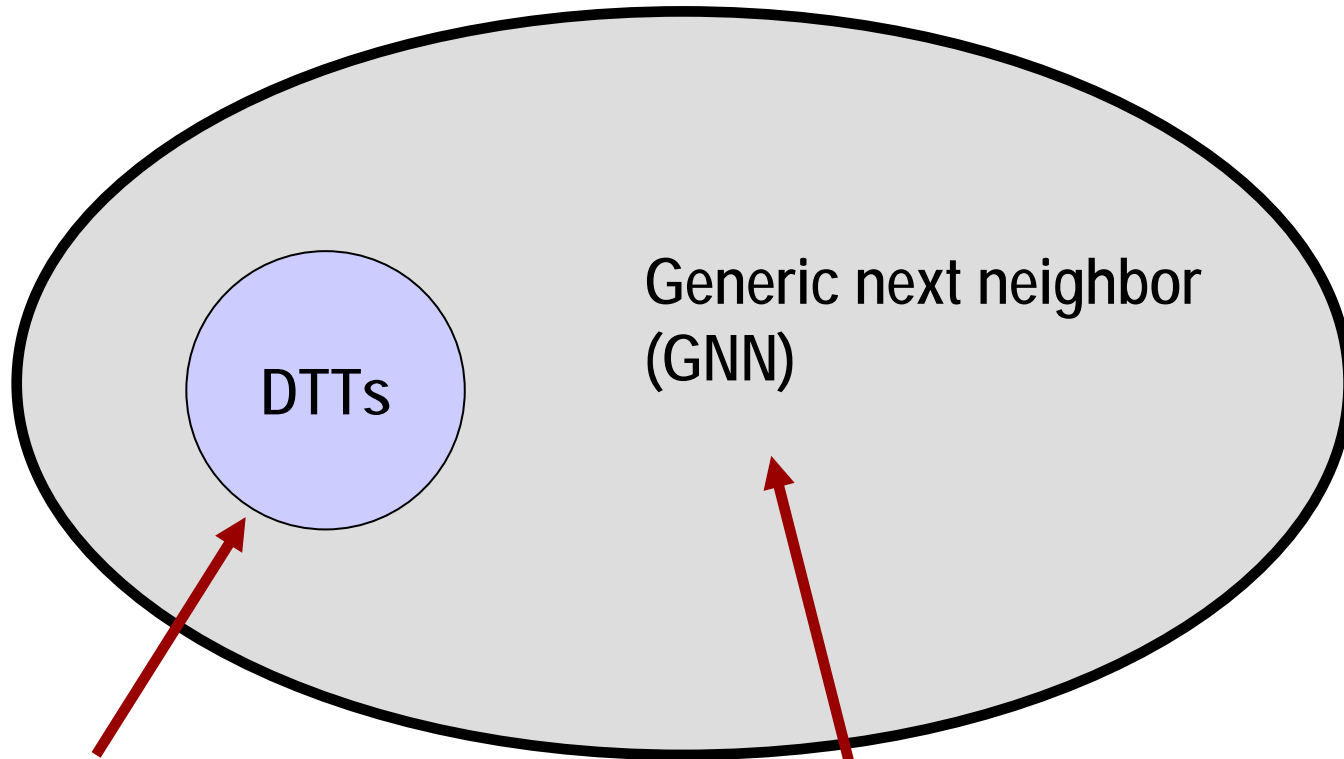


# Application Study of Known and Novel Transforms

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# Introduction



Chebyshev  
polynomials

Orthogonal  
polynomials

Properties of DTTs in image processing applications are well-studied

We aim to study related GNN transforms in this domain

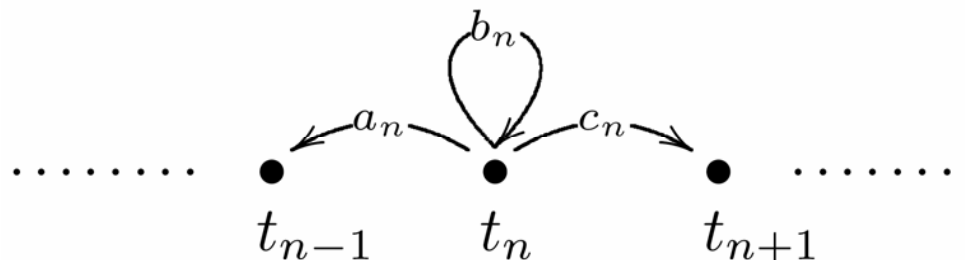
# Overview

- Background
- Approach
- Experimental Results
- Conclusions

# Generic Next Neighbor Signal Model

[Pueschel and Moura, 2006]

## Generalized Shift Model



(Visualization from [Pueschel and Moura, 2006])

## Associated Orthogonal Polynomial Model

$$P_{n+1} = \frac{x - b_n}{c_n} P_n - \frac{a_n}{c_n} P_{n-1}$$



# Overview

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# GNNs and Associated Polynomials

- Recall: DTTs associated with Chebyshev polynomials

$$\mathcal{A} = \mathcal{M} = \mathbb{C}[x]/p(x)$$

(  $p(x)$  expressed in terms of  $T, U, V, W$  )

- Idea: examine GNN transforms associated with other well-known polynomials

# Polynomials

## Hermite:

$$P_0(x) = 1$$

$$P_1(x) = 2x$$

$$P_{n+1}(x) = 2xP_n(x) - 2nP_{n-1}(x), \text{ for } n > 0$$

$$a_i = i, \quad b_i = 0, \quad c_i = 0.5$$

## Laguerre:

$$P_0^{(\alpha)}(x) = 1$$

$$P_1^{(\alpha)}(x) = -x + \alpha + 1$$

$$P_{n+1}^{(\alpha)}(x) = \frac{x - (2n + 1 + \alpha)}{-n - 1} P_n^{(\alpha)}(x) - \frac{-n - \alpha}{-n - 1} P_{n-1}^{(\alpha)}(x), \text{ for } n > 0,$$

$$a_i^{(\alpha)} = -i - \alpha, \quad b_i^{(\alpha)} = 2i + 1 + \alpha, \quad c_i^{(\alpha)} = -i - 1$$



