18-799F Algebraic Signal Processing Theory
Spring 2007
Solutions: Assignment 5

Filter: $h = H(z) = \sum_{n\in\mathbb{Z}} h_n z^{-n}$, where $(h_n)_{n\in\mathbb{Z}} \in l^1(\mathbb{Z})$, i.e. $\sum_{n\in\mathbb{Z}} h_n < \infty$. Thus, $\mathcal{A} = \Phi(l^1(\mathbb{Z}))$.

Signal: $s = S(z) = \sum_{n\in\mathbb{Z}} s_n z^{-n}$, where $(s_n)_{n\in\mathbb{Z}} \in l^2(\mathbb{Z})$, i.e. $\sum_n |s_n|^2 < \infty$. Thus, $\mathcal{M} = \Phi(l^2(\mathbb{Z}))$.

Filtering: $hs = H(z)S(z) = \sum_{n\in\mathbb{Z}} h_n z^{-n} \cdot \sum_{n\in\mathbb{Z}} s_n z^{-n} = \sum_{n\in\mathbb{Z}} \sum_{k+l=n} h_k s_l z^{-n}$.

Impulse: $b_i = z^i$.

Impulse response: $h \cdot z^i = \sum_{n\in\mathbb{Z}} h_n z^{-n} z^i = \sum_{n\in\mathbb{Z}} h_n z^{-n+i}$.

Fourier transform:

$$\begin{aligned} \Delta : \quad \mathcal{M} &\to \bigoplus_{w\in(-\pi,\pi]} \mathcal{M}_w \\ s = S(z) = \sum_{n\in\mathbb{Z}} s_n z^{-n} &\mapsto (S(e^{jw})E_w(z))_{w\in(-\pi,\pi]} = w \mapsto S(e^{jw})E_w(z), w \in (-\pi,\pi] \end{aligned}$$

where $j = \sqrt{-1}$ and $E_w(z) = \sum_{n\in\mathbb{Z}} e^{jwn} z^{-n}$.

Spectrum of signal: $\Delta(s) = (S(e^{jw})E_w(z))_{w\in(-\pi,\pi]} = w \mapsto S(e^{jw})E_w(z), w \in (-\pi,\pi]$.