x = i)p(y = j)p(z = k)} = SC^{xy}_{ij}SC^{xz}_{ik}SC^{yz}_{jk}. \tag{12}$$

Proposition 4: For every pair of signals (x, y) the following equations hold:

$$\sum_{j=0,1} SC^{xy}_{ij}p(y = j) = 1 \quad \forall i = 0, 1$$
$$\sum_{i=0,1} SC^{xy}_{ij}p(x = i) = 1 \quad \forall j = 0, 1. \tag{13}$$

The set of four equations and four unknowns $SC^{xy}_{ij}(i, j = 0, 1)$ is indeterminate; the matrix of the system has rank ≤ 3 in all nontrivial cases (i.e., when none of the signal probabilities is 1). □

Our approach is more general: to capture the spatial correlations between signals, for each pair of signals (x, y) and for all possible transitions, we consider instead *transition correlation coefficients (TC's).*

Definition 5 (Transition Correlation Coefficients): We define the $TC's$ for any two signals x, y as

$$TC^{xy}_{ij,kl} = \frac{p(x_{i\to k} \cap y_{j\to l})}{p(x_{i\to k})p(y_{j\to l})} \tag{14}$$

where $i, j, k, l = 0, 1$.

Note: If the signals a and b in Fig. 1 are spatially correlated, then based on $TC's$ defined above, we have

$$p(a_{0\to 1}b_{1\to 0}) = p(a_{0\to 1})p(b_{1\to 0})TC^{ab}_{01,10}.$$

Definition 6: We define the $TC's$ among three signals x, y, z as

$$TC^{xyz}_{ijk,lmn} = \frac{p(x_{i\to l} \cap y_{j\to m} \cap z_{k\to n})}{p(x_{i\to l})p(y_{j\to m})p(z_{k\to n})} \tag{15}$$

where $i, j, k, l, m, n = 0, 1$.

Neglecting higher order correlations, we therefore assume that the following holds for any signals x, y, z and any values $i, j, k, l, m, n = 0, 1$.

$$TC^{xyz}_{ijk,lmn} = TC^{xy}_{ij,lm}TC^{yz}_{jk,mn}TC^{xz}_{ik,ln}. \tag{16}$$

Proposition 5: For every pair of signals (x, y) the following equations hold:

$$\sum_{j,l=0,1} TC^{xy}_{ij,kl} p(y_{j\to l}) = 1 \quad \forall i, k = 0, 1$$
$$\sum_{i,k=0,1} TC^{xy}_{ij,kl} p(x_{i\to k}) = 1 \quad \forall j, l = 0, 1. \tag{17}$$

The above set of eight equations and 16 unknowns $TC^{xy}_{ij,kl}$ $(i, j, k, l = 0, 1)$ is indeterminate; the matrix of the system has rank ≤ 7 in all nontrivial cases. □

The last two propositions are very important from a practical point of view. The set of equations involving $SC's$ may be solved knowing only SC^{xy}_{11} for example, and this was the approach taken by Ercolani *et al.* in [7] (although, no similar analysis appeared in their original paper). In the more complex case involving $TC's$, we need to know at least nine out of 16 coefficients to deduce all other values.

IV. Propagation Mechanisms

Having already described the analytical model for dependencies, we present subsequently the mechanism for propagating spatiotemporal correlations from the primary inputs throughout the target circuit. To this end, in what follows, we ignore higher order correlations, that is, correlations between any number of signals are expressed only in terms of pairwise correlation coefficients.

Definition 6 and (16) may be easily extended to any number of signals. Based on the above assumption, we use an OBDD-based procedure for computing the transition probabilities and for propagating the $TC's$ throughout the network. The main reason for using the OBDD representation for a signal is that it is a canonical representation of a Boolean function and it offers a disjoint cover which is essential for our purposes. Depending on the set of signals with respect to which we represent a node in the boolean network, two approaches may be used.

- A *global approach*: for each node, we build the OBDD in terms of the primary inputs of the circuit.
- An *incremental approach*: for each node, we build the OBDD in terms of its immediate fanin and propagate the transition probabilities and the $TC's$ through the boolean network.

The first approach is more accurate, but requires much more memory and run time; indeed, for large circuits, it is nearly impractical. The second one offers good results whilst being more efficient as far as memory requirements and running time are concerned. However, the propagation mechanisms we present subsequently are equally applicable to both global and incremental approaches.

A. Computation of Transition Probabilities

Let f be a node in the boolean network represented in terms of n (immediate fanin or primary input) variables $x_1, x_2, \cdots, x_n; f$ may be defined through the following two sets of OBDD paths:

1) Π_1—the set of all OBDD paths in the ON-set of f;
2) Π_0—the set of all OBDD paths in the OFF-set of f.

