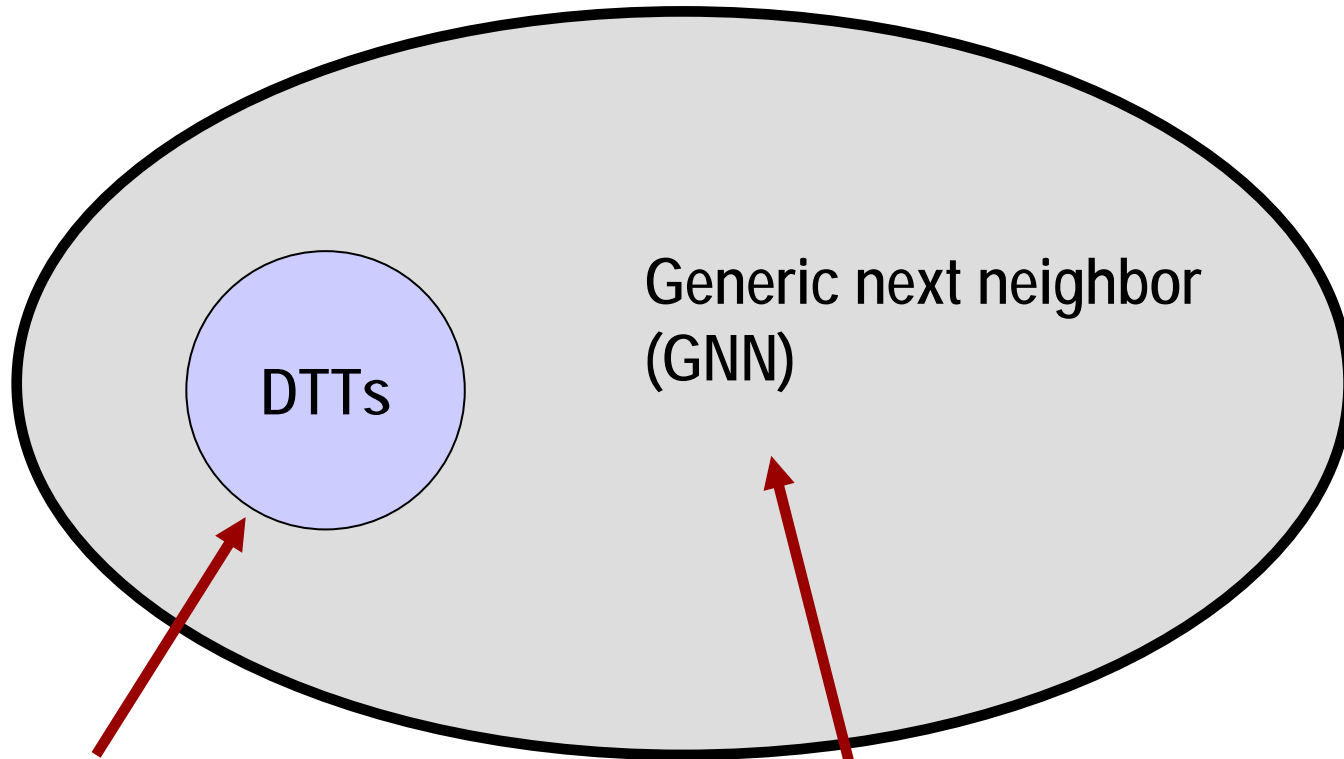# Application Study of Known and Novel Transforms

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Introduction

DTTs

Generic next neighbor (GNN)

Chebyshev polynomials

Orthogonal polynomials

Properties of DTTs in image processing applications are well-studied

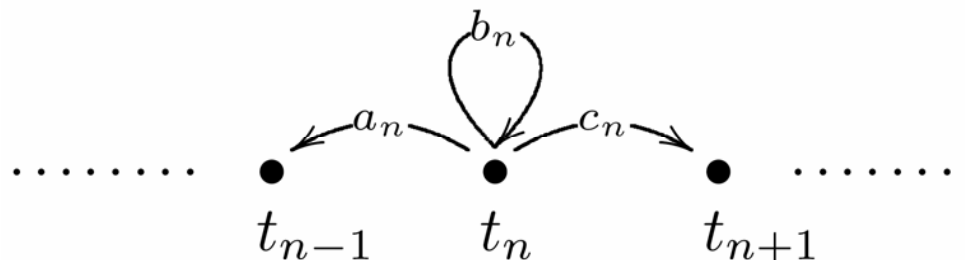We aim to study related GNN transforms in this domain

Overview

- **Background**

- Approach

- Experimental Results

- Conclusions

Generic Next Neighbor Signal Model

[Pueschel and Moura, 2006]

Generalized Shift Model

(Visualization from [Pueschel and Moura, 2006])

Associated Orthogonal Polynomial Model

$$P_{n+1} = \frac{x - b_n}{c_n} P_n - \frac{a_n}{c_n} P_{n-1}$$

GNN: Relationship with DTTs

GNNs:

$$\phi(x) = \begin{bmatrix} b_0 & a_1 & & & & \\ c_0 & b_1 & a_2 & & & \\ & c_1 & b_2 & \cdot & & \\ & & c_2 & \cdot & a_{n-2} & \\ & & & \cdot & b_{n-2} & a_{n-1} \\ & & & & c_{n-2} & b_{n-1} \end{bmatrix}$$

DTTs:

$$\phi(x) = \frac{1}{2} \cdot \begin{bmatrix} \beta_1 & 1 & & & \\ \beta_2 & 0 & 1 & & \\ 0 & 1 & 0 & \cdot & \\ & & 1 & \cdot & 1 \\ & & & \cdot & 0 & \beta_3 \\ & & & & 1 & \beta_4 \end{bmatrix}$$

