Algorithms and Computation in Signal Processing

special topic course 18-799B
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MMM versus MVM

Matrix-Vector Multiplication (MVM)

- MMM:
  - BLAS3
  - $O(n^2)$ data (input), $O(n^3)$ computation, implies $O(n)$ reuse per number (More precise on blackboard)
- MVM: $y = Ax$
  - BLAS2
  - $O(n^2)$ data, $O(n^2)$ computation
  - explain which optimizations remain useful (partially blackboard)
    - cache blocking?
    - register blocking?
    - unrolling?
    - scalar replacement?
    - add/mult interleaving, skewing?

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  - explain which optimizations remain useful (partially blackboard)
    - cache blocking? yes, but reuse of x and y only
    - register blocking? yes, but reuse of x and y only
    - unrolling? yes
    - scalar replacement? x and y only
    - add/mult interleaving, skewing? yes
    - expected gains smaller

MMM vs. MVM: Performance

- Performance for 2000 x 2000 matrices
- Best code out of ATLAS, vendor lib., Goto

Processor and compiler	Clock (MHz)	Data cache sizes	DGEMV (MFLOPS)	DGEMM (MFLOPS)
Sun UltraSPARC IIi Sun C v6.0	333	L1: 16 KB L2: 2 MB	58	425
Intel Pentium III Mobile (Coppermine) Intel C v6.0	800	L1: 16 KB L2: 256 KB	147	590
IBM Power4 IBM xlc v6	1300	L1: 64 KB L2: 1.5 MB L3: 32 MB	915	3500
Intel Itanium 2 Intel C v7.0	900	L1: 16 KB L2: 256 KB L3: 3 MB	1330	3500

Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

Sparse Matrix-Vector Multiplication (Sparsity, Bebop)

Sparse MVM

- y = Ax, A sparse but known

- Important routine in:
  - finite element methods
  - PDE solving
  - physical/chemical simulation (e.g., fluid dynamics)
  - linear programming
  - scheduling
  - signal processing (e.g., filters)
  - …

- In these applications, y = Ax is performed many times
  - justifies one-time tuning effort

Storage of Sparse Matrices

- Standard storage (as 2-D array) inefficient (many zeros are stored)
- Several sparse storage formats are available
- Explain compressed sparse row (CSR) format (blackboard)
  - advantage: arrays are accessed consecutively for y = Ax
  - disadvantage: no reuse of x and y, inserting elements costly

Direct Implementation y = Ax, A in CSR

```
void smvm_1x1( int m, const double* value, const int* col_idx,
               const int* row_start, const double* x, double* y )
{
        int i, jj;

        /* loop over rows */
        for( i = 0; i < m; i++ ) {
                double y_i = y[i];

                /* loop over non-zero elements in row i */
                for( jj = row_start[i]; jj < row_start[i+1];
                     jj++, col_idx++, value++ ) {
                        y_i += value[0] * x[col_idx[0]];
                }
                y[i] = y_i;
        }
}
```

scalar replacement
(only y is reused)

indirect array addressing
(problem for compiler opt.)

Code Generation/Tuning for Sparse MVM

- Sparsity/Bebop *link*

- Paper used: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004 (can be found on above website)

Impact of Matrix-Sparsity on Performance

- Adressing overhead (dense MVM vs. dense MVM in CSR):
  - ~ 2x slower (mflops, example only)
- Irregular structure
  - ~ 5x slower (mflops, example only) for "random" sparse matrices
- Fundamental difference between MVM and sparse MVM (SMVM):
  - sparse MVM is input dependent (sparsity pattern of A)
  - changing the order of computation (blocking) requires change of data structure (CSR)

Bebop/Sparsity: SMVM Optimizations

- Register blocking
- Cache blocking

Register Blocking

- Idea: divide SMVM y = Ax into micro (dense) MVMs of matrix size r x c
  - store A in r x c block CSR (r x c BCSR)
- Explain on blackboard
  - Advantages:
    - reuse of x and y (as for dense MVM)
    - reduces index overhead
  - Disadvantages:
    - computational overhead (zeros added)
    - storage overhead (for A)