# Polynomials

Legendre:

$$P_0(x) = 1$$

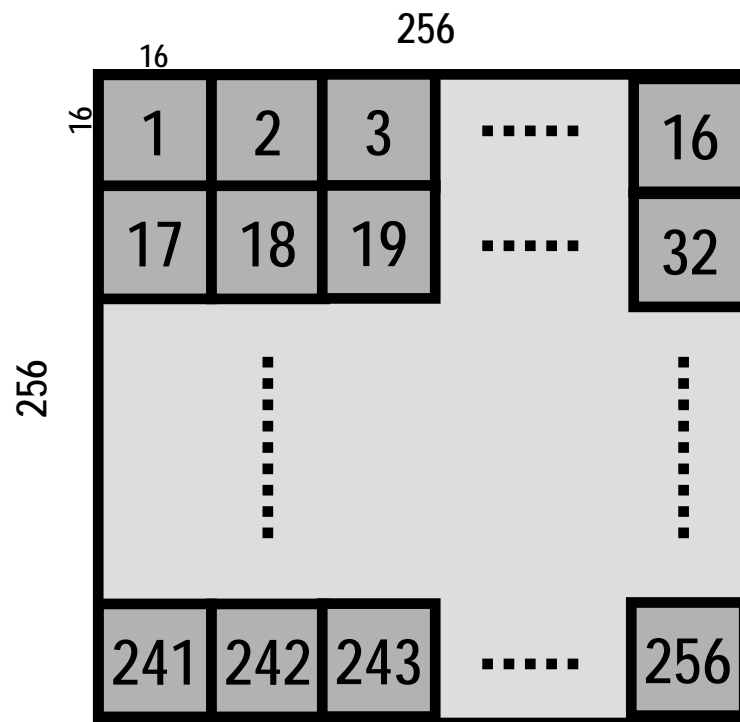
$$P_1(x) = x$$

$$P_{n+1}(x) = \frac{x}{\frac{n+1}{2n+1}} P_n(x) - \frac{n}{n+1} P_{n-1}(x), \text{ for } n > 0$$

$$a_i = \frac{i}{2i+1}, \quad b_i = 0, \quad c_i = \frac{i+1}{2i+1}$$

# Image Processing

- 256x256 image with 16x16 blocking
- Why blocking?
  - Used in JPEG (8x8 blocking)
  - GNN limitations
- Apply a 16x16 Transform on each block separately

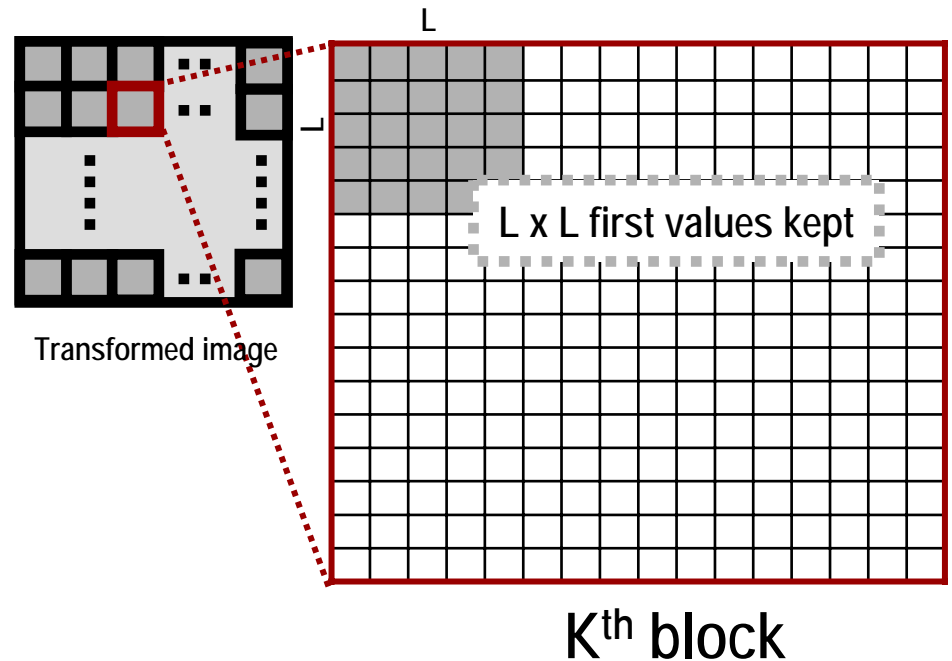


# Energy Compaction & Reconstruction

- Let  $E_{L,K}$  be the energy of the first  $L \times L$  values of the  $K^{\text{th}}$  transformed block
- Energy compaction of the whole image due to first  $L \times L$  values is

$$E_L = \frac{\sum_{K=1}^{256} E_{L,K}}{\sum_{K=1}^{256} E_{16,K}}$$

- Reconstruction is also done by keeping the first  $L \times L$  values of each block.



# Images used : Tree, Desk, Book, and ...



## TO OUR VISITORS

*This pictorial review booklet was printed some time after the "Great Storm" of March 6 and 7, 1962, and a word of explanation should accompany the disaster photos published herein. Rehoboth Beach, Lewes Beach, Dewey Beach and the coastal area are now being rebuilt and most all of the damage is well under way toward being repaired and homes and business places rebuilt. The boardwalk at Rehoboth Beach is expected to be replaced by July, according to Mayor Juel C. Stamper and City Manager Frank Buck. Sand is being replaced by the U. S. Corps of Army Engineers where it was washed and beaten away by the high waves, and all the debris has been cleared from the beachfront and safe bathing is assured to all once again.*

*The Coastal Highway, Route 14, is open to all traffic but the Indian River Inlet Bridge will remain closed for repairs until June. Bethany Beach and Fenwick Island are well on the way to recovery and Lewes Beach is now ready for its summer season, with the bridge across the Lewes-Rehoboth Canal being repaired through the efforts of the Otis H. Smith, Mayor of Lewes, and the State Highway Department.*