(Visualizations and shift matrices from [Pueschel and Moura, 2006])

Overview

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GNNs and Associated Polynomials

- Recall: DTTs associated with Chebyshev polynomials

$$\mathcal{A} = \mathcal{M} = \mathbb{C}[x]/p(x)$$

($p(x)$ expressed in terms of T, U, V, W)

- Idea: examine GNN transforms associated with other well-known polynomials

Polynomials

Hermite:

$$
\begin{aligned}
P_0(x) &= 1 \\
P_1(x) &= 2x \\
P_{n+1}(x) &= 2xP_n(x) - 2nP_{n-1}(x), \text{ for } n > 0
\end{aligned}
$$

$$a_i = i, \quad b_i = 0, \quad c_i = 0.5$$

Laguerre:

$$
\begin{aligned}
P_0^{(\alpha)}(x) &= 1 \\
P_1^{(\alpha)}(x) &= -x + \alpha + 1 \\
P_{n+1}^{(\alpha)}(x) &= \frac{x - (2n + 1 + \alpha)}{-n - 1}P_n^{(\alpha)}(x) - \frac{-n - \alpha}{-n - 1}P_{n-1}^{(\alpha)}(x), \text{ for } n > 0,
\end{aligned}
$$

$$a_i^{(\alpha)} = -i - \alpha, \quad b_i^{(\alpha)} = 2i + 1 + \alpha, \quad c_i^{(\alpha)} = -i - 1$$

Polynomials

Legendre:

$$
\begin{aligned}
P_0(x) &= 1 \\
P_1(x) &= x \\
P_{n+1}(x) &= \frac{x}{\frac{n+1}{2n+1}}P_n(x) - \frac{n}{n+1}P_{n-1}(x), \text{ for } n > 0
\end{aligned}
$$

$$
a_i = \frac{i}{2i+1}, \quad b_i = 0, \quad c_i = \frac{i+1}{2i+1}
$$

Image Processing

- **256x256 image with 16x16 blocking**
- **Why blocking?**
  - Used in JPEG (8x8 blocking)
  - GNN limitations
- **Apply a 16x16 Transform on each block separately**

256

16

16

| 1 | 2 | 3 | ····· | 16 |
| 17 | 18 | 19 | ····· | 32 |

256

| 241 | 242 | 243 | ····· | 256 |
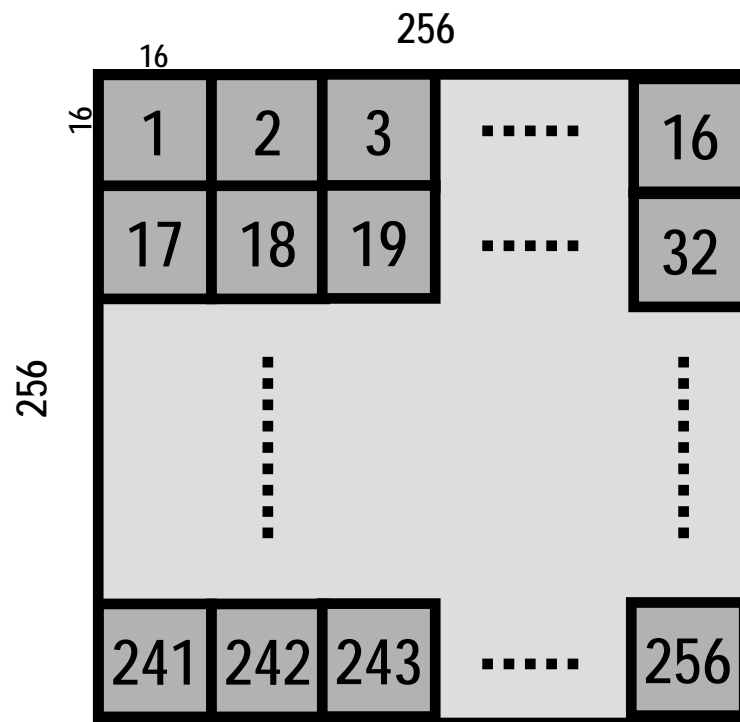
Energy Compaction & Reconstruction

- Let $E_{L,K}$ be the energy of the first L x L values of the Kth transformed block

- Energy compaction of the whole image due to first L x L values is

$$E_L = \frac{\sum\limits_{K=1}^{256} E_{L,K}}{\sum\limits_{K=1}^{256} E_{16,K}}$$

- Reconstruction is also done by keeping the first L x L values of each block.

Transformed image

L

L x L first values kept

K^{th} block
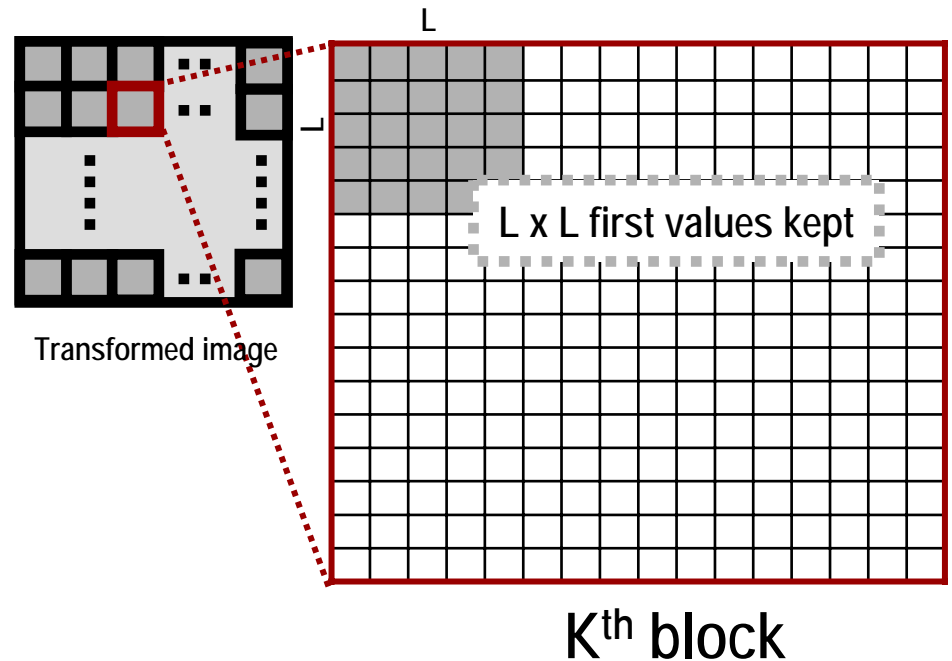
Images used : Tree, Desk, Book, and …