Some of the approaches reported in the literature (e.g., [10]), use the XOR-OBDD of f at two consecutive time steps to compute the transition probabilities. We consider instead only the OBDD of f and through a dynamic programming approach, we compute the transition probabilities more efficiently. The probability of the event "f switches from value i to value j" $(i, j = 0, 1)$ may be written as

$$p(f_{i\to j}) = p\left(\bigcup_{\pi\in\Pi_i} \bigcup_{\pi'\in\Pi_j} \bigcap_{k=1}^{n} x_{k_{i_k\to j_k}}\right) \tag{18}$$

where i_k, j_k are the values of variable x_k on the paths π and π', respectively, (i.e., $x_k = i_k$ for the path $\pi, x_k = j_k$ for the path π' where $i_k, j_k = 0, 1, 2$, and 2 stands for *don't care* values) for each $k = 1, 2, \cdots, n$. In other words, this is the probability of the event which represents the *union* over all possible switches from a path $(i_1, i_2, \cdots, i_n)$ to a path $(j_1, j_2, \cdots, j_n)$.

Applying the property of disjoint events (which is satisfied by the collection of paths in the OBDD), the above formula becomes

$$p(f_{i\to j}) = \sum_{\pi\in\Pi_i} \sum_{\pi'\in\Pi_j} p\left(\bigcap_{k=1}^{n} x_{k_{i_k\to j_k}}\right). \tag{19}$$

However, since the variables x_k may *not* be spatially independent, the probability of a path to "switch" from $(i_1, i_2, \cdots, i_n)$ to $(j_1, j_2, \cdots, j_n)$ *cannot* be simply expressed as the product of the transition probabilities of the individual variables. Instead, we will use the following result which holds if we neglect higher order correlations.

Proposition 6: If (16) is true for any three signals in the set $\{x_1, x_2, \cdots, x_n\}$, then the transition probability of a signal f from state i to state j $(i, j = 0, 1)$ is

$$p(f_{i\to j}) = \sum_{\pi\in\Pi_i} \sum_{\pi'\in\Pi_j} \prod_{k=1}^{n} \left(p(x_{k_{i_k\to j_k}}) \prod_{1\leq k<l\leq n} TC^{x_k x_l}_{i_k i_l, j_k j_l} \right). \tag{20}$$

Although this expression seems to be very complicated, its complexity is within reasonable bounds. We will show that it is not necessary to enumerate all *pairs* of paths in the OBDD (which would provide a quadratic complexity in the number of paths in the OBDD), but for a fixed path in Π_i the computation may be done in linear time in terms of the OBDD nodes.

While for the global approach (20) can be applied knowing only the $TC's$ of the primary inputs, for the incremental approach we need a mechanism not only for computing the transition probabilities, but also for propagating the $TC's$ through the boolean network. For a given node in the circuit, it is only necessary to propagate the $TC's$ of the output with respect to the signals on which the inputs depend.

B. Propagation of Transition Correlation Coefficients

Let f be a node with immediate inputs $x_1, x_2, \cdots, x_n$ and x a signal on which at least one of the inputs $x_1, x_2, \cdots, x_n$ depends. Since the transition probabilities for f and x are already computed, the only problem now is to compute the probability of both f and x switching from i to j and from p to q, respectively. We have the following result.

Proposition 7: The TC between signals f and x, for any values $i, j, p, q = 0, 1$ may be expressed as in (21) located at the bottom of the page. In the incremental approach, (20) and (21) are applied in a recursive manner until all probabilities and $TC's$ become known.

C. Complexity Issues

To assess the complexity claimed in Section IV-A, let us define $f_{\pi \to j} = \cup_{\pi' \in \Pi_j} \ \cap_{k=1}^{n} \ x_{k_{i_k \to j_k}}$, such that $f_{i \to j} = \cup_{\pi \in \Pi_i} f_{\pi \to j}$ $(i, j = 0, 1$ and i_k, j_k are the values of variable x_k on paths π, π' respectively). Using the disjointness property of the paths in the OBDD, the corresponding probability is

$$p(f_{\pi \to j}) = \sum_{\pi' \in \Pi_j} p\left(\bigcap_{k=1}^{n} x_{k_{i_k \to j_k}}\right). \tag{22}$$

Since the path π is fixed, the above probability may be computed using the OBDD in the same way as a signal probability. Using Shannon decomposition, the above probability may be computed in linear time in the number of OBDD nodes. Indeed, $f_{\pi \to j}$ may be written as $f_{\pi \to j} = x_{k_{i_k \to 0}} f_{\pi \to j}^{\overline{x}_k} + x_{k_{i_k \to 1}} f_{\pi \to j}^{x_k}$ where $f_{\pi \to j}^{\overline{x}_k}, f_{\pi \to j}^{x_k}$ are the cofactors with respect to $\overline{x}_k$ and x_k, respectively. Based on this recursive decomposition, we may write a similar relation for the corresponding probabilities, taking also into account all possible existing correlations

$$\begin{aligned} p(f_{\pi \to j}) = {} & p\big(x_{k_{i_k \to 0}}\big) p\big(f_{\pi \to j}^{\overline{x}_k}\big) \prod_{k < l \le n} TC_{i_k i_l, 0 j_l}^{x_k x_l} \\ & + p\big(x_{k_{i_k \to 1}}\big) p\big(f_{\pi \to j}^{x_k}\big) \prod_{k < l \le n} TC_{i_k i_l, 1 j_l}^{x_k x_l}. \end{aligned} \tag{23}$$

