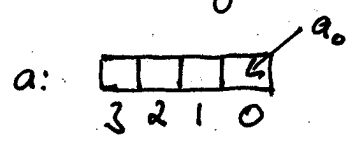


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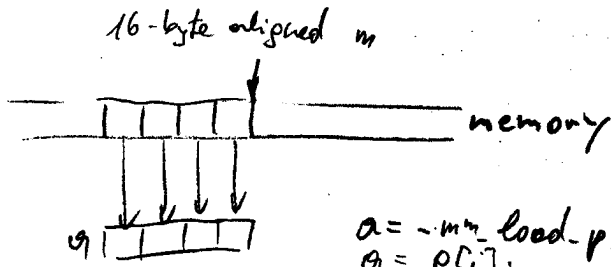
Instructions: 1-7  
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 - loads & stores 8-12  
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 - arithmetic 16-18

vector indexing:



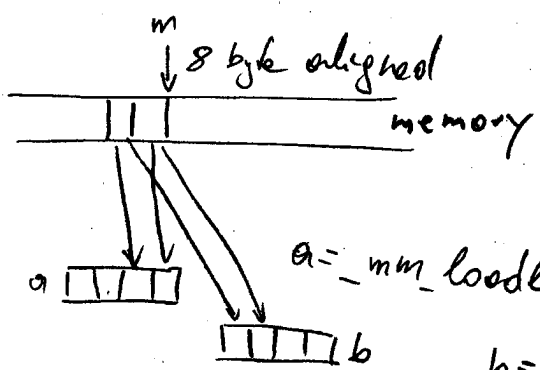
in most instructions the order of operands matters  $\frac{D}{0}$

# Load Instructions (SSE and later)

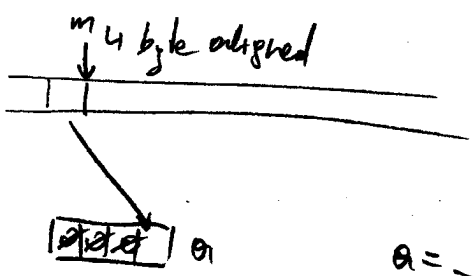


$a = \_mm\_load\_ps(m);$   
 $a = p[0];$   
 $a = \_mm\_loadu\_ps(m);$

aligned, explicit  
 - ~~4~~, implicit  
 unaligned



$a = \_mm\_loadl\_pi(a, m)$   
 $b = \_mm\_loadh\_pi(b, m)$

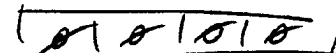
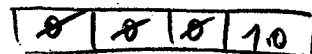
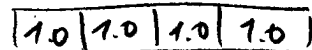
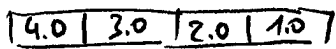


$a = \_mm\_load\_ss(m)$

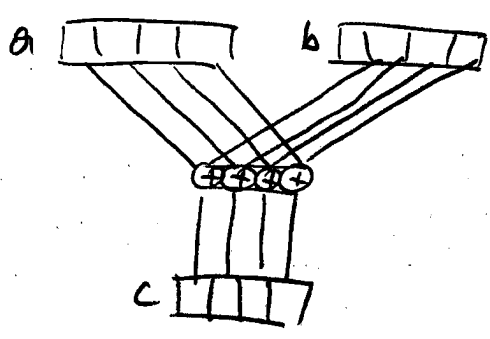
stores are analogous

# Constants (Special load instructions) (SSE and later)

$c = \_mm\_set\_ps(1.0, 2.0, 3.0, 4.0);$   
 $d = \_mm\_set1\_ps(1.0);$   
 $e = \_mm\_set\_ss(1.0);$   
 $f = \_mm\_set\_zero\_ps();$