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Pure Frequencies : 1-D DCT DCs
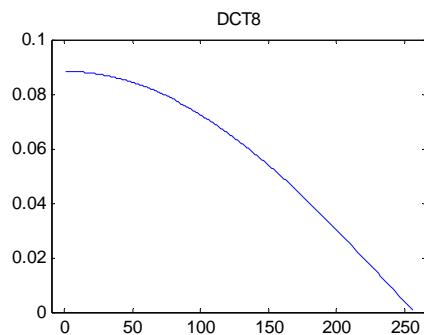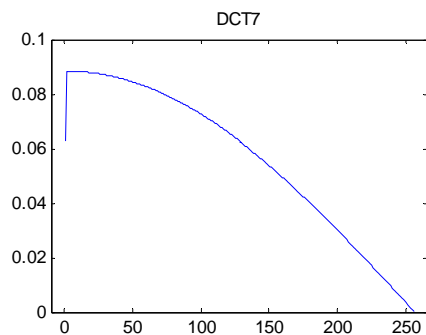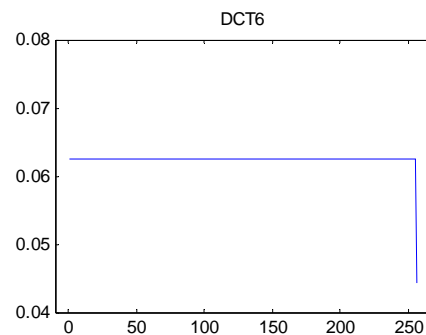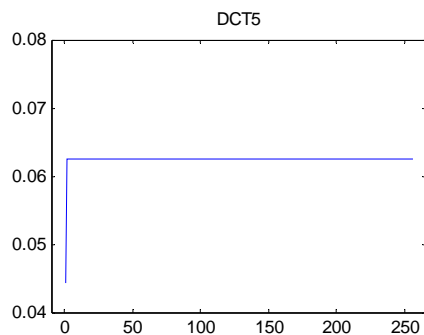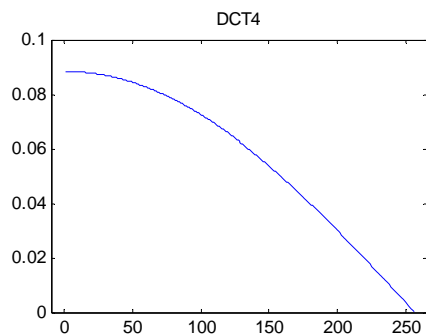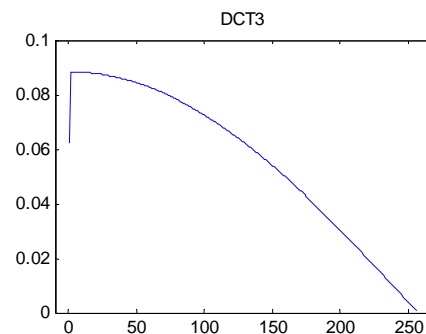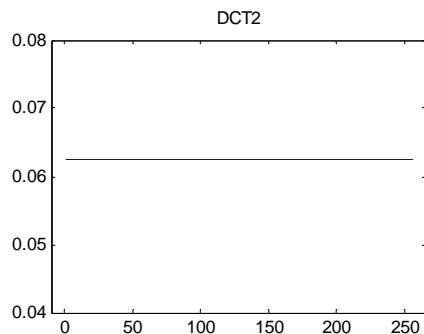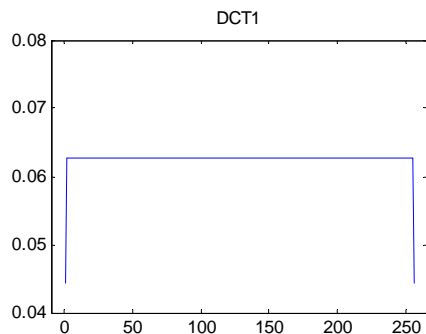
Pure Frequencies : 1-D DST DCs