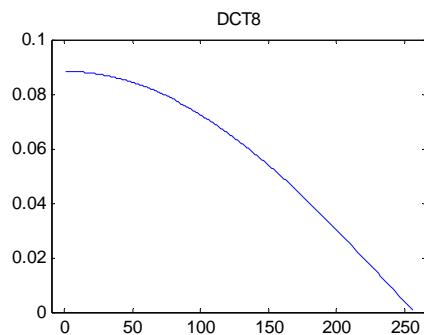
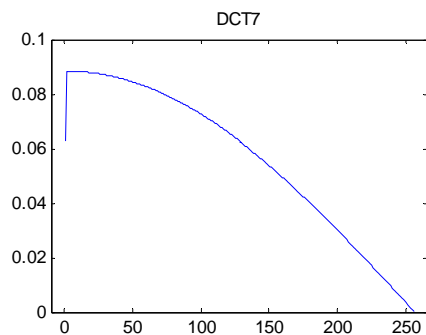
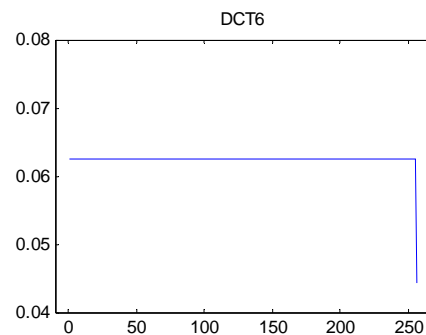
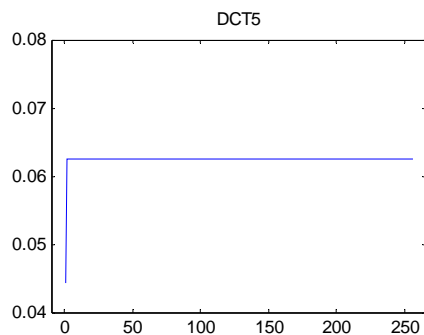
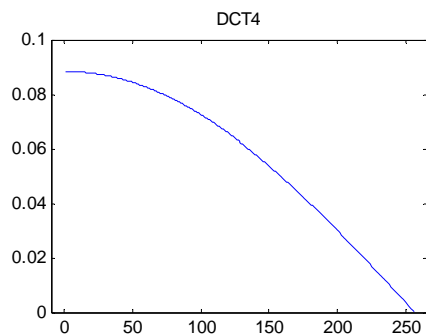
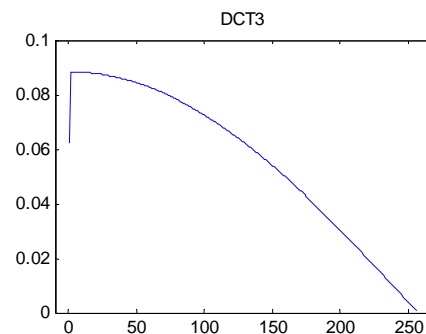
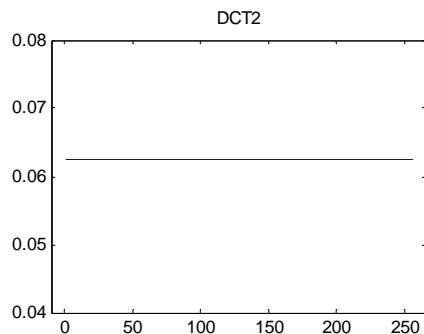
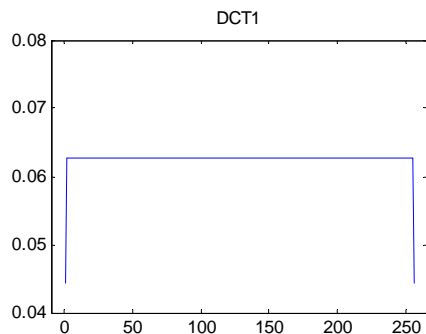
*As may be noted on the last page of this pictorial re-*



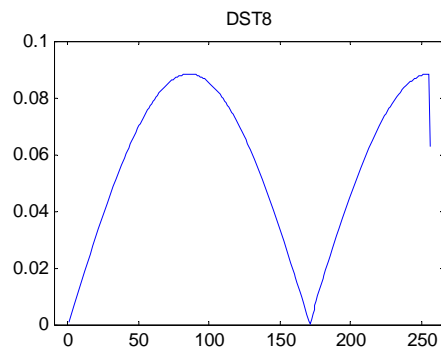
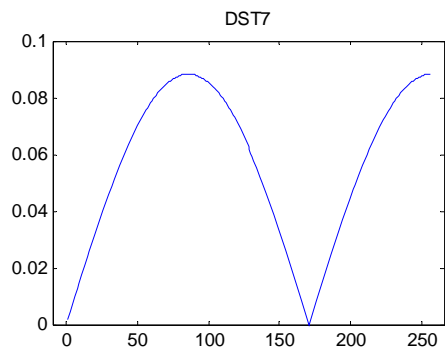
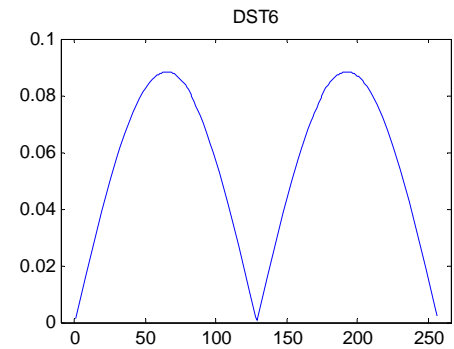
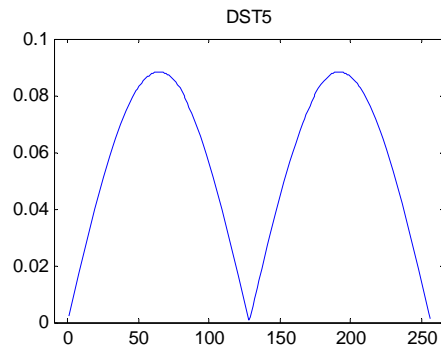
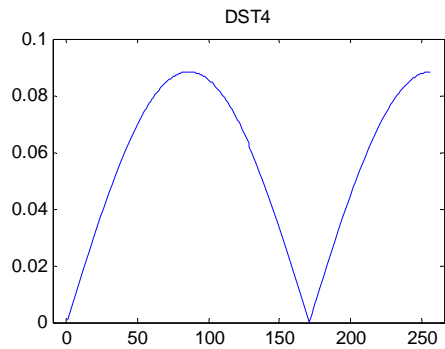
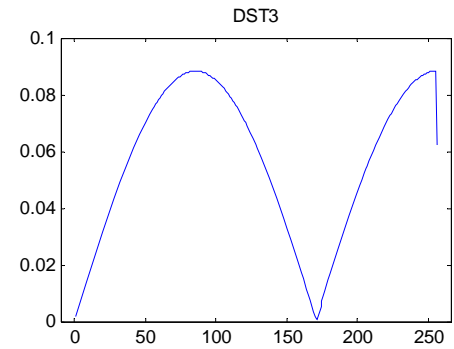
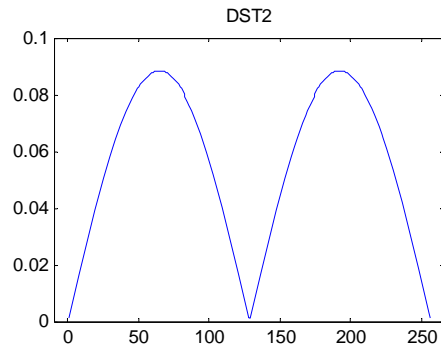
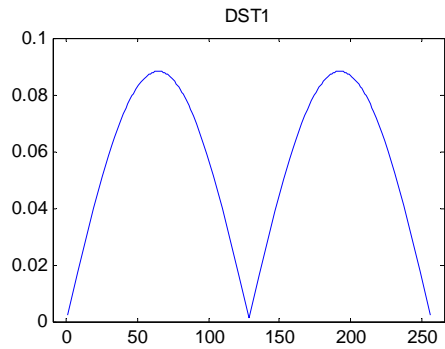
# Overview