# Vector arithmetic (SSE and later)



$$c = \_mm\_addps(a, b)$$

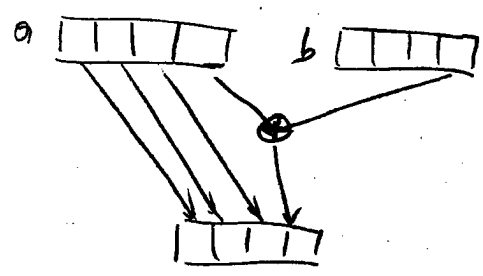
Same:

$$c = \_mm\_subps(a, b)$$

$$c = \_mm\_mulps(a, b)$$

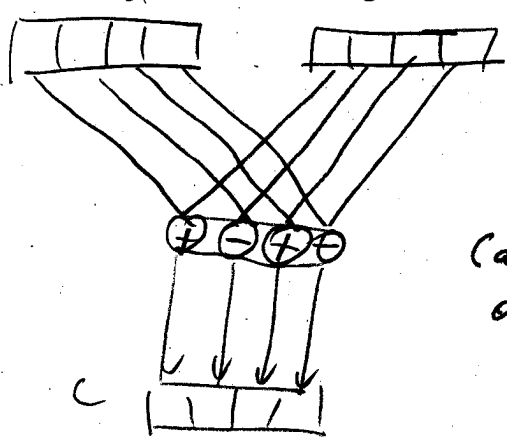
⋮

# Scalar arithmetic (SSE and later)



$$c = \_mm\_addss(a, b)$$

# Add Sub (SSE3 and later)



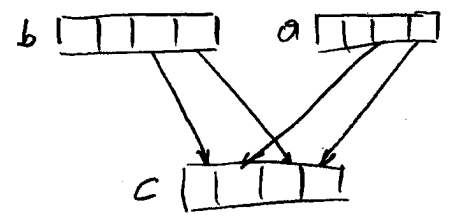
$$c = \_mm\_addsubps(a, b)$$

(alternate add & sub of vector elements)

# Reorders Instructions (SSE and later)

## Unpack lo

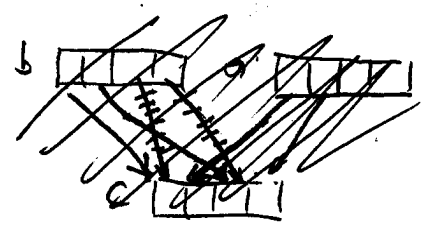
$$c = \text{\_mm\_unpacklo\_ps}(a, b)$$



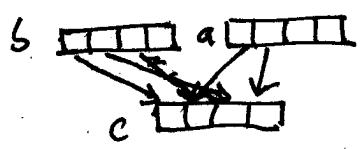
$$\begin{aligned} c_0 &= a_0 \\ c_1 &= b_0 \\ c_2 &= a_1 \\ c_3 &= b_1 \end{aligned}$$

## Unpack hi

$$c = \text{\_mm\_unpackhi\_ps}(a, b)$$

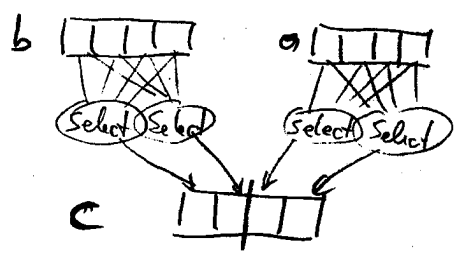


$$\begin{aligned} c_0 &= a_2 \\ c_1 &= b_2 \\ c_2 &= a_3 \\ c_3 &= b_3 \end{aligned}$$



## Shuffle

$$c = \text{\_mm\_shuffle\_ps}(a, b, \text{\_MM\_SHUFFLE}(i, k, j, l))$$



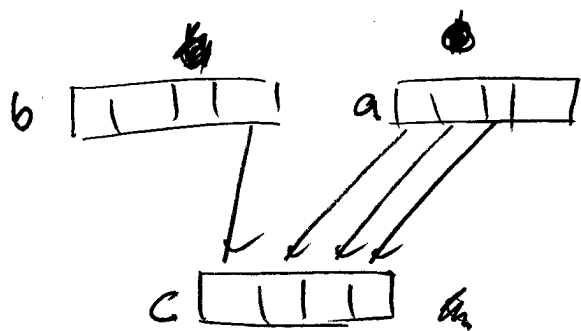
$$\begin{aligned} c_0 &= a_i \\ c_1 &= a_j \\ c_2 &= b_k \\ c_3 &= b_l \end{aligned}$$

any element of b      any element of a

$i, j, k, l \in \{0, \dots, 3\}$   
immediate

align (SSE3 and later)