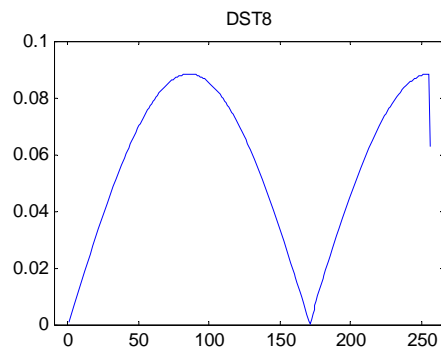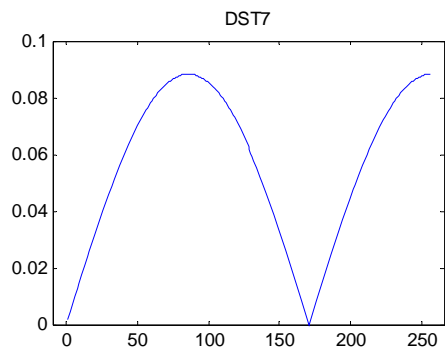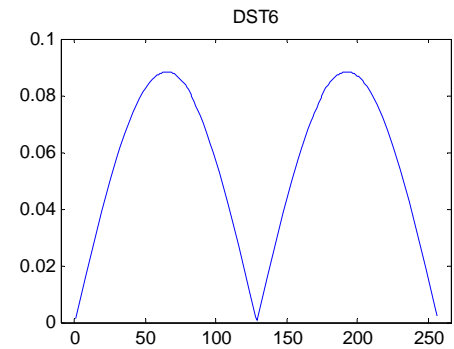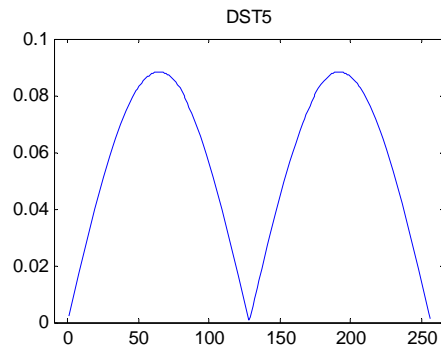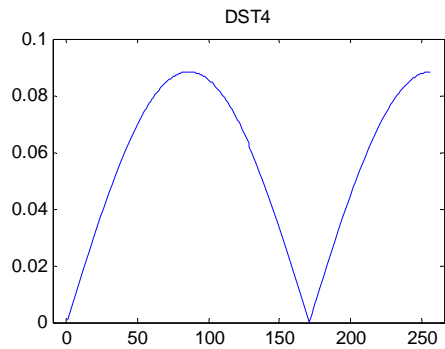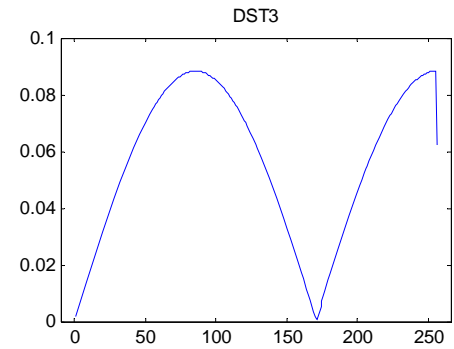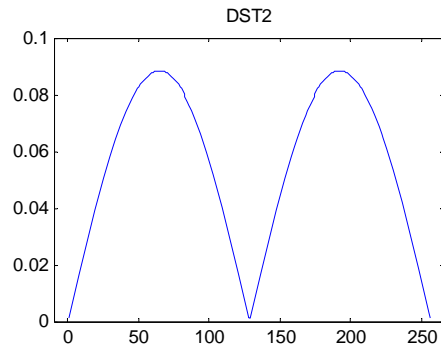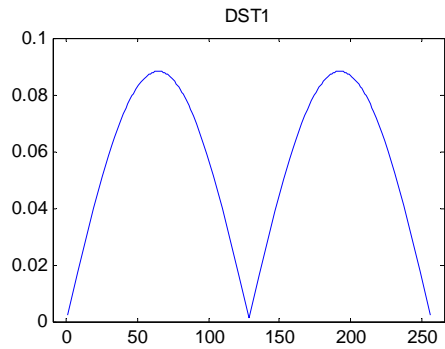
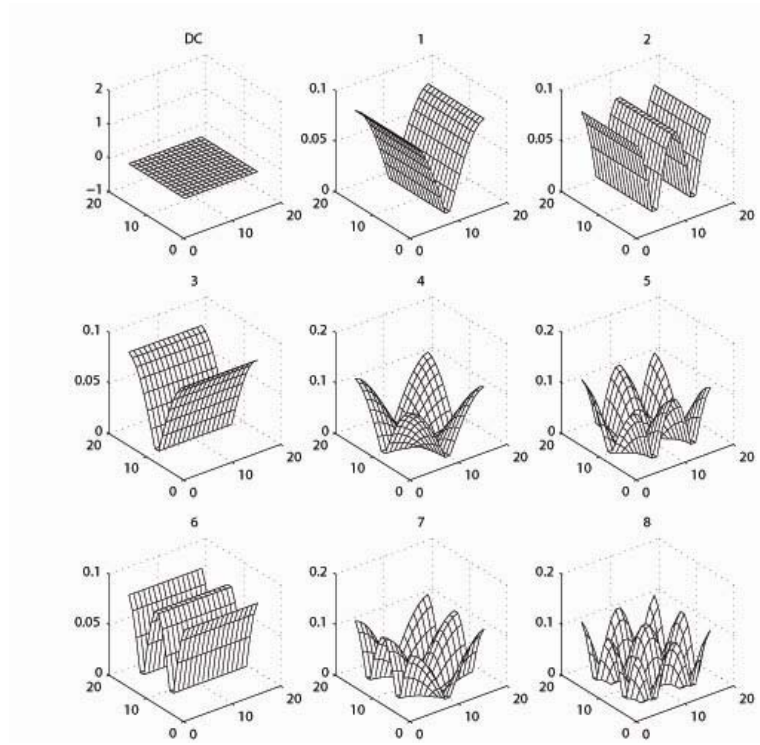# Pure Frequencies : 1-D GNN DCs
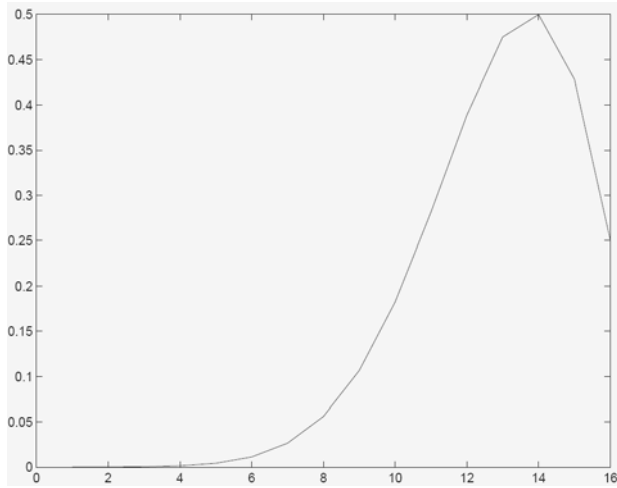
Hermite
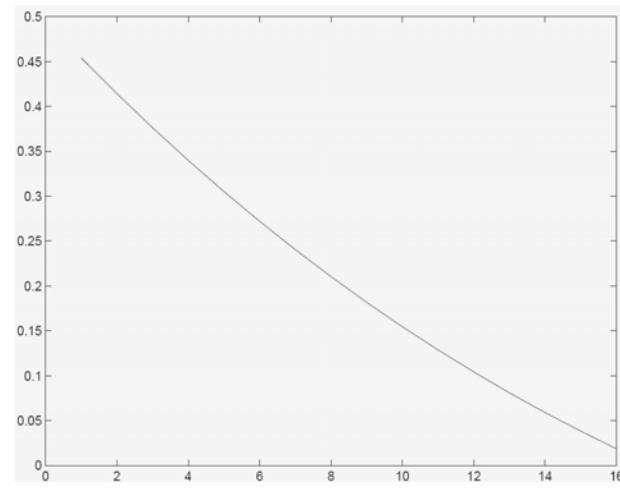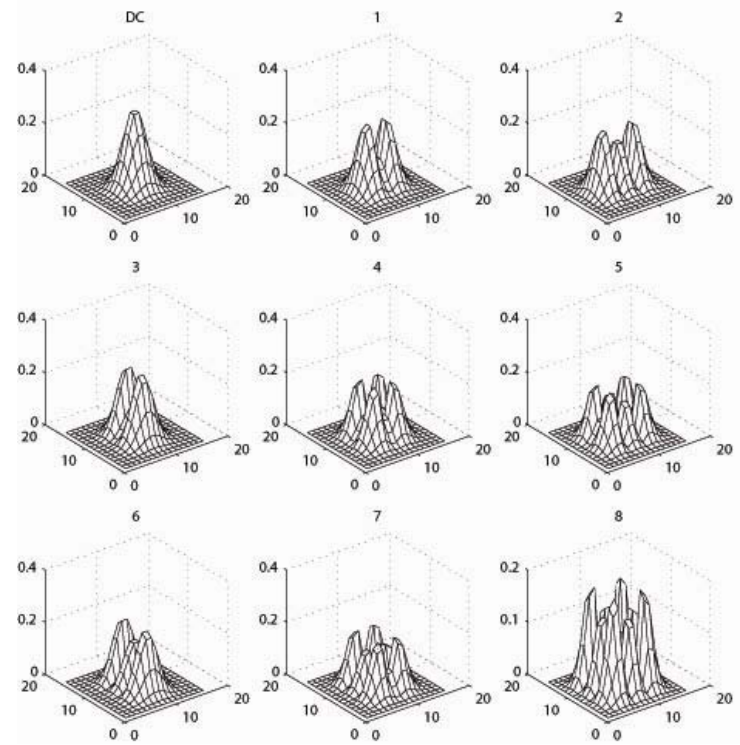