Having computed this probability for each path π we immediately get the corresponding transition probabilities and hence the switching activity. Thus, for a fixed path π, the complexity is $O(n^2 N)$ where n is the number of variables and N is the number of nodes in the OBDD. (The n^2 factor comes from keeping track of the $TC's$ involved on each path. There is a number of $\binom{n}{2}$ factors in the product, thus the complexity is quadratic in the number of variables.) Hence, for all paths in Π_i, the time complexity is $O(n^2 N P)$ where P is the number of paths in the OBDD. In the incremental approach, this is within reasonable limits since usually n does not exceed three or four variables in the immediate fanin of the node.

Example 2: Let us consider the following function: $f = x_1 \oplus x_2 \oplus x_3$ and its OBDD representation in Fig. 4. Suppose $i = 0, j = 1$ and $\pi = (0\ 1\ 1)$ is a fixed path in the OFF-set Π_0 of f. We can compute the probability given in (23) by using a bottom-up parsing of the OBDD from the leaf labeled with "1" to the root. We adopt a dynamic programming approach in which at each level we use the results computed at the lower levels. Some partial results are shown in Fig. 4. For instance, in the case of node A, the variable x_3 must change from 1 to 0, and therefore node A is labeled with $p(x_{3_{1 \to 0}})$. At node B there are two alternatives: either x_2 switches from 1 to 1 and x_3 from 1 to 0, or x_2 switches from 1 to 0 and x_3 from 1 to 1. These transitions are not independent; thus, for each alternative we use the corresponding $TC's$ as shown in Fig. 4. The same operations are performed for any other path in Π_0. A similar approach can be further used to propagate the $TC's$ between f and any other signal x.

$$p\big(f_{\pi \to j}^{\overline{x}_1}\big) = p(x_{2_{1 \to 1}}) p(x_{3_{1 \to 0}}) TC_{11,10}^{x_2 x_3} + p(x_{2_{1 \to 0}}) p(x_{3_{1 \to 1}}) TC_{11,01}^{x_2 x_3}$$

$$p\big(f_{\pi \to j}^{x_1 \overline{x}_2}\big) = p\big(f_{\pi \to j}^{\overline{x}_1 x_2}\big) = p(x_{3_{1 \to 0}})$$

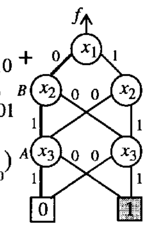


Fig. 4. Probability calculations for $f = x_1 \oplus x_2 \oplus x_3$.

V. An Axiomatic Approach to Conditional Probability

In this section, we discuss some practical limitations regarding the propagation mechanism described in Section IV. In particular, we introduce two new concepts: conditional independence and signal isotropy, which will help us to overcome these limitations.

A. Issues in Performance Management

In real examples, we may have to estimate power consumption in large circuits such as C6288, C7552, 32-bit multipliers, etc. where global approaches are totally impractical. In such cases, incremental approaches based on correlation coefficients are applicable, although they may require large running times for switching activity analysis [2]. Surprisingly enough, there are other circuits, much simpler in terms of their gate count and internal structure, which create a lot of problems in terms of running time. For example, for tree circuits, the incremental approaches need a large number of backtracks to compute the correlations among different signals and they almost "degenerate" to global approaches (as far as the running time is concerned).

The second issue is related to the degree in which signals are correlated. In general, large values of the correlation coefficients cause a lot of problems in the propagation of the coefficients due to the approximate formulas used throughout the calculations. As a consequence, the accuracy of computing the switching activity gets worse for highly correlated signals.

$$TC_{ip,jq}^{fx} = \frac{\displaystyle\sum_{\pi \in \Pi_i} \sum_{\pi' \in \Pi_j} \prod_{k=1}^{n} \left(TC_{i_k p, j_k q}^{x_k x} p\big(x_{k_{i_k \to j_k}}\big) \prod_{1 \le k < l \le n} TC_{i_k i_l, j_k j_l}^{x_k x_l} \right)}{p(f_{i \to j})} \tag{21}$$

These two limitations, namely excessive running time and accuracy degradation for highly correlated signals, stimulated us to further investigate stronger concepts able to overcome these drawbacks.