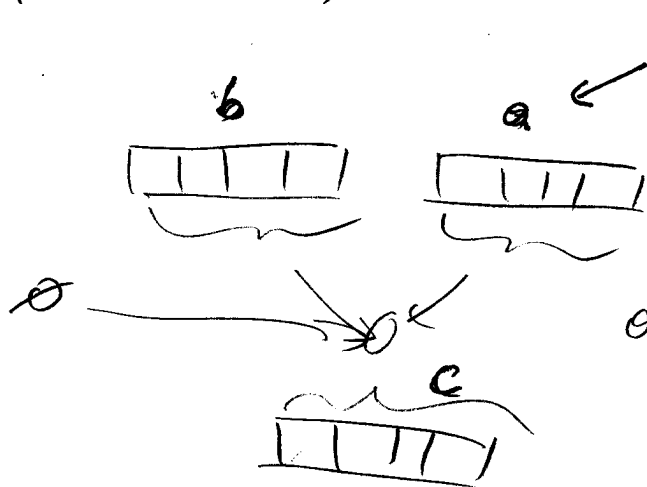
5



"any consecutive 4 elements of the concatenation of a and b goes into c"

```
c = _mm_castps_si128 ( _mm_alignr_epi8 (
    _mm_castsi128_ps(a),
    _mm_castsi128_ps(b) ) );
```

shuffle (SSE3 and later)



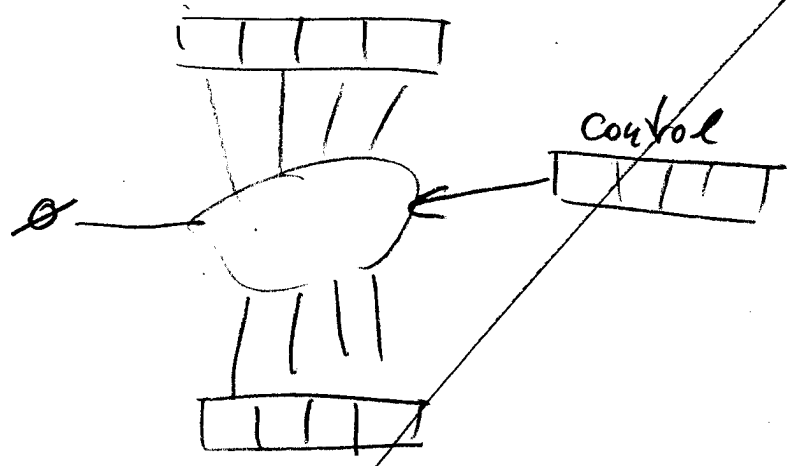
"c is filled with any element of b or 0" (specified by a)

```
_mm_shuffle_epi8 ( )
```

# shuffle (SSSE3 and later)

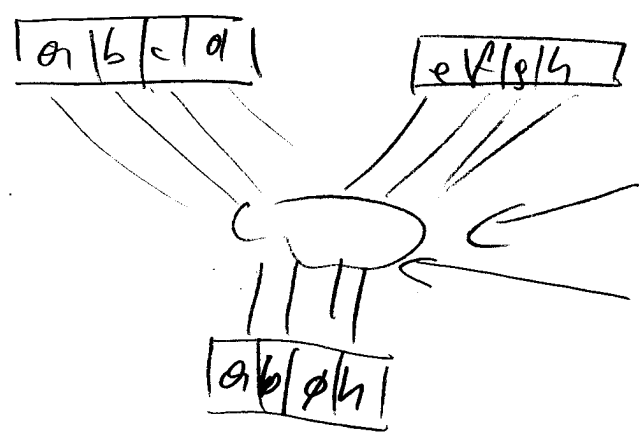
6

- wrong last time
- correct



- not the same as Altivec

## Blend instruction



"c is filled with elements of a or b at the same position on 0"

