FFTW again

$$DFT_{km} = (DFT_k \otimes I_m)\, T\, (I_k \otimes DFT_m)\, L^{km}_k$$

$$(\text{output})\ Y \xleftarrow{\text{in place}} Y \xleftarrow{\text{out of place}} x\ (\text{input})$$

interfaces: $\mathrm{DFT}(\underset{\text{size}}{n}, \underset{\text{in=out}}{*x}, \underset{\text{twiddles}}{*t}, \underset{\text{stride}}{s})$ $\qquad$ $\mathrm{DFT}(n, \underset{\text{in}}{*x}, \underset{\text{out}}{*y}, \underset{\text{in stride}}{s\_in}, \underset{\text{out stride}}{s\_out})$

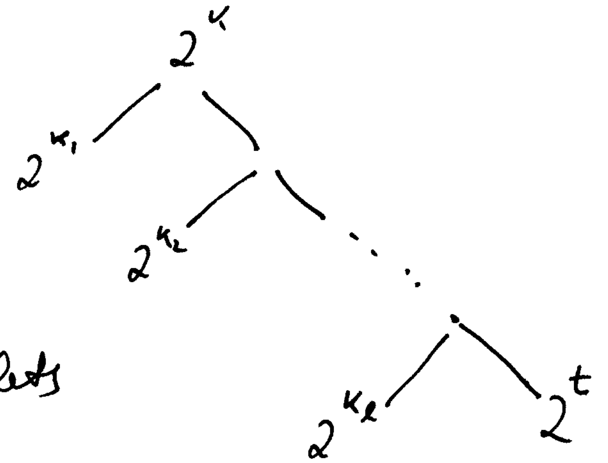
Pseudocode for Cooley-Tukey: in = *x, out = *y

```
for i = 0:k-1
    DFT(m, *x+i, *y+im, k, 1)      } recursed until codelet
for j = 0:m-1
    DFT(k, *x+j, *t, m)            } codelets
```

Optimizations:

1.) Fusing iterative steps (for locality)
2.) Precomputing constants (twiddles)
3.) Optimized basic blocks (codelets, generated)
4.) Adaptivity through search over relevant algorithm space