B. Conditional Independence and Signal Isotropy

Definition 7 (Conditional Independence): Given the set of n signals $\{x_1, x_2, \cdots, x_n\}$ and an index i $(1 \leq i \leq n)$, we say that the subset $\{x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n\}$ is *conditionally independent* with respect to x_i if the following holds:

$$p\left(\bigcap_{1 \leq j \leq n, j \neq i} x_j | x_i\right) = \prod_{1 \leq j \leq n, j \neq i} p(x_j | x_i). \tag{24}$$

Note: We note that if the set $\{x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n\}$ is conditionally independent with respect to x_i, it might not be conditionally independent with respect to $\overline{x}_i$. However, the corresponding set in which *any* variable (or subset of variables) is complemented, is still conditionally independent with respect to x_i if the conditions in Definition 7 are met.

Using the notion of support of a boolean function (i.e., the set of variables on which the function depends), we give the following definition.

Definition 8 (Logic Independence): Two boolean functions f and g are *logically independent* (denoted by $f \perp g$) if and only if $Sup(f) \cap Sup(g) = \emptyset$; if they are not logically independent then f and g must share at least one common input variable.

Note: It can be seen from the above definition that logic independence is a *functional* concept and does not use any information about the statistics of the inputs.

For boolean functions, we give the following property.

Proposition 8: Let f and g be two boolean functions and f^c, g^c the cofactors of f and g with respect to a common variable c; if $f^c \perp g^c$ and the variables in their support sets are independent, then f and g are conditionally independent with respect to c, that is

$$p(fg|c) = p(f|c)p(g|c). \tag{25}$$

Example 3: In the circuit in Fig. 1, signals a, b are conditionally independent with respect to c because $p(ab|c) = p(xyc)/p(c) = p(x)p(y)$ and $p(a|c)p(b|c) = p(xc)(yc)/p^2(c) = p(x)p(y)$.

It is worthwhile to note that, to compute $p(abc)$, if a and b are conditionally independent with respect to c, we may use only pairwise signal probabilities. Indeed, $p(abc) = p(ab|c)p(c) = p(a|c)p(b|c)p(c) = p(ac)p(bc)/p(c)$ which reduces the problem of evaluating the probability of three correlated signals to the one of considering only pairwise correlated signals.

As a conclusion, the concept of conditional independence can lead to efficient computations even in very complex situations. In fact, Proposition 8 gives us a *sufficient* condition for conditional independence and this is very useful from a practical point of view. However, the general problem to determine a variable x_i from a set of n signals $\{x_1, x_2, \cdots, x_n\}$ such that the remaining set of $(n-1)$ signals is conditionally independent with respect to x_i is a NP-complete problem.

Proposition 9 (Conditional Independence Problem): Given a set of n boolean functions $\{x_1, x_2, \cdots, x_n\}$, an index i and $k \leq n-1$, deciding whether there are at least k signals from the remaining subset conditionally independent with respect to x_i is a NP-complete problem.

Hint: We prove that conditional independence problem (CIP) is NP-complete using a reduction from the set packing problem [15]. ■

Because CIP is NP-complete, we need another concept to make the conditional independence relationship applicable in practice. To this end, we introduce the concept of *signal isotropy* which can be used in an approximate form as it will be shown subsequently.

Definition 9 (Signal Isotropy): Given the set of n signals $\{x_1, x_2, \cdots, x_n\}$, we say that the conditional independence relation is *isotropic* if it is true for all signals $x_1, x_2, \cdots, x_n$. More precisely, taking out all x_i's one at a time, the subset of the remaining $(n-1)$ signals is conditionally independent with respect to the taken x_i.

Returning to our circuit in Fig. 1, given the set of signals $\{a, b, c\}$, we have that $\{a, b\}$ is conditionally independent with respect to c but the sets $\{a, c\}$ or $\{b, c\}$ are not conditionally independent with respect to b or a, respectively; conditional independence is not isotropic in this particular case.

The concept of isotropy defined above is restrictive by its very nature. To make this concept more practical, we propose the following approximation.

Definition 10 (ε-Isotropy): The property of conditional independence for a set of n signals $\{x_j\}_{1 \leq j \leq n}$ is called ε-*isotropic* if there exists some ε $(\varepsilon \geq 0)$ such that

$$\left| \frac{\prod_{1 \leq j \leq n, j \neq i} p(x_j|x_i)}{p\left(\bigcap_{1 \leq j \leq n, j \neq i} x_j | x_i\right)} - 1 \right| \leq \varepsilon \quad \text{for any } i = 1, 2, \cdots, n. \tag{26}$$

Differently stated, ε-isotropy is an approximation of pure isotropy within given bounds of relative error. A natural question is then, how often is it appropriate to consider ε-isotropy as an approximation of pure isotropy? To answer this question, we consider in Fig. 5 three common situations involving the set of signals $\{u, v, w\}$ and the relative position of their logic cones (each cone illustrates the dependence of signals u, v, w on the primary inputs). Whilst the isotropy is completely satisfied only in (b), the ε-isotropy concept is applicable in all other cases. More precisely, the conditional independence relation is partially satisfied in (a) with respect to w and in (c) with respect to u and v.