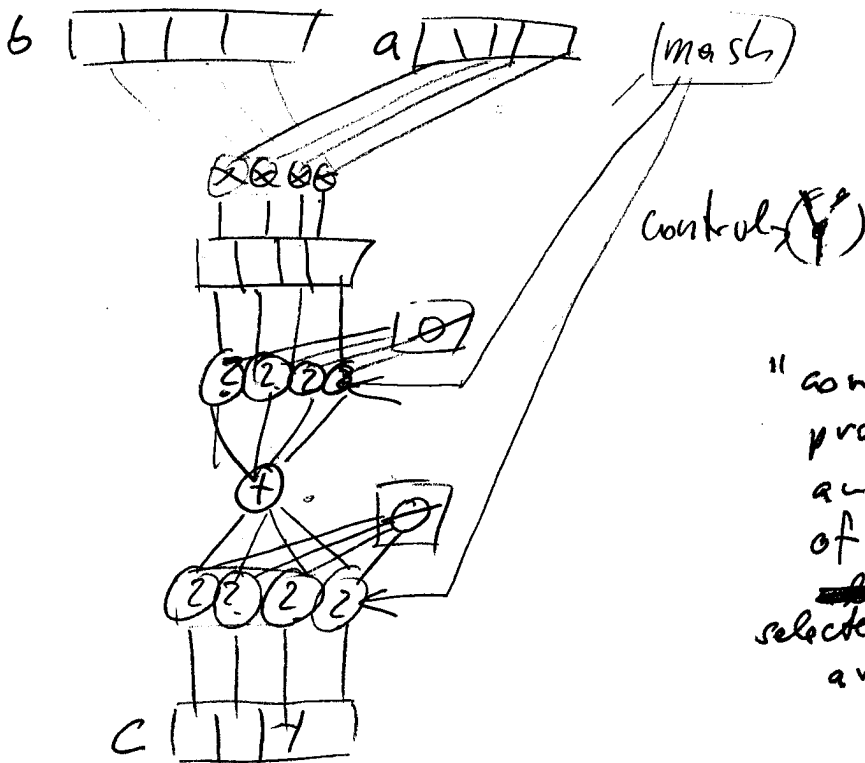
mask (int or vector)

# Matrix-Vector Product (SSE4 and later)

(7)

