recap: finite, 1-D, shift-invariant signal models

midterm

- $\mathbb{C}[x]/q(x)$ is a $\mathbb{C}[x]/p(x)$ - module (w.r.t. standard op.)

$\iff q \mid p$

Derivation of Signal Models

shift $\rightarrow$ signal model $(\mathcal{A}, \mathcal{M}, \Phi)$

Steps:

1.) definition of shift

2.) linear extension

3.) realization

Infinite time $\dots \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \dots$

Finite time

- Definition: signal extension and monomial sig. ext.
- 1.) - 3.) seem to lead to $\underset{x^0}{\bullet} \rightarrow \underset{x^1}{\bullet} \dots \underset{x^{n-2}}{\bullet} \rightarrow \underset{x^{n-1}}{\bullet}$?

  $\mathcal{M} = \mathbb{C}_{n-1}[x]$

  but this no module (not closed under shift)

- Boundary condition and signal extension
- left boundary
- summary: right b.c. $\Rightarrow$ right and left sig. ext.
- "nicest" case: - x invertible

  - mon. sig. ext.

  $\Rightarrow \mathcal{A} = \mathcal{M} = \mathbb{C}[x]/x^n - a$

  compute signal extension

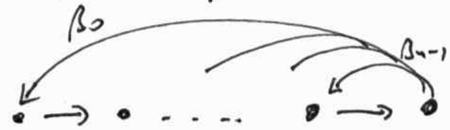
Finite time models:

- generic case: $\mathcal{A} = \mathcal{M} = \mathbb{C}[x]/p(x)$, $p(x)$ distinct zeros (so spectrum is nice)

$$\Phi: \mathbb{C}^n \to \mathcal{M}, \quad \hat{s} \mapsto \sum_{k=0}^{n-1} s_k x^k$$

so $b = \{1, x, \dots, x^{n-1}\}$ "time-basis"

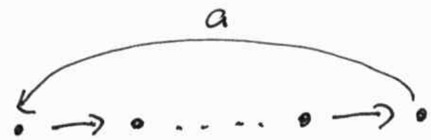
$$\varphi(x) = \begin{pmatrix} & & & +\beta_0 \\ 1 & & & \vdots \\ & \ddots & & \\ & & 1 & +\beta_{n-1} \end{pmatrix}$$ where $p(x) = x^n - r(x)$, $r(x) = \sum_{k=0}^{n-1} \beta_k x^k$

visualization: 

$\mathcal{F} = \mathcal{P}_{b,\alpha}$ = Vandermonde matrix = $[\alpha_k^\ell]_{k,\ell}$ α_k zeros of p

- nicest case: $p(x) = x^n - a$

$$\varphi(x) = \begin{pmatrix} & & & a \\ 1 & & & \\ & \ddots & & \\ & & 1 & \end{pmatrix}$$

visualization: 

$\mathcal{F} = \mathcal{P}_{b,\alpha} = DFT_n \cdot D$, D diagonal

usual choice: $a = 1$

$\Rightarrow$ $\varphi(x)$ = cyclic shift, vis. = directed circle

sig. ext. = periodic, $\mathcal{F} = \mathcal{P}_{b,\alpha} = DFT_n$.

- Discuss b.c.'s: $x^n = 0$, $x^n = x^{n-1}$