Based on the previous definition, we get the following result.

Proposition 10: Given an ε-isotropic set of signals $\{x_j\}_{1 \leq j \leq n}$, the probability of the composed signal $p(\bigcap_{j=1}^{n} x_j)$

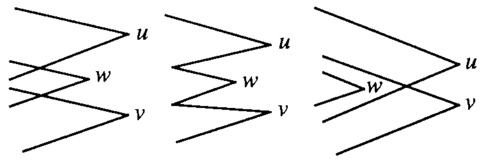


Fig. 5. An example to illustrate pure and ε-isotropy.

may be estimated within ε-relative error as

$$p\left(\bigcap_{j=1}^{n} x_j\right) = \frac{\left(\prod_{1\le i<j\le n} p(x_i x_j)\right)^{2/n}}{\left(\prod_{i=1}^{n} p(x_i)\right)^{(n-2)/n}}. \tag{27}$$

This proposition provides us with a strong result: given that n signals are ε-isotropic, the probability of their conjunction may be estimated within ε-relative error using only the probabilities for pairs of signals, thus reducing the problem complexity from exponential to quadratic.

C. Computation of Transition Probabilities Using ε-Isotropic Signals

If the property of ε-isotropy is satisfied, Proposition 10 may be easily extended to boolean functions represented by OBDD's. Let f be a boolean function of n variables $x_1, x_2, \cdots, x_n$ which may be defined through the ON- and OFF-sets as in Section IV. In the global approach, f is represented in terms of the primary inputs, while in the incremental approach it depends only on its immediate fanin variables. Based on this representation, we have the following result.

Proposition 11: Given f, a boolean function of variables $x_1, x_2, \cdots, x_n$, the following hold.

a) If the set $\{x_j\}_{1\le j\le n}$ (where every variable is either direct or complemented) is ε-isotropic, then the *signal probability* $p\ (f = i)$ (with $i = 0, 1$) may be expressed within ε-relative error as

$$p(x=i) = \sum_{\pi\in\Pi_i} \frac{\left(\prod_{1\le k<l\le n} p(x_k = i_k \cap x_l = i_l)\right)^{2/n}}{\left(\prod_{k=1}^{n} p(x_k = i_k)\right)^{(n-2)/n}} \tag{28}$$

where i_k is the value taken by the variable x_k in the cube $\pi \in \Pi_i$.

b) If the set $\{x_{j_{k\to l}}\}_{1\le j\le n, k,l=0,1}$ is ε-isotropic, then the *transition probability* $p\ (f_{i\to j})$ (with $i, j = 0, 1$) may be expressed within ε-relative error as

$$p(f_{i\to j}) = \sum_{\pi\in\Pi_i}\sum_{\pi'\in\Pi_j} \cdot \frac{\left(\prod_{1\le k<l\le n} p\left(x_{k_{i_k\to j_k}} \cap x_{l_{i_l\to j_l}}\right)\right)^{2/n}}{\left(\prod_{k=1}^{n} p\left(x_{k_{i_k\to j_k}}\right)\right)^{(n-2)/n}} \tag{29}$$

where i_k, j_k are the values taken by the variable x_k in cubes $\pi \in \Pi_i$ and $\pi' \in \Pi_j$. □

The above result may be reformulated using signal and transition correlation coefficients; it can be used in signal probability and switching activity estimation if the ε-isotropy conditions are met.

Corollary 1: Given a set of signals $\{x_j\}_{1\le j\le n}$ as in Proposition 11 and a boolean function f of variables $\{x_j\}_{1\le j\le n}$, the following relations hold within ε-relative error:

$$p(f=i) = \sum_{\pi\in\Pi_i}\left(\prod_{1\le k<l\le n} SC_{i_k i_l}^{x_k x_l}\right)^{2/n} \cdot \prod_{k=1}^{n} p(x_k = i_k)$$

$$p(f_{i\to j}) = \sum_{\pi\in\Pi_i}\sum_{\pi'\in\Pi_j}\left(\prod_{1\le k<l\le n} TC_{i_k i_l, j_k j_l}^{x_k x_l}\right)^{2/n} \cdot \prod_{k=1}^{n} p\left(x_{k_{i_k\to j_k}}\right). \tag{30}$$

For the incremental approach, this result can also be extended to the calculation of correlation coefficients $(SC's, TC's)$ between any two signals in the circuit. In practice, this becomes an important piece in the propagation of probabilities and coefficients through the boolean network.