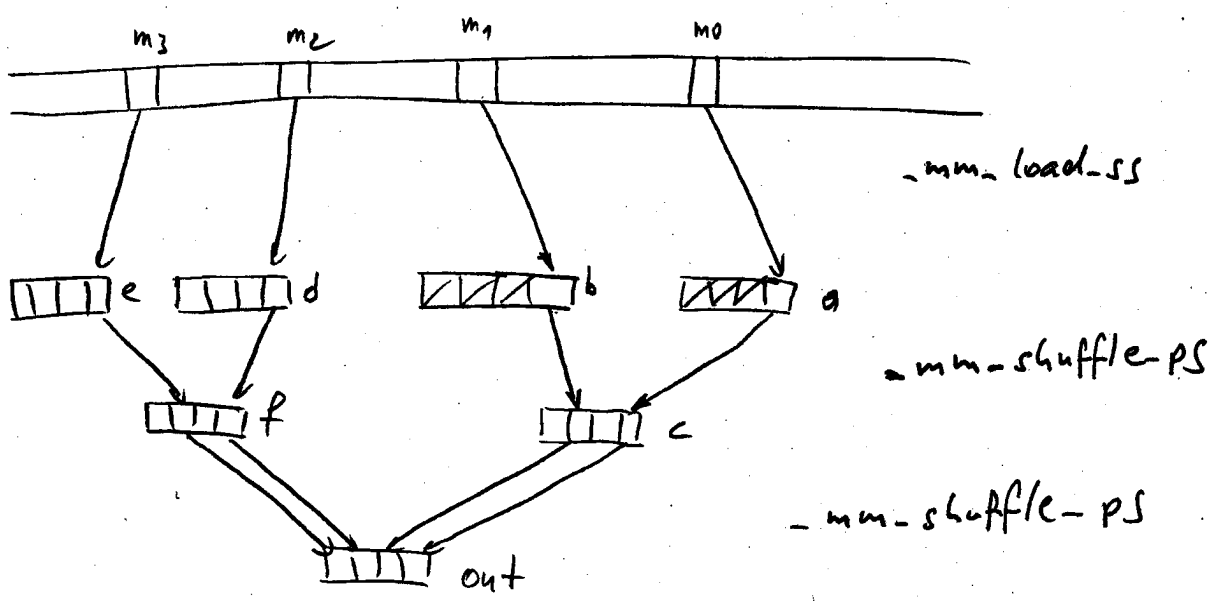
## Dot-product instruction

- mm\_dp-ps(a, b, mask) magic



"computes the pointwise product of a and b and writes an arbitrary sum of the resulting numbers into ~~all~~ selected elements of c (the others are set to 0)"

# Example: load 4 real numbers from arbitrary memory locations (SSE)



```
# define SCALAR_LOAD (out, m0, m1, m2, m3)
{
  a = _mm_load_ss(m0);
  b = _mm_load_ss(m1);
  c = _mm_shuffle_ps(a, b, _MM_SHUFFLE(1, 0, 1, 0));
  d = _mm_load_ss(m2);
  e = _mm_load_ss(m3);
  f = _mm_shuffle_ps(d, e, _MM_SHUFFLE(1, 0, 1, 0));
  out = _mm_shuffle_ps(c, f, _MM_SHUFFLE(1, 0, 1, 0));
}
```

7 instructions

Note: - Whenever possible avoid this by ~~aligning~~ restructuring the algorithm or data format to have aligned vector loads (see page 1)

- ~~can~~ equivalent to macro on page 11 (but the above is "safer")



Other gather/scatter implementation (BAD) Don't do it like this (10)

float f[20] = { ... };

--declspec(align(16)) g[4];

--m128 vf;

g[0] = f[3];

g[1] = f[5];

g[2] = f[12];

g[3] = f[17];

vf = mm\_loadps(g);  
// operations

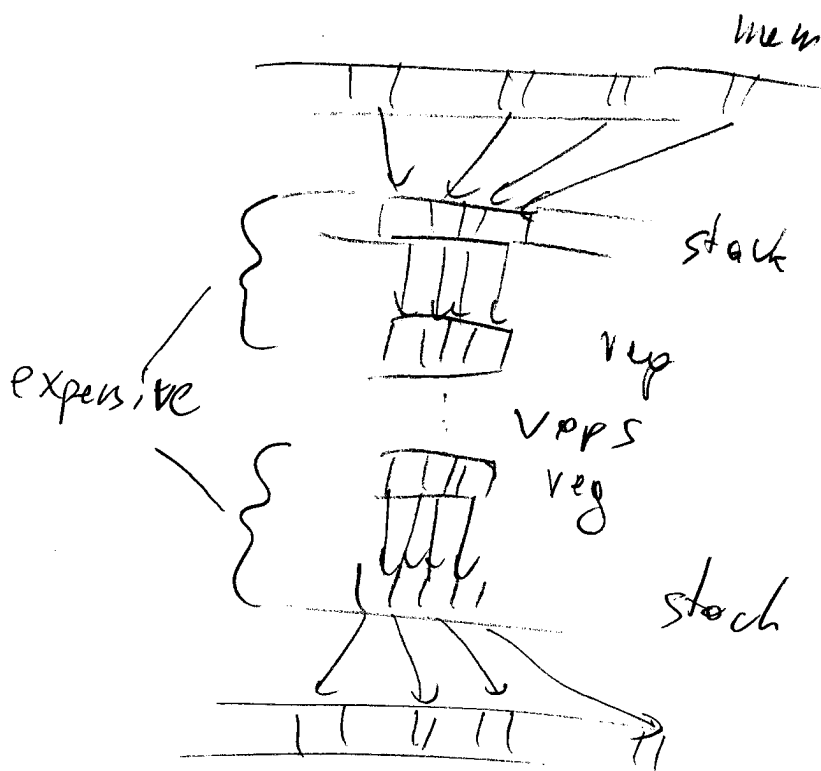
--mm\_storeps(g, vf);

f[7] = g[0];

f[13] = g[1];

f[11] = g[2];

f[17] = g[3];



loads innocent, but is really bad  
most people try that at some point  
look at the assembly to see for yourself