- Background
- Approach
- **Experimental Results**
- Conclusions

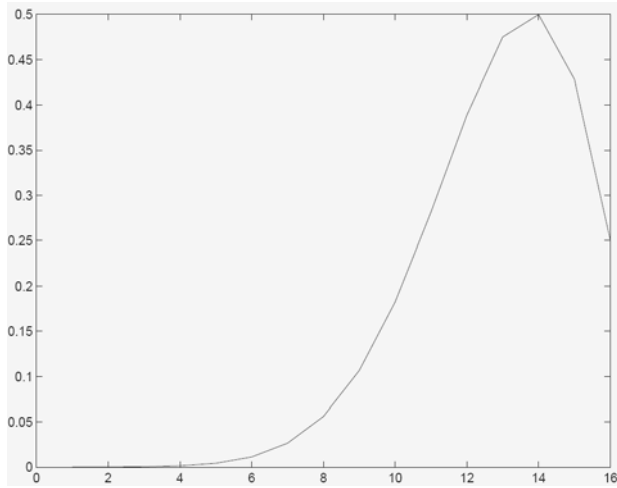
# Pure Frequencies : 1-D DCT DCs



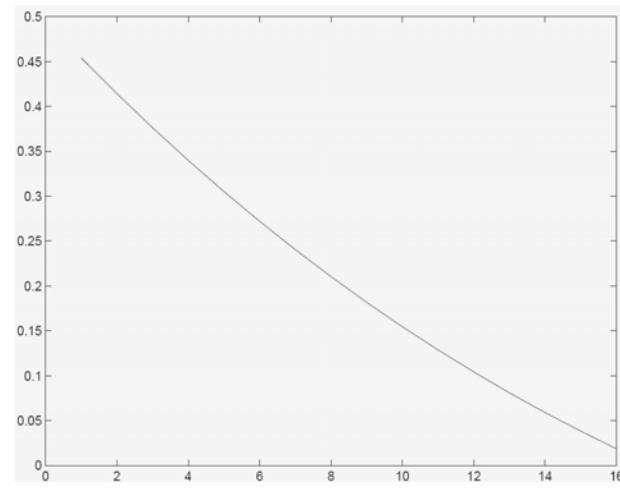
# Pure Frequencies : 1-D DST DCs



# Pure Frequencies : 1-D GNN DCs



Hermite