D. Computation of TC's Using ε-Isotropic Signals

Proposition 12: Given a set of signals $\{x_j\}_{1\le j\le n}$, a boolean function f of variables $\{x_j\}_{1\le j\le n}$ and x a signal from the circuit, if $\{x_1, x_2, \cdots, x_n, x\}$ is a set as in Proposition 11, then the correlation coefficients $(SC's$ and $TC's)$ can be

$$\text{a)}\quad SC_{ij}^{fx} = \frac{\sum_{\pi\in\Pi_i}\left(\prod_{1\le k<l\le n} SC_{i_k i_l}^{x_k x_l}\right)^{2/(n+1)} \prod_{k=1}^{n}\left(p(x_k = i_k)\left(SC_{i_k j}^{x_k x}\right)^{2/(n+1)}\right)}{p(f=i)}$$

$$\text{b)}\quad TC_{ip,jq}^{fx} = \frac{\sum_{\pi\in\Pi_i}\sum_{\pi'\in\Pi_j}\left(\prod_{1\le k<l\le n} TC_{i_k i_l, j_k j_l}^{x_k x_l}\right)^{2/(n+1)} \prod_{k=1}^{n}\left(p\left(x_{k_{i_k\to j_k}}\right)\left(TC_{i_k p, j_k p}^{x_k x}\right)^{2/(n+1)}\right)}{p(f_{i\to j})} \tag{31}$$

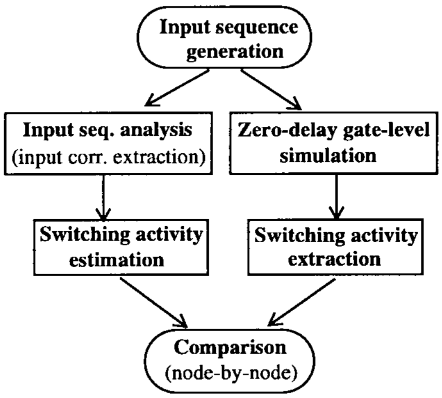


Fig. 6. The experimental setup.

expressed within ε-relative error[2] (as in (31) at the bottom of the previous page), where where $i, j, p, q = 0, 1$. □

These results lead to a new heuristic algorithm for signal and transition probability estimation under input streams which exhibit spatiotemporal correlations. We may thus see (30) as the improvement of (20) by using the concepts of conditional independence and signal isotropy. Compared to the heuristic proposed in Section IV, this new approach based on conditional independence has also the advantage that it supplies bounds for error estimation provided that the input signals are ε-isotropic. This bounding value could not be provided using the spatiotemporal hypothesis alone. Finally, in the incremental approach, the model introduced above provides a way to improve the run time requirements.

Proposition 13: If C_j is a correlation coefficient (SC or TC) at level j (given by a topological ordering from the inputs to the outputs of the circuit), then it is related to $C_{j-l} (0 < l < j)$ by a proportionality relationship having the form $C_j \propto (C_{j-l})^{2/(n+1)^l}$ where n represents the average fan-in value in the circuit. Moreover, if $l \to \infty$ then $(C_{j-l})^{2/(n+1)^l} \to 1$ (the signals on level $j - l$ behave as uncorrelated). □

In other words, we do not need to compute the coefficients which are beyond some level l in the circuit; instead, we may assume them equal to 1 without significantly decreasing the level of accuracy. Also, *the larger the average fanin n of the circuit, the smaller value for l* may be used. It is worthwhile to note that the conditional independence relationship, more specifically the concept of ε-isotropy, is essential for this conclusion. The approach presented in [2], based *only* on spatiotemporal correlations, does not provide a rationale for using such a limit. This is a very important heuristic to use in practice and its impact on the run time is huge; limiting the number of calculations for each node in the boolean network to a fixed amount (which depends on the value set as threshold for l) reduces the problem of coefficients estimation from *quadratic* (in the worst case) to *linear* complexity in the size of the circuit.

[2]This ε is the maximum over all values that occur during the incremental propagation process.

VI. Experimental Results

All experiments were performed using the SIS environment on an Ultra SPARC 2 workstation with 64 Mbytes of memory. The working procedure is shown below in Fig. 6.

As input sequences, we use highly correlated vector streams produced by different strategies: modified LFSR generators, generating pseudorandom vectors at the inputs of some circuit A and then cascading A with the target circuit B, using the state bit lines of different types of counters. For large circuits, we tried to keep time/space requirements of the simulation at a reasonable level and used up to 2^{20} input vectors during the actual logic simulation.

As standard measure for power estimation, we use the average switching activity at each node of the circuit calculated as in (9). We are interested in measuring the accuracy of the model in estimating the switching activity locally (at each internal node) and globally (for the entire circuit) given a set of inputs with spatiotemporal correlations. To report error, we use the standard measures for accuracy: maximum error (MAX), mean error (MEAN), root-mean square (RMS), and standard deviation (STD); we deliberately excluded the relative error from this picture due to its misleading prognostic for small values.