Laguerre

Legendre

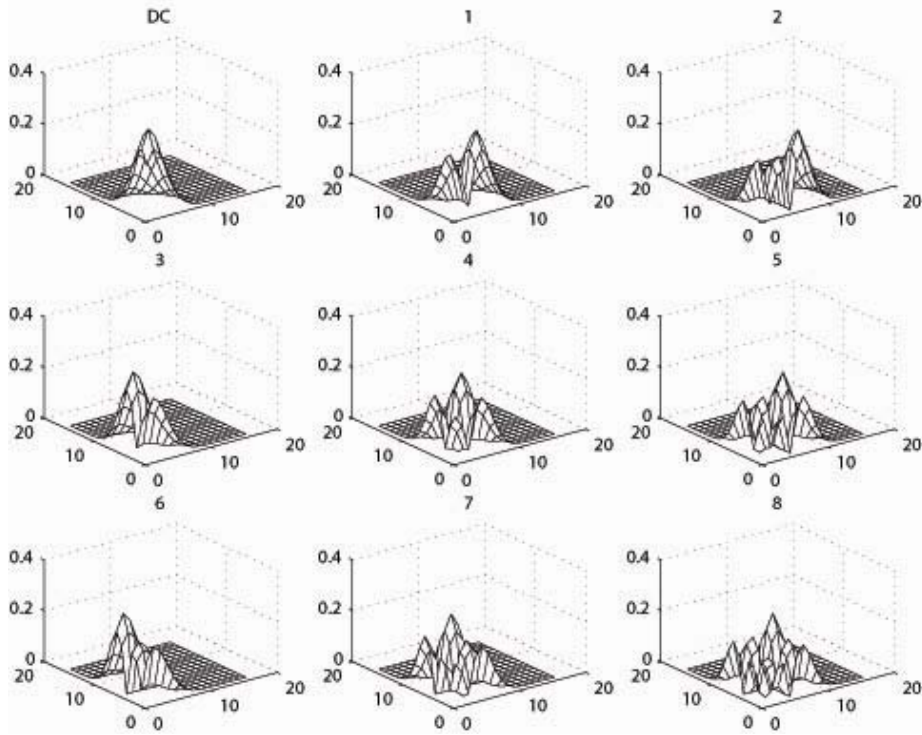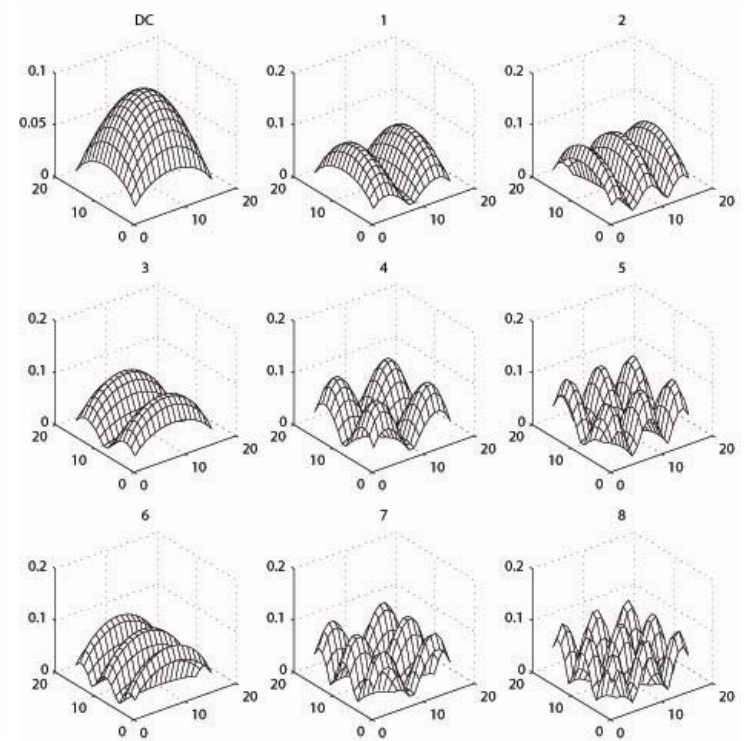# Pure Frequencies : 2-D