Algorithm space: $n = 2^k$

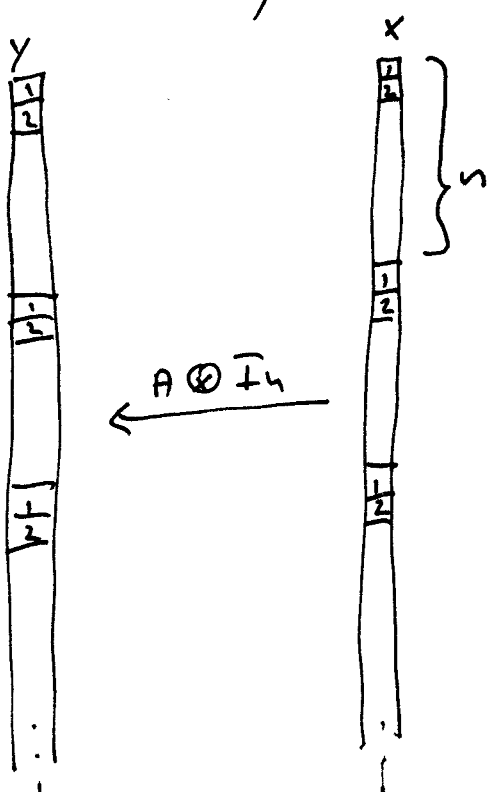


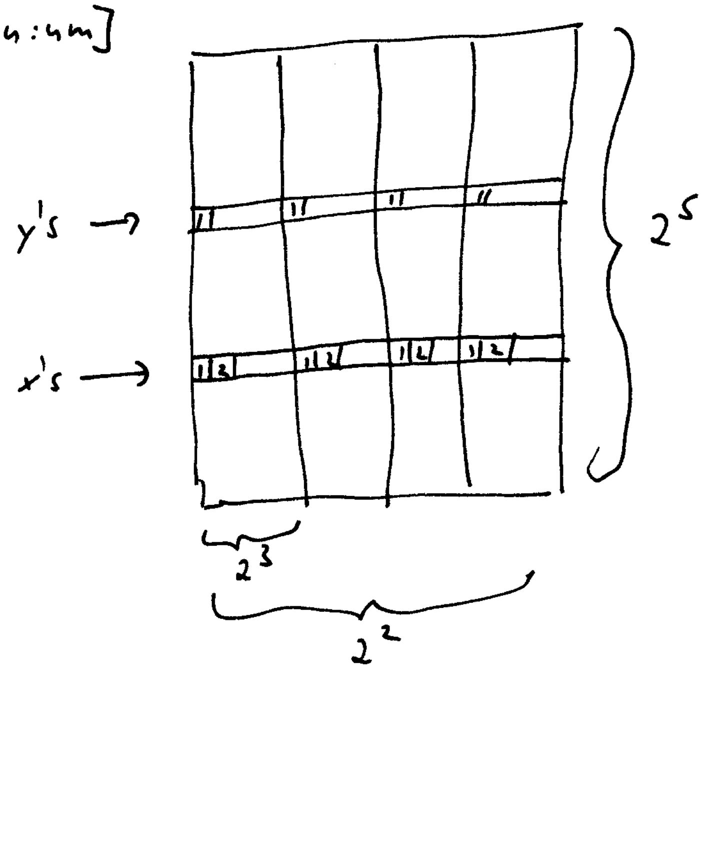
$k = k_1 + \dots + k_\ell + t$

The "$A \otimes I_n$" Problem

Assume:
- $8KB = 2^{13} B = 2^{10}$ doubles cache
  $64B = 2^3$ doubles cacheline
  4-way set associative
- A_m is $m \times m$
  $n = 2^k$, $k \geq 5$

$A_m \otimes I_n \Leftrightarrow$ for $i = 0:n-1$
  $y[i:n:nm] = A\,x[i:n:nm]$





problems:

- in each iteration, only 4 (associativity) cache location are usable for in- and output of A, respectively.
- no cacheline reuse ~~the $\boxed{1}$'s are gone once~~

Solution 1:

$$A_m \otimes I_n = L^{nm}_{m} \, (\underbrace{I_n \otimes A_m}_{\text{perfect access}}) \, L^{nm}_{n}$$

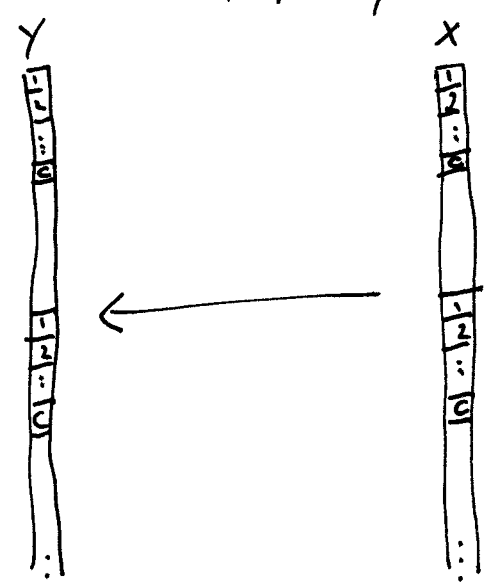
expensive runtime permutations

- requires efficient implementation of L, e.g., L can be blocked

Solution 2: Copy strided in/output to consecutive memory locations. ~~and~~

Solution 3: Interleave loop bodies for cacheline reuse

$$A_m \otimes I_n \rightarrow \underbrace{(A_m \otimes I_c)}_{\text{loop body}} \otimes \underbrace{I_{n/c}}_{\text{loop}}$$



Interleaving $A \otimes I_c$

instead of	do
1. op. on $\boxed{1}$'s	1. op on $\boxed{1}$'s
2. op on $\boxed{1}$'s	1. op on $\boxed{2}$'s
⋮	⋮
last op on $\boxed{1}$'s	1. op on $\boxed{c}$'s
1. op on $\boxed{2}$'s	2. op on $\boxed{1}$'s
⋮	⋮
last op on $\boxed{2}$'s	2. op on $\boxed{c}$'s
⋮	⋮
1. op on $\boxed{c}$'s	last op. on $\boxed{1}$'s
⋮	⋮
last op on $\boxed{c}$'s	last op. on $\boxed{c}$'s

- c can be a search parameter, or
- choose c = cacheline size (in doubles)

The "$A \otimes I_n$" problem occurs in
- Cooley-Tukey FFT for 2-power size
- WHT (see assignment)
- any 2D transform (2-power size)

$$T_{n \times n} = T_n \otimes T_n = (T_n \otimes I_n)(I_n \otimes T_n)$$