To illustrate the impact of correlations, we consider the benchmark $f51m$[3] and generate the inputs using the state lines of an 8-bit counter. The estimated values of the switching activity are compared against the exact values obtained by logic simulation; all internal nodes and primary outputs have been taken into consideration. The results are reported in Fig. 7 where, on the x axis, we plot the absolute error of switching activity, that is $|sw_{\text{exact}} - sw_{\text{estimated}}|$.

As the results show, the level of correlation on the primary inputs strongly impacts the quality of estimation. Specifically, it makes completely inaccurate the global approach based on input independence (despite the fact that internal dependencies due to reconvergent fanout are accounted by building the global OBDD). This is visible in the topmost diagram in Fig. 7, where less than 20% of the nodes are estimated with a precision higher than 0.1. On the other hand, even if temporal correlations are taken into account, but the inputs are assumed to be spatially uncorrelated (as in [13]), only 80% of the nodes are estimated with an error less than 0.1 (middle diagram). Accounting for spatiotemporal correlations provides excellent results for highly correlated inputs; in the lowest diagram, 100% nodes are estimated with a precision better than or equal to 0.1 and for 90% of the nodes the error is even less than 0.05.

These results clearly demonstrate that power estimation is a strongly pattern dependent problem, therefore accounting for dependencies (at the primary inputs and internally, among the different signal lines) is mandatory if the accuracy is important. From this perspective, considering spatiotemporal correlations and using signal isotropy seems to be the best candidate to date.

Using some ISCAS'85 benchmarks, we further performed the following types of experiments:

[3]To compare our approach with techniques that use global OBDD's, we had to choose a small circuit.

TABLE I
HIGHLY CORRELATED INPUTS ($f_{\rm clk} = 20$ MHz; $V_{dd} = 5$ V)

		WITH conditional independence (ε-isotropy)						WITHOUT conditional independence					
Circuit	Exact power [μW]	MAX	MEAN	RMS	STD	Estimated power [μW]	Time [sec]	MAX	MEAN	RMS	STD	Estimated power [μW]	Time [sec]
C432	395.50	0.2234	0.0133	0.0448	0.0432	351.37	57.03	0.8499	0.1058	0.2274	0.2032	1390.07	48.75
C499	2378.86	0.1552	0.0391	0.0692	0.0572	2260.24	81.77	0.4254	0.0387	0.0933	0.0851	2543.99	72.22
C880	248.75	0.0103	0.0009	0.0024	0.0022	243.28	64.18	0.7853	0.0471	0.1630	0.1571	482.60	53.32
C1355	1912.44	0.2603	0.0293	0.0665	0.0599	1813.74	84.42	0.4722	0.0516	0.1252	0.1144	2561.76	64.50
C1908	2821.10	0.3155	0.0236	0.0575	0.0525	2831.73	96.28	0.4903	0.0459	0.1000	0.0892	3201.10	88.10
C3540	169.55	0.0268	0.0002	0.0025	0.0025	167.32	550.15	0.5463	0.0280	0.0365	0.0365	207.38	495.49
C6288	6753.63	0.1366	0.0154	0.0154	0.0275	6800.83	769.37	0.5639	0.1092	0.1995	0.1685	19428.91	666.33
mul16	28221.04	0.2942	0.0356	0.0607	0.0492	30018.84	4111.93	0.9863	0.2198	0.3048	0.2112	47996.24	3968.76

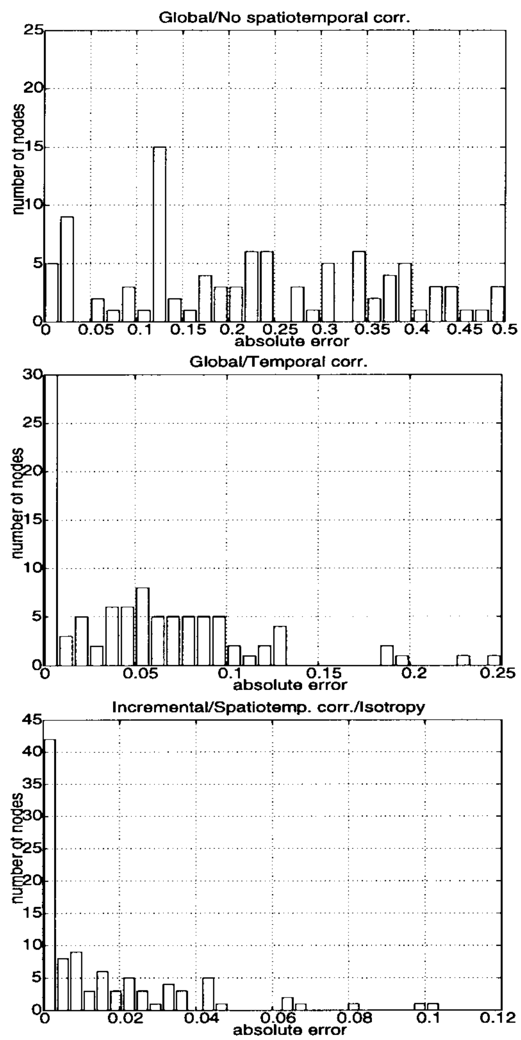


Fig. 7. The impact of the correlation level in $f51m$.