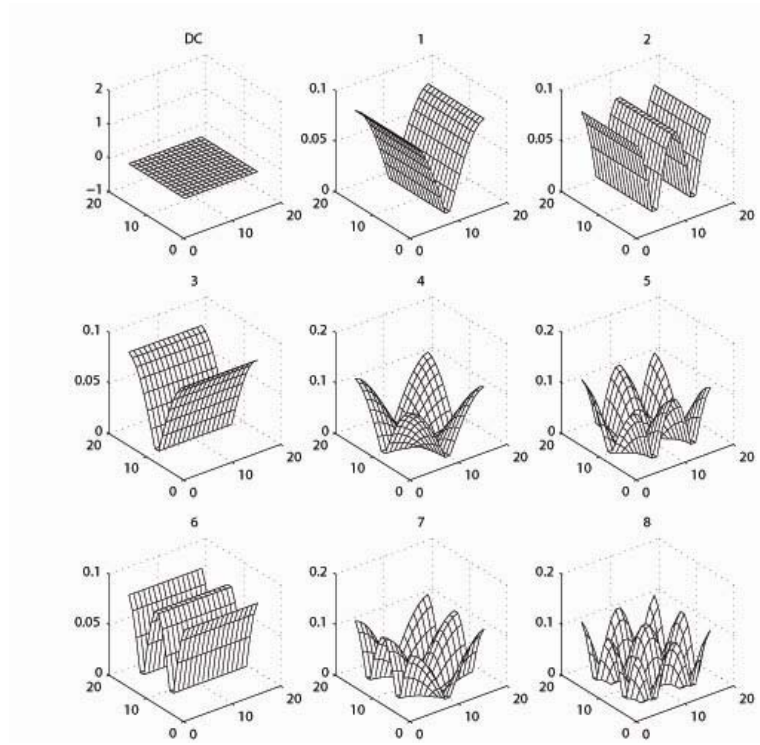
Laguerre



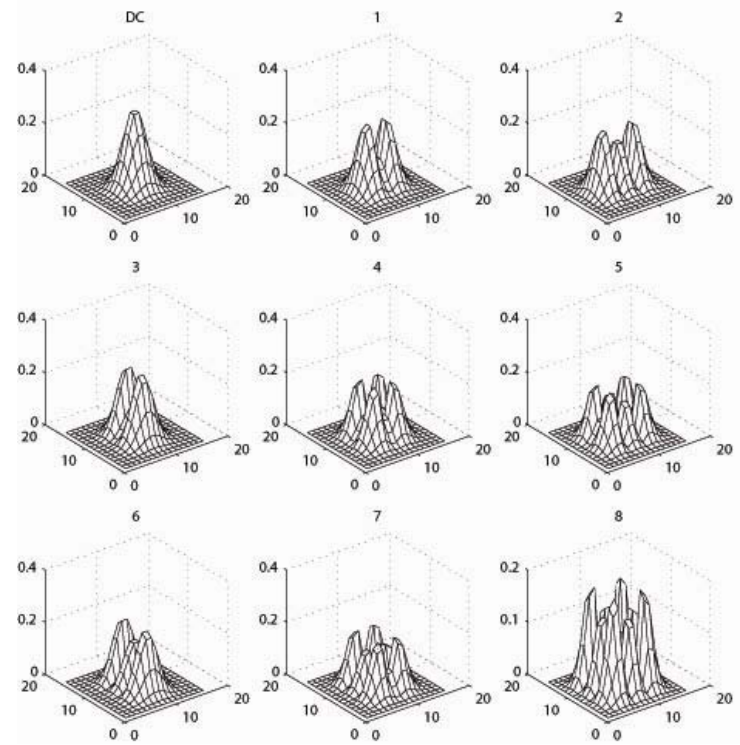
Legendre



# Pure Frequencies : 2-D

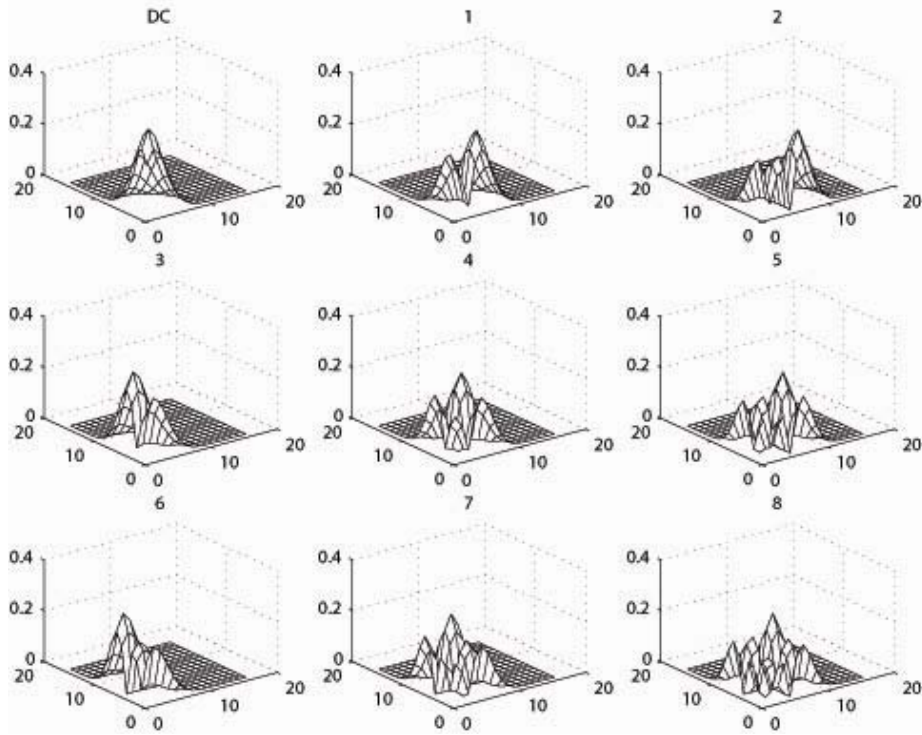


DCT2