Example: y = Ax in 2 x 2 BCSR

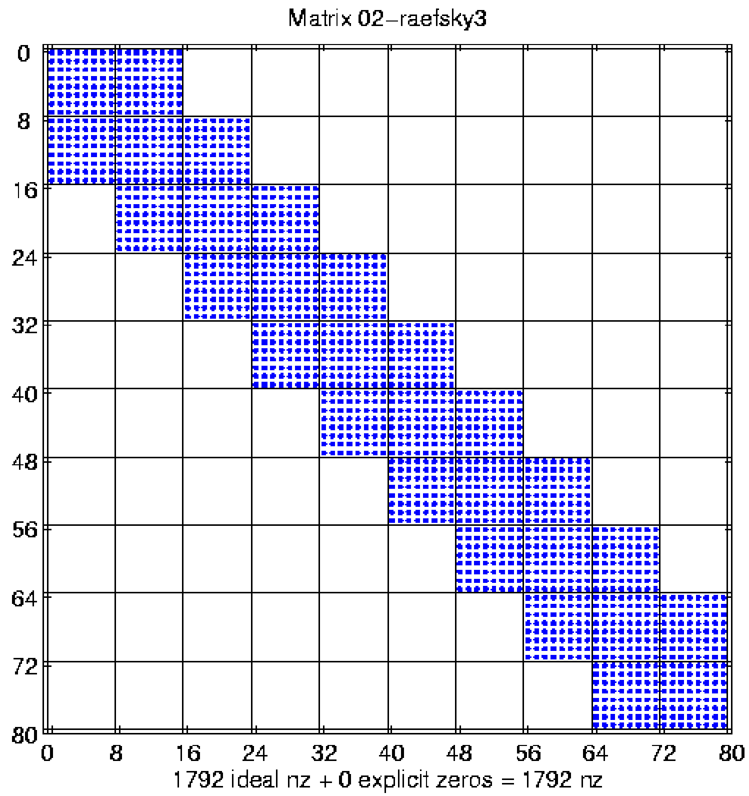
```
void smvm_2x2( int bm, const int *b_row_start, const int *b_col_idx,
               const double *b_value, const double *x, double *y )
{
        int i, jj;

        /* loop over block rows */
        for( i = 0; i < bm; i++, y += 2 ) {
                register double d0 = y[0];
                register double d1 = y[1];

                /* dense micro MVM */
                for( jj = b_row_start[i]; jj < b_row_start[i+1];
                     jj++, b_col_idx++, b_value += 2*2 ) {
                        d0 += b_value[0] * x[b_col_idx[0]+0];
                        d1 += b_value[2] * x[b_col_idx[0]+0];
                        d0 += b_value[1] * x[b_col_idx[0]+1];
                        d1 += b_value[3] * x[b_col_idx[0]+1];
                }
                y[0] = d0;
                y[1] = d1;
        }
}
```

scalar replacement
(y is reused)

Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

Which Block Size (r x c) is Optimal?

- Example: ~20,000 x 20,000 matrix with perfect 8 x 8 block structure, 0.33% non-zero entries
- In this case:
  no overhead when blocked r x c, with r,c divides 8