1) experiments to validate the model with conditional independence and ε-isotropy;
2) experiments to assess the impact of the limiting technique based on Proposition 13.

Once again, the switching activity values and power consumption were estimated at *each* internal node and primary output and compared with the exact values obtained from logic simulation. We found that power estimation for the entire circuit is not a real measure to use in low-power design where the switching activity at *each* node has to be accurately estimated.

1) Accuracy Results: The experiments were performed on large examples using highly correlated inputs obtained from counted sequences of length 2^{20}. To report the error, all estimations were verified against logic simulation performed with SIS. To show the impact of conditional independence, in the high-correlation scenario in Table I, we also present the results obtained if the conditional independence is not used. More specifically, the results in columns 3–8 are obtained as in Section V (i.e., applying (30), (31) recursively as many times as needed), while columns 9–14 are calculated as in Section IV [using (20), (21)].

As we can see, by using conditional independence and signal isotropy, the accuracy in node-by-node analysis improves on average by an order of magnitude; on the other hand, by not using conditional independence at all, the total power consumption for highly correlated inputs is overestimated by 100% on average.

2) Run Time Improvement: The heuristic proposed in Section V-C is important in practice not only for achieving a level of accuracy similar to that when the threshold limit is set to infinity, but also for substantially reducing the run time. We present in Table II[4] the results obtained for some benchmarks using the limit $l = 4$ in $TC's$ calculation (that is, the allowed number of recursive calls of (31) is limited to 4). By comparing these results with those obtained for $l = \infty$ in Table I (columns 3–8), we can see that the quality of the estimates remains basically the same while the run time is significantly improved.

We can see in Fig. 8 that the speed up is about three to five times for less complex circuits, but it may become 15 to 20 times for large examples.

VII. CONCLUSION

We have proposed an original approach for switching activity estimation in combinational logic modules under pseudo-

[4] Similar results have been obtained for pseudorandom inputs.

TABLE II
HIGHLY CORRELATED INPUTS WITH LIMIT $l = 4$ ($f_{\mathrm{clk}} = 20$ MHz; $V_{dd} = 5$ V)

Circ.	MAX	MEAN	RMS	STD	Power [μW]	Time [sec]
C432	0.2342	0.0141	0.0465	0.0447	368.97	10.40
C499	0.1566	0.0421	0.0760	0.0634	2283.03	8.18
C880	0.2478	0.0265	0.0591	0.0529	263.13	13.16
C1355	0.0068	0.0009	0.0021	0.0019	1865.81	5.01
C1908	0.3157	0.0251	0.0613	0.0561	2826.10	8.11
C3540	0.0250	0.0002	0.0024	0.0024	168.33	42.17
C6288	0.0741	0.0138	0.0268	0.0232	7360.98	22.54
mult16	0.2943	0.0445	0.0720	0.0566	31143.68	23.74
mult32	0.3648	0.0531	0.0918	0.0749	145263.44	103.87

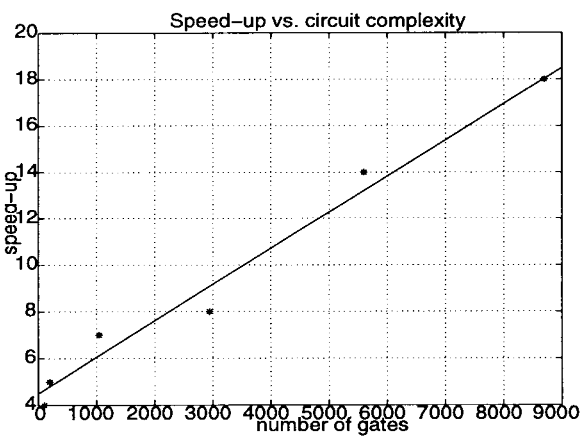


Fig. 8. The speed-up factor versus circuit complexity.

random or highly biased input streams. Using the zero-delay hypothesis, we have derived a probabilistic model based on lag-one Markov chains which supports spatiotemporal correlations among the primary inputs and internal lines of the circuit under consideration. From this perspective, the new concepts of conditional independence and signal isotropy are used in a uniform manner to fulfill practical requirements for fast and accurate estimation. Under general assumptions, the conditional independence problem has been shown to be NP-complete; consequently, efficient heuristics have been provided for probabilities and correlation coefficients calculation.

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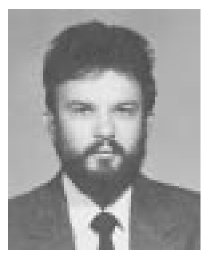

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