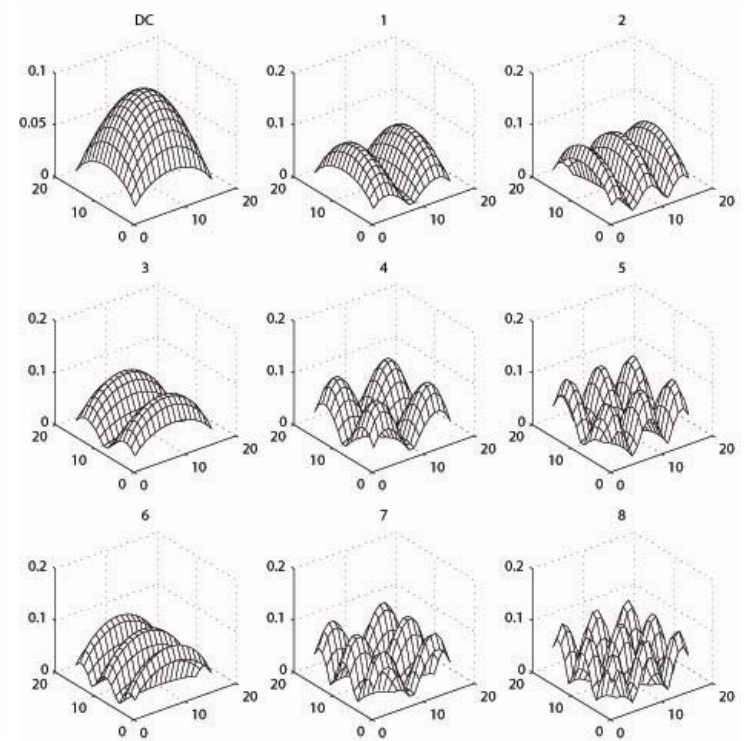


Hermite

# Pure Frequencies : 2-D



Laguerre

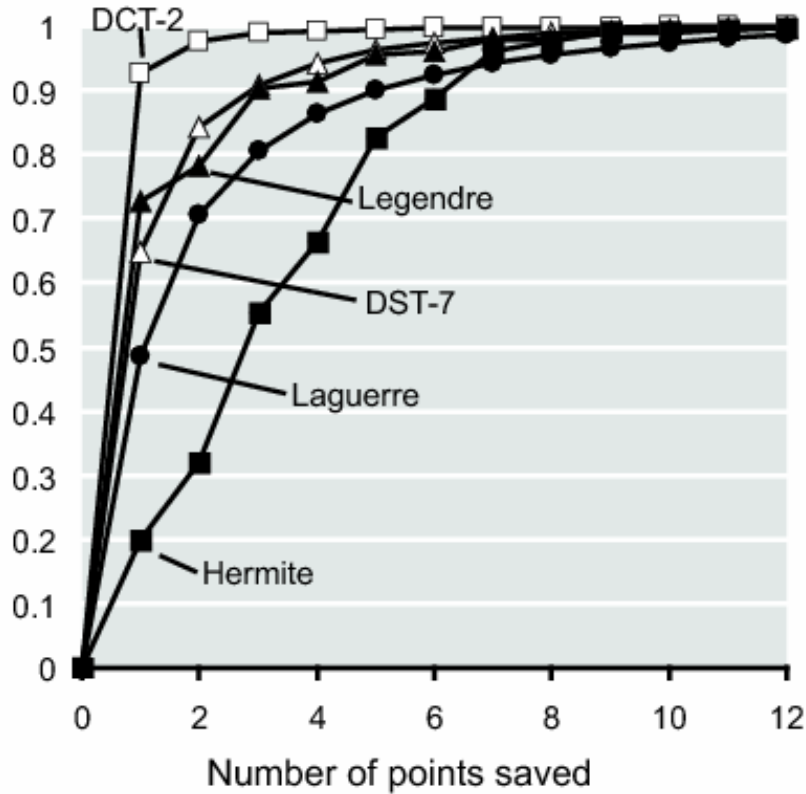


Legendre

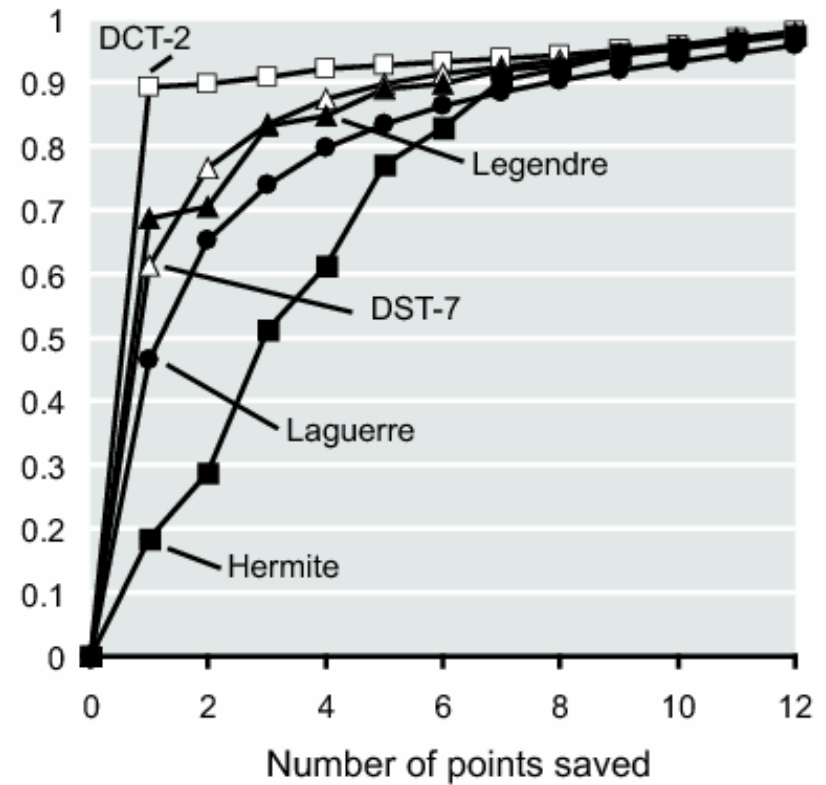
# Energy Compaction

Energy compaction on face (left) and book (right)