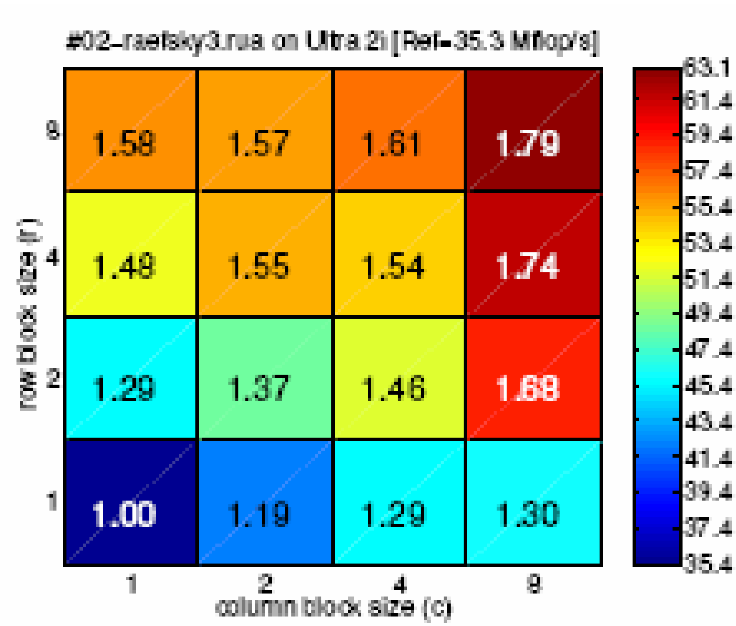


source: R. Vuduc, LLNL

Speed-up through r x c Blocking

Ultra 2i

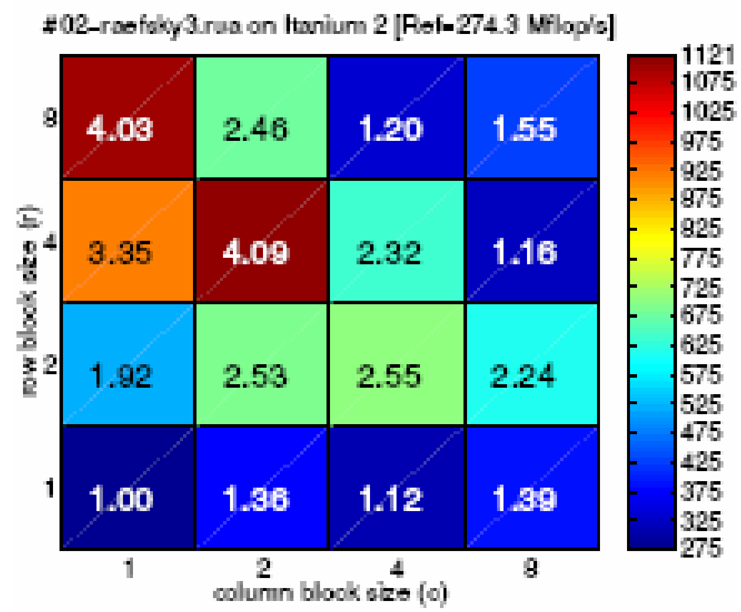
row block size r



col. block size c

Itanium 2

row block size r



col. block size c

- machine dependence
- hard to predict

Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

How to Find the Best Register Blocking for given A?

- Best blocksize hard to predict (see previous slide)
- Searching over all r x c (within a range, say 1..12) BCSR expensive
  - conversion of A in CSR to BCSR roughly as expensive as 10 SMVMs
- Solution: Performance model for given A

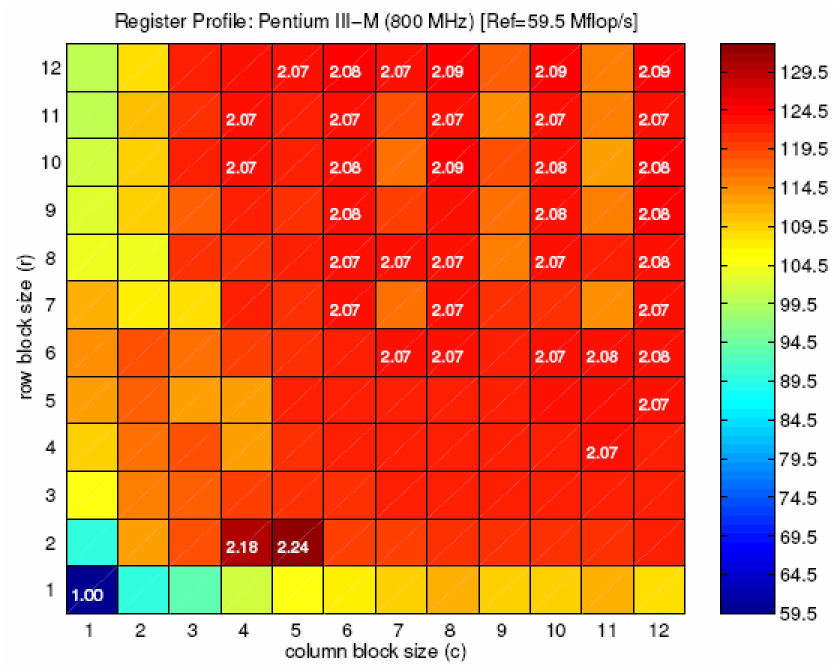
Performance Model for given A

- Model for given A built from
  - Gain of blocking:
    $G_{r,c}$ = Performance r x c BCSR/performance CSR for **dense** MVM
    machine dependent, independent of matrix A
  - Computational overhead:
    $O_{r,c}$ = size of A in r x c BCSR/size of A in CSR
    machine independent, dependent on A
    computed by scanning only a fraction of the matrix
    (blackboard example)
- Model: Performance gain from r x c blocking of A:
  $P_{r,c} = G_{r,c}/O_{r,c}$
- For given A, use this model to search over all r, c in {1,…,12}