Set instruction ~~type and usage~~ (SSE and later)

(1)

You can do: (see page 1)

```
_mm128 vf = _mm_set_ps(0.0, 3.0, 2.0, 1.0);
```

→ 1 vector load of 128 bit constant

Compiler lets you do this type of use

```
float f[20] = { ... };
```

```
_mm128 vf = _mm_set_ps(f[3], f[5], f[12], f[17]);
```

however internally: 4 loads, 3 shuffles

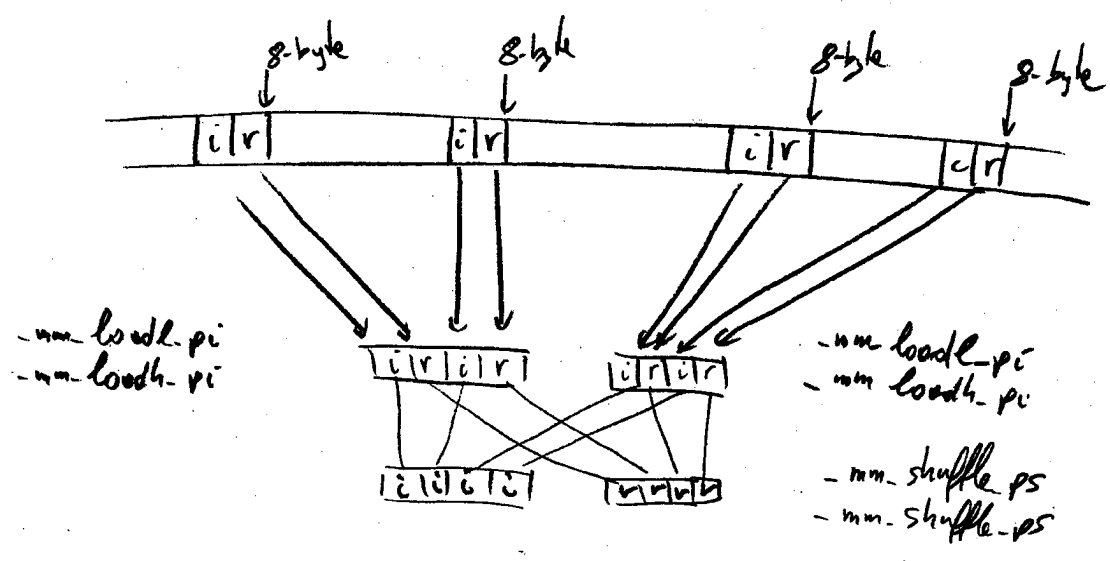
equivalent to page 8

Do not use `_mm_set_ps()` on variables  
if you can avoid it! (see page 8)

Example: load 4 complex numbers (load 4 pairs of numbers)

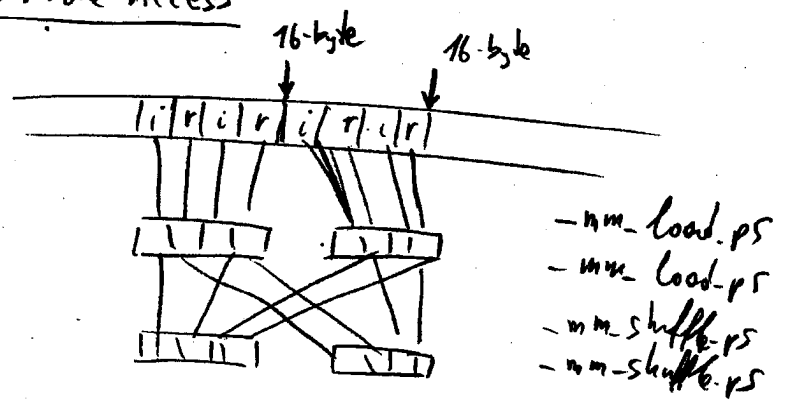
(SSE and /etc)