Relative energy

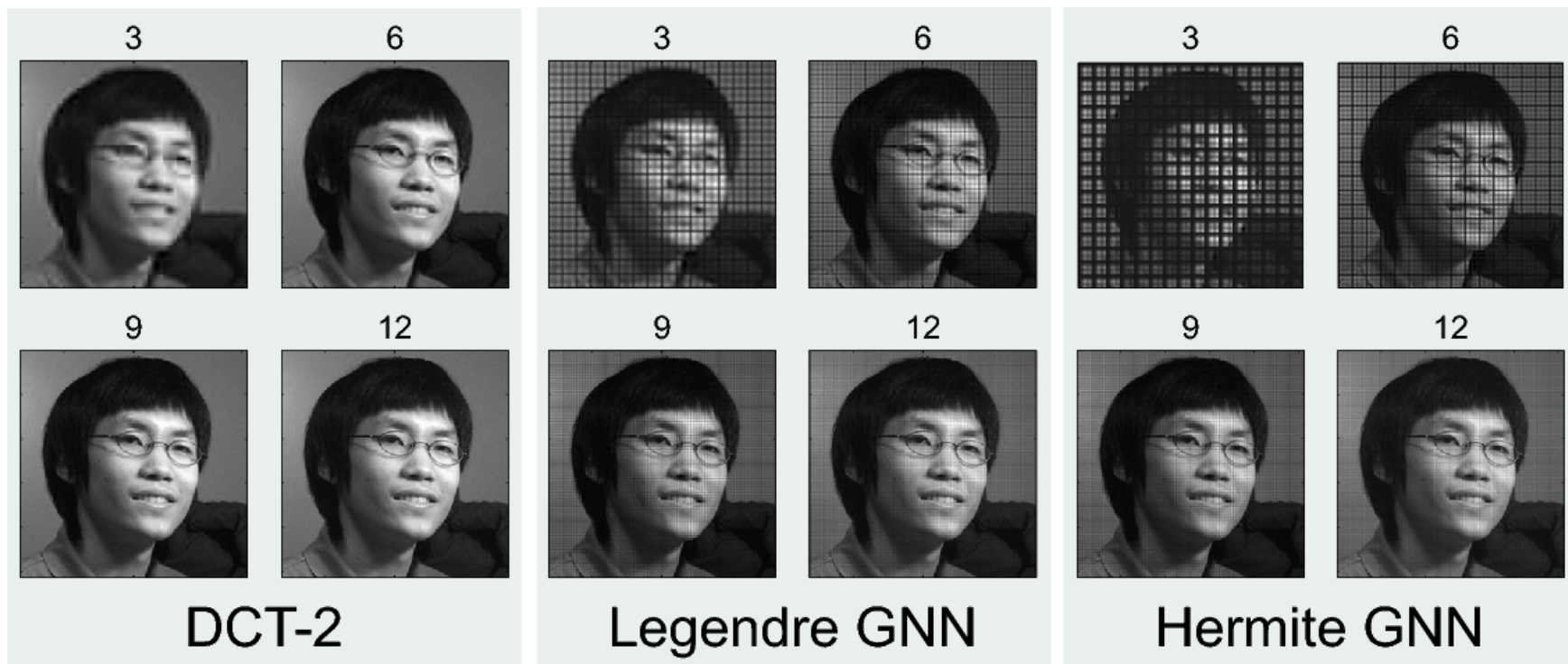


Relative energy



New transforms provide less compaction than DCT-2,  
but compare with DST-7

# Image Reconstruction



**Quality of reconstruction matches energy compaction**

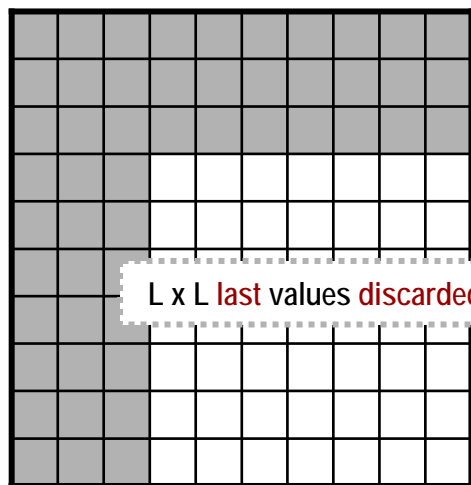
- Blocking has no effect on DCT-2
- DCT-2 gains resolution as more samples are kept

# Overview

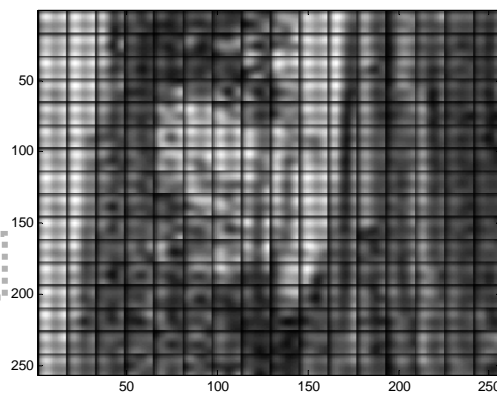
- Background
- Approach
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# Conclusion

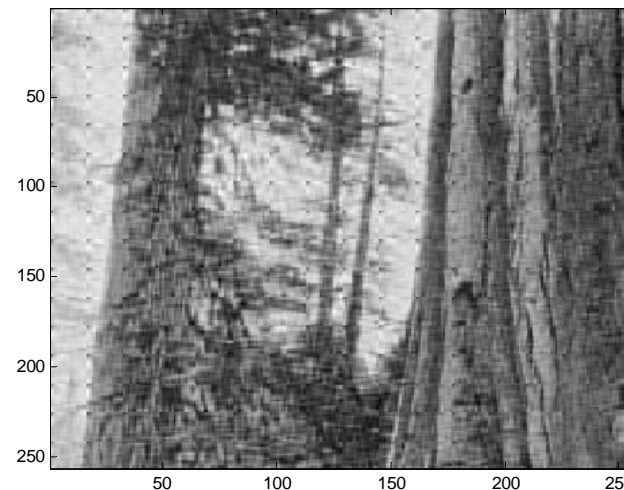
- DCT-2 is still best suited for image compression
  - Immune to blocking effects
  - Best energy compaction
- But...
  - There are much more GNNs we have not tried.
  - Other possible reconstruction schemes are possible. For example