DCT2   Hermite
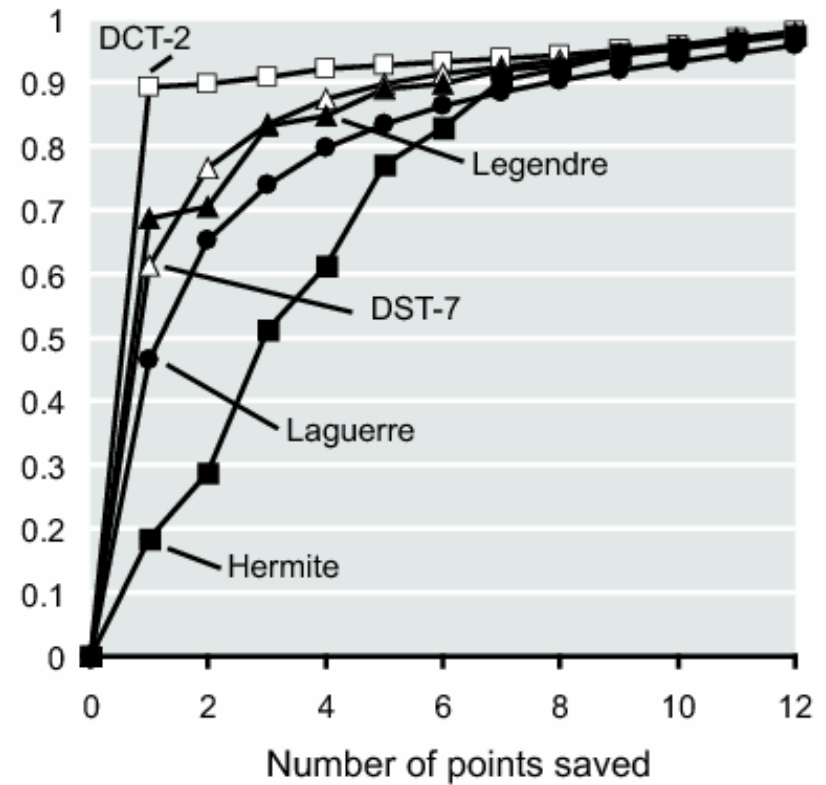
Pure Frequencies : 2-D

Laguerre   Legendre

Energy Compaction

Energy compaction on face (left) and book (right)
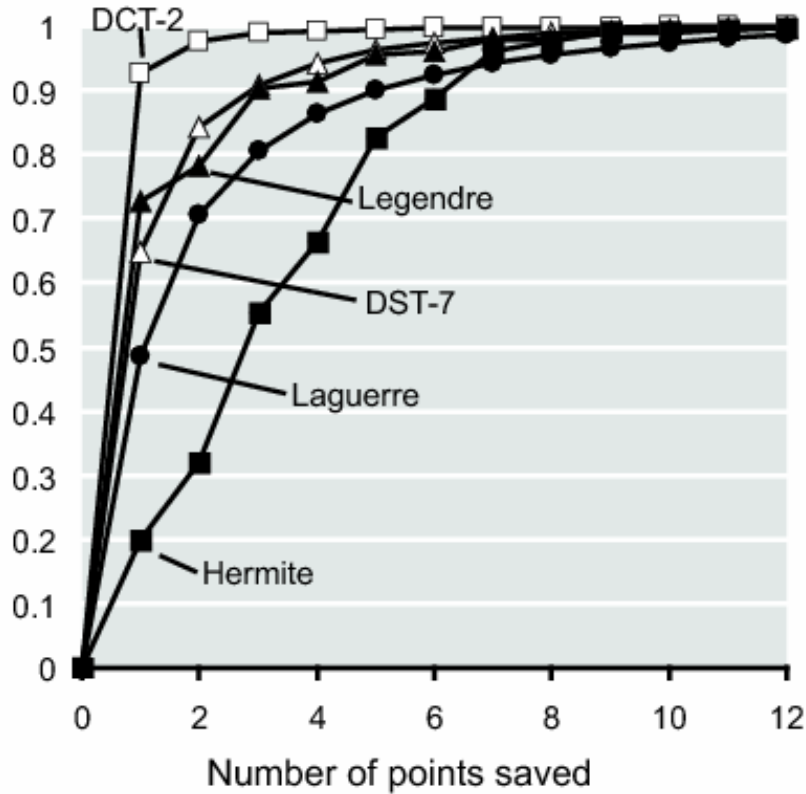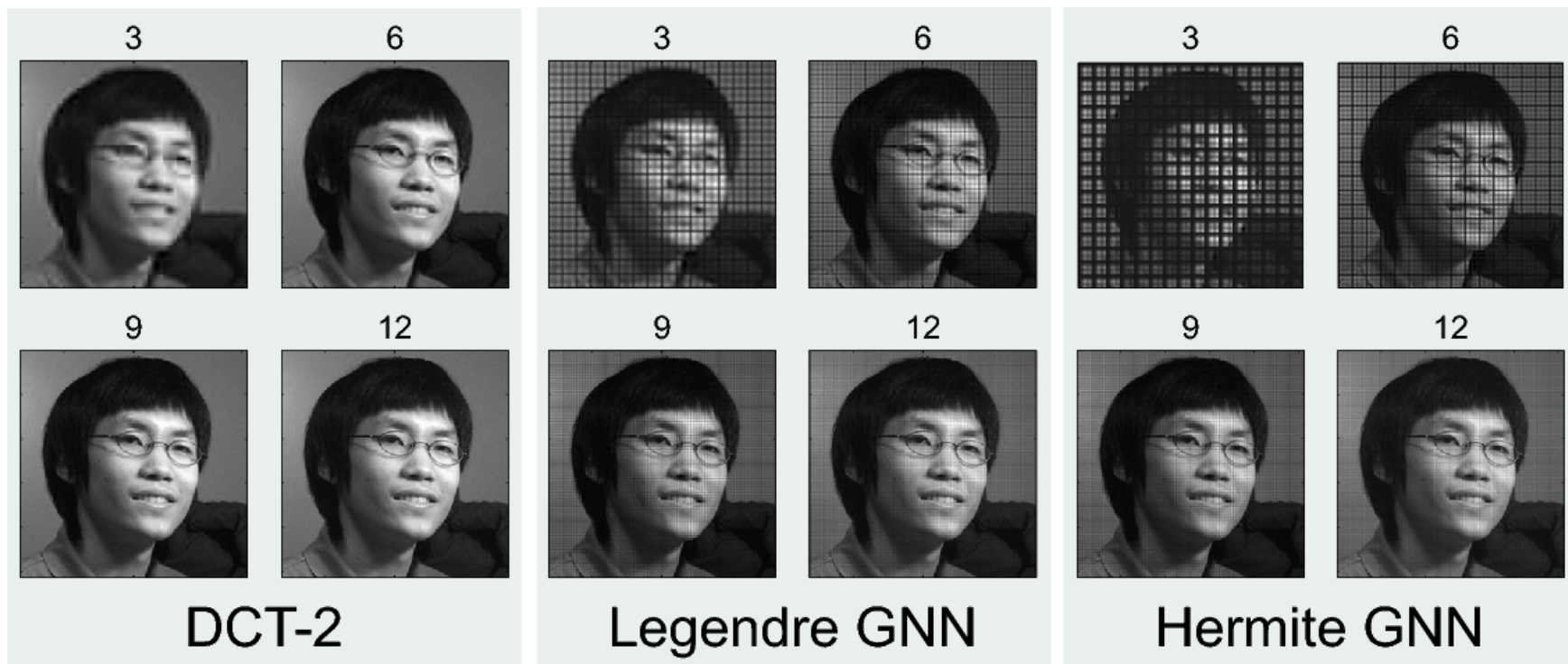
New transforms provide less compaction than DCT-2,
but compare with DST-7

Image Reconstruction

DCT-2   Legendre GNN   Hermite GNN

Quality of reconstruction matches energy compaction

- Blocking has no effect on DCT-2
- DCT-2 gains resolution as more samples are kept
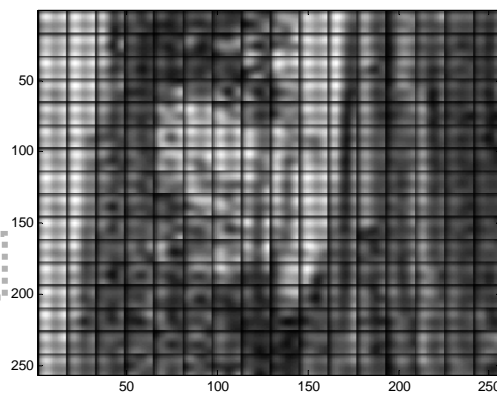
Overview