Gain from Blocking (Dense Matrix in BCSR)

Pentium III

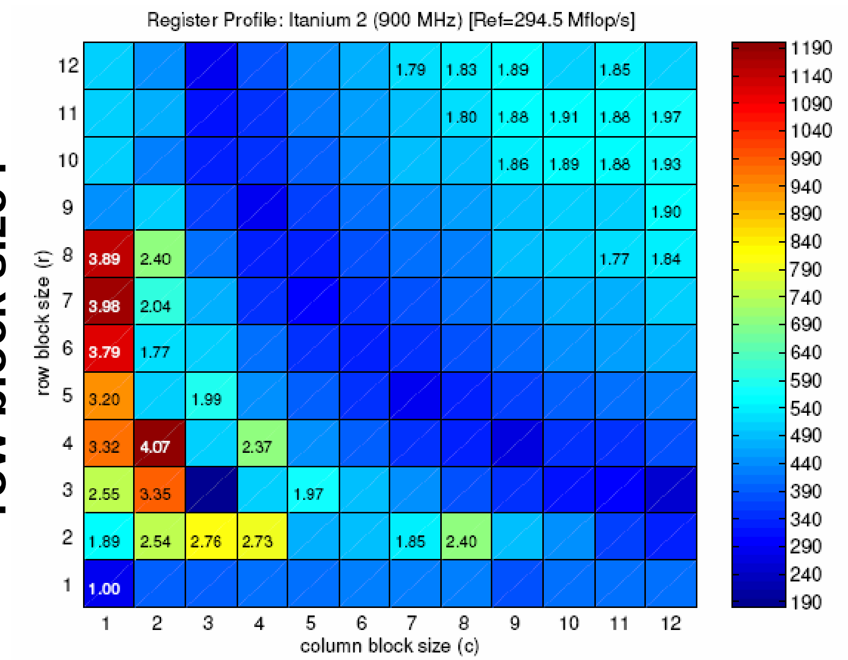
row block size r



col. block size c

Itanium 2

row block size r



col. block size c

- machine dependence
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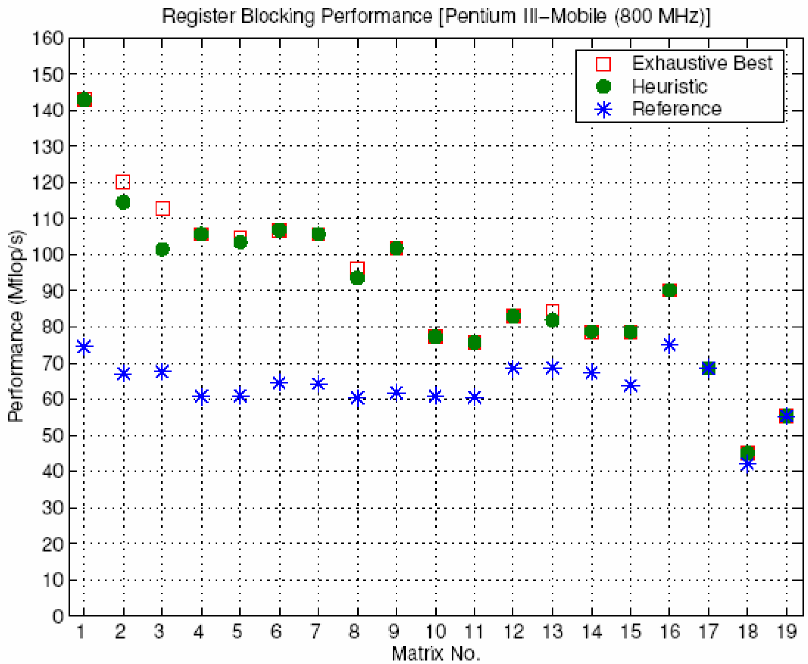
Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