$K^{\text{th}}$  Block



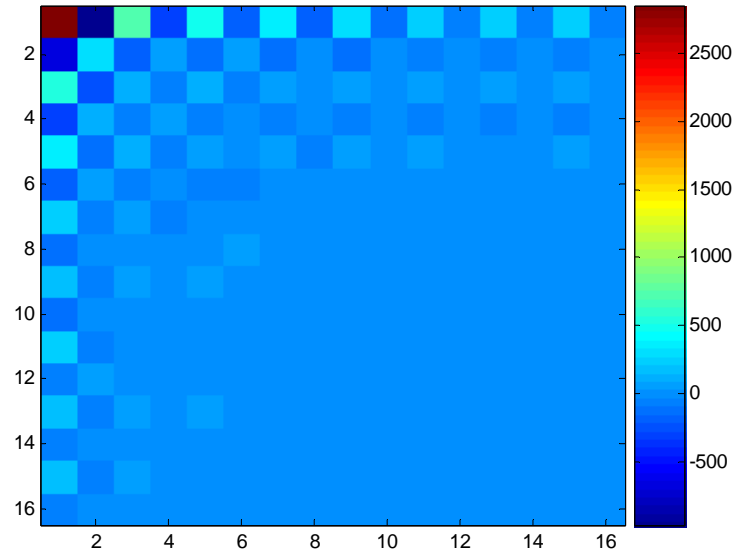
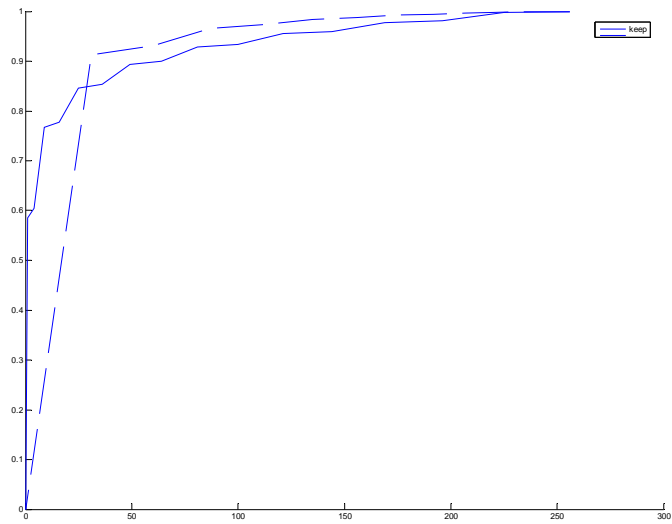
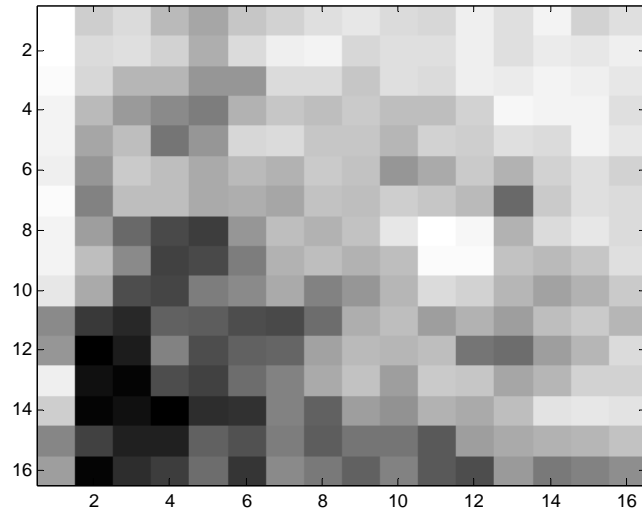
DST7 with old scheme



New scheme with the same  
number of points kept

# Questions?

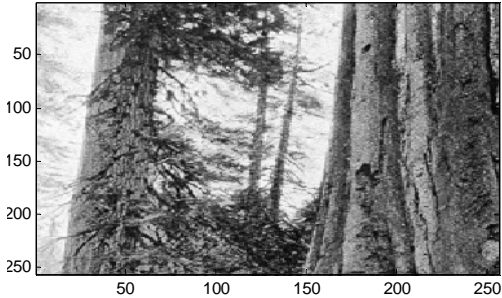
# DST7



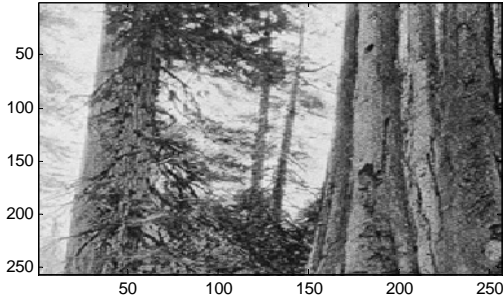


# DCT2

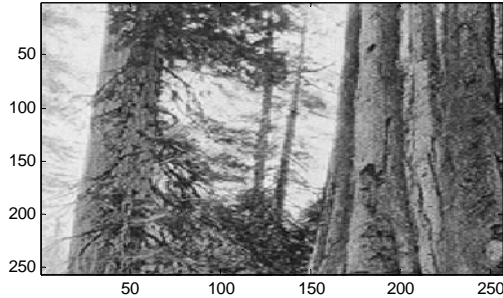
16



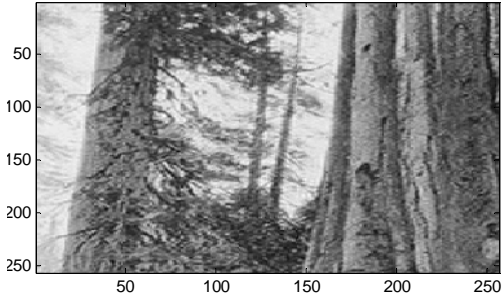
14



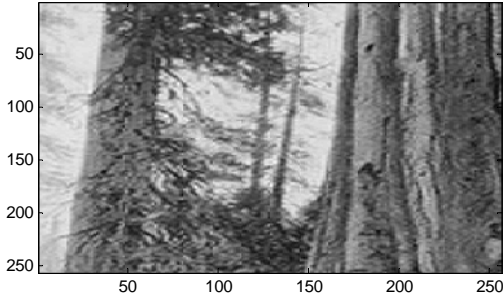
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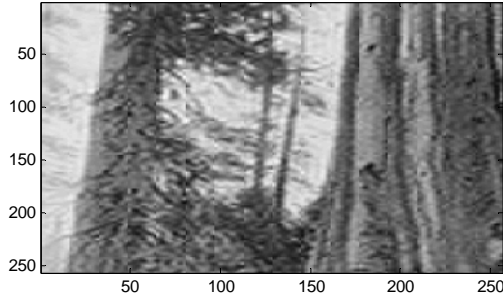
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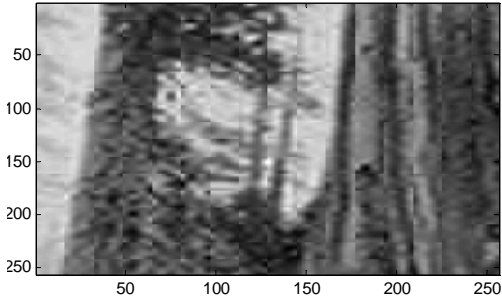
8



6



4



2