- Background

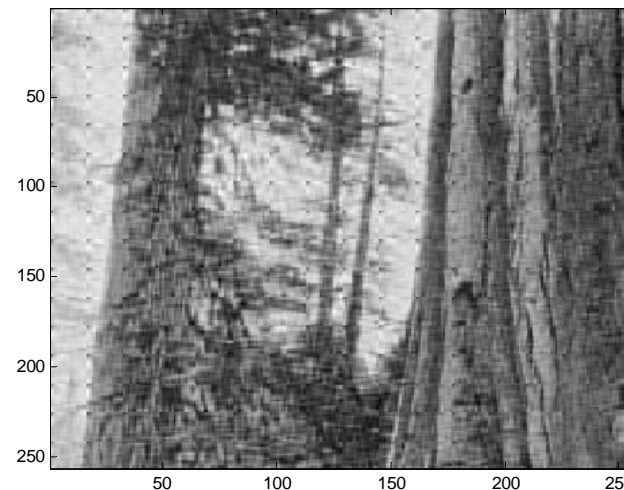- Approach

- Experimental Results

- Conclusions

Conclusion

- **DCT-2 is still best suited for image compression**
  - Immune to blocking effects
  - Best energy compaction
- **But…**
  - There are much more GNNs we have not tried.
  - Other possible reconstruction schemes are possible. For example
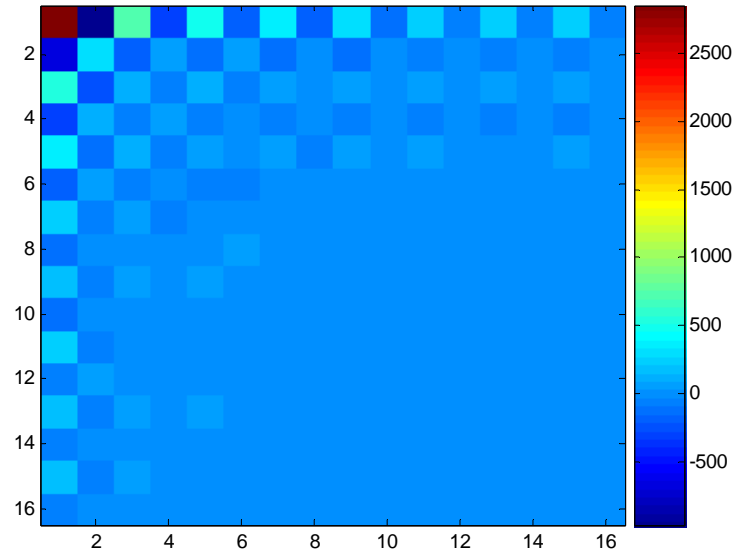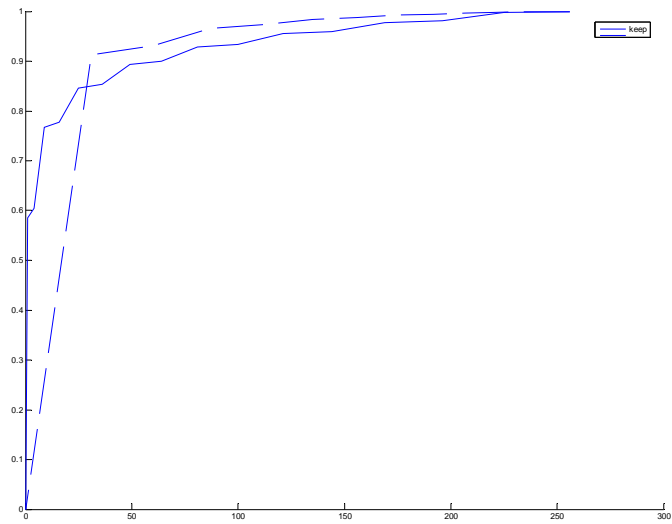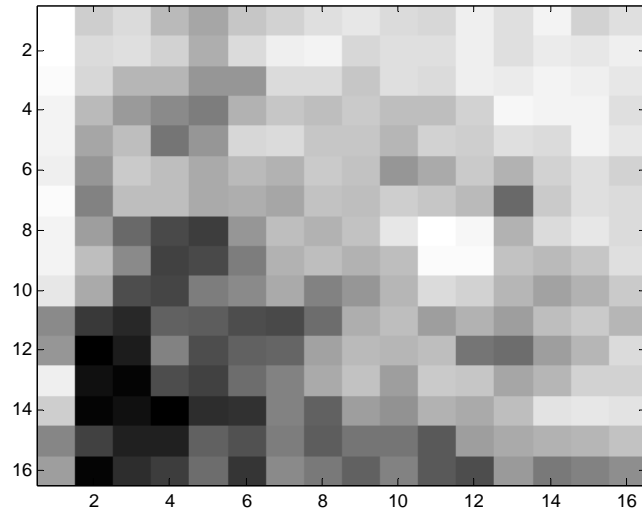

L x L last values discarded
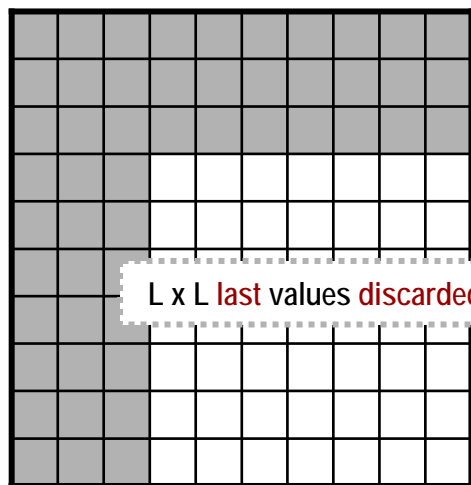
K^{th} Block

DST7 with old scheme

New scheme with the same number of points kept

Questions?

DST7

DCT2