Register Blocking: Experimental results

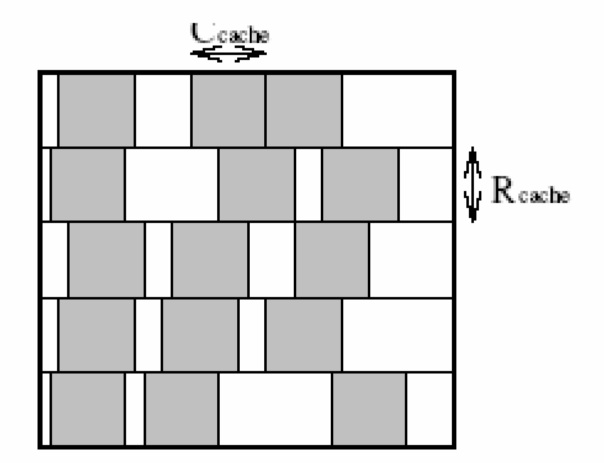
- Paper applies method to a large set of sparse matrices
- Performance gains between 1x (no gain) for very unstructured matrices and 4x



Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

Cache Blocking

- Idea: divide sparse matrix into blocks of sparse matrices

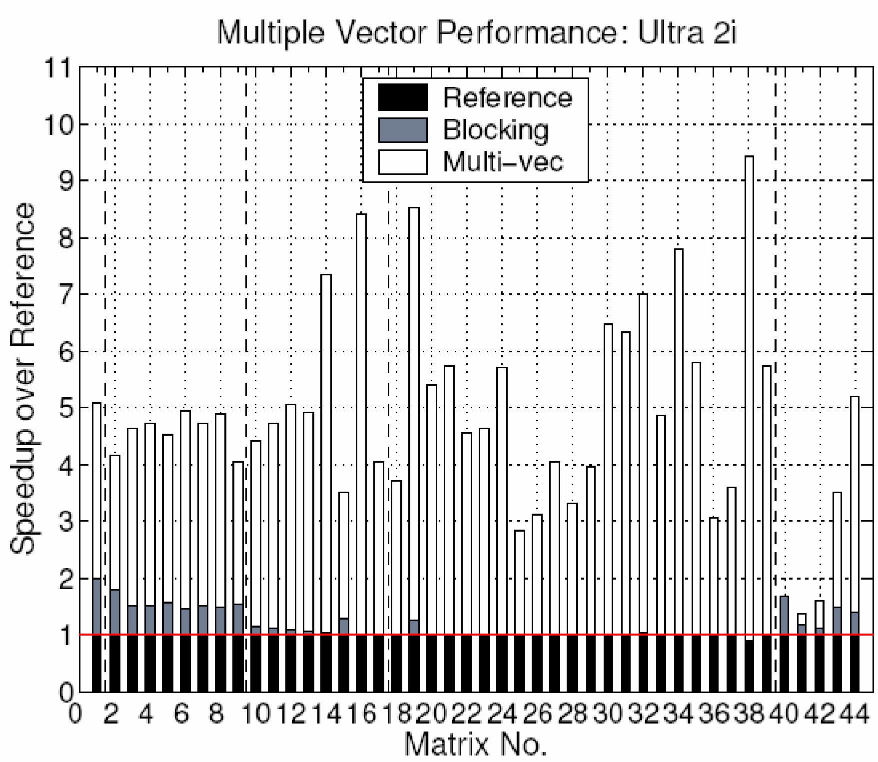


- Experiments:
  - requires very large matrices (x and y do not fit into cache)
  - speed-up up to 80%, speed-up only for few matrices, with 1 x 1 BCSR

Figure Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

Multiple Vector Optimization

- Blackboard
- Experiments: up to 9x speedup for 9 vectors



Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

Principles in Bebop/Sparsity Code Generation

- Optimization for memory hierarchy = increasing locality
  - Blocking for registers (micro-MMMs) + change of data structure for A
  - Less important: blocking for cache
  - Optimizations are input dependent (on sparse structure of A)
- Fast basic blocks for small sizes (micro-MMM):
  - Loop unrolling (reduce loop overhead)
  - Some scalar replacement (enables better compiler optimization)
- Search for the fastest over a relevant set of algorithm/implementation alternatives (= r, c)
  - Use of performance model (versus measuring runtime) to evaluate expected gain

red = different from ATLAS