Strided access



6 instructions

Unit stride Access

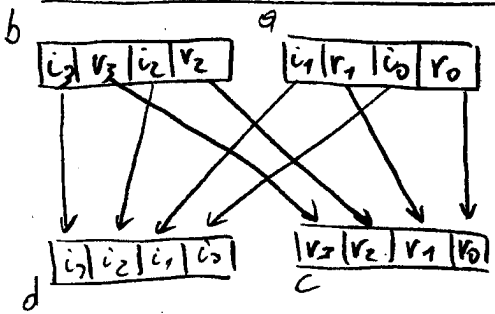


4 instructions  
(Benefit of consecutive data)

Same for store ops

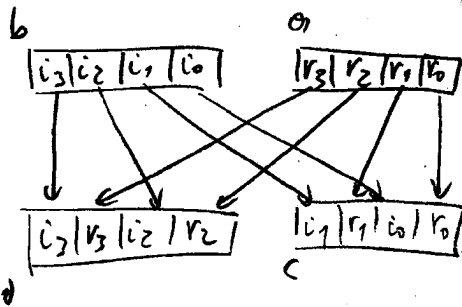
# Reorder Instructions - Examples (SSE and later)

Interleaved Complex  $\rightarrow$  split complex:  $L_2^8$



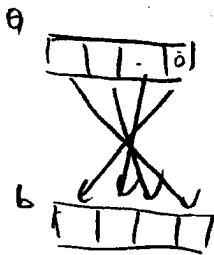
```
c = _mm_shuffle_ps(a, b, _MM_SHUFFLE(2, 0, 2, 0));
d = _mm_shuffle_ps(a, b, _MM_SHUFFLE(3, 1, 3, 1));
```

Split Complex  $\rightarrow$  interleaved complex:  $L_4^8$



```
c = _mm_unpacklo_ps(a, b);
d = _mm_unpackhi_ps(a, b);
```

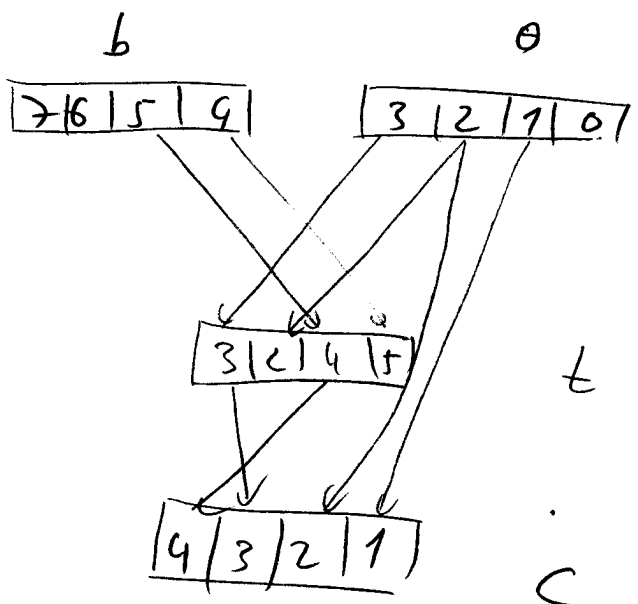
Reverse Vector:  $J_4$



```
b = _mm_shuffle_ps(a, a, _MM_SHUFFLE(0, 1, 2, 3));
```

~~Shift by M~~

Shift by 1 (SSE)



2 instructions

$$t = \text{mm\_shuffle\_ps}(b, a, \text{MM\_SHUFFLE}(3, 2, 1, 0));$$

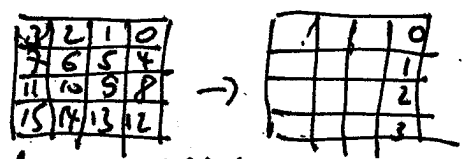
$$c = \text{mm\_shuffle\_ps}(b, b, \text{MM\_SHUFFLE}(1, 3, 2, 1));$$

SSE3:  $\text{palign} \quad \text{mm\_alignr\_epi8}()$

1 instruction

~~$\text{psrnb} \quad \text{mm\_shuffle\_epi8}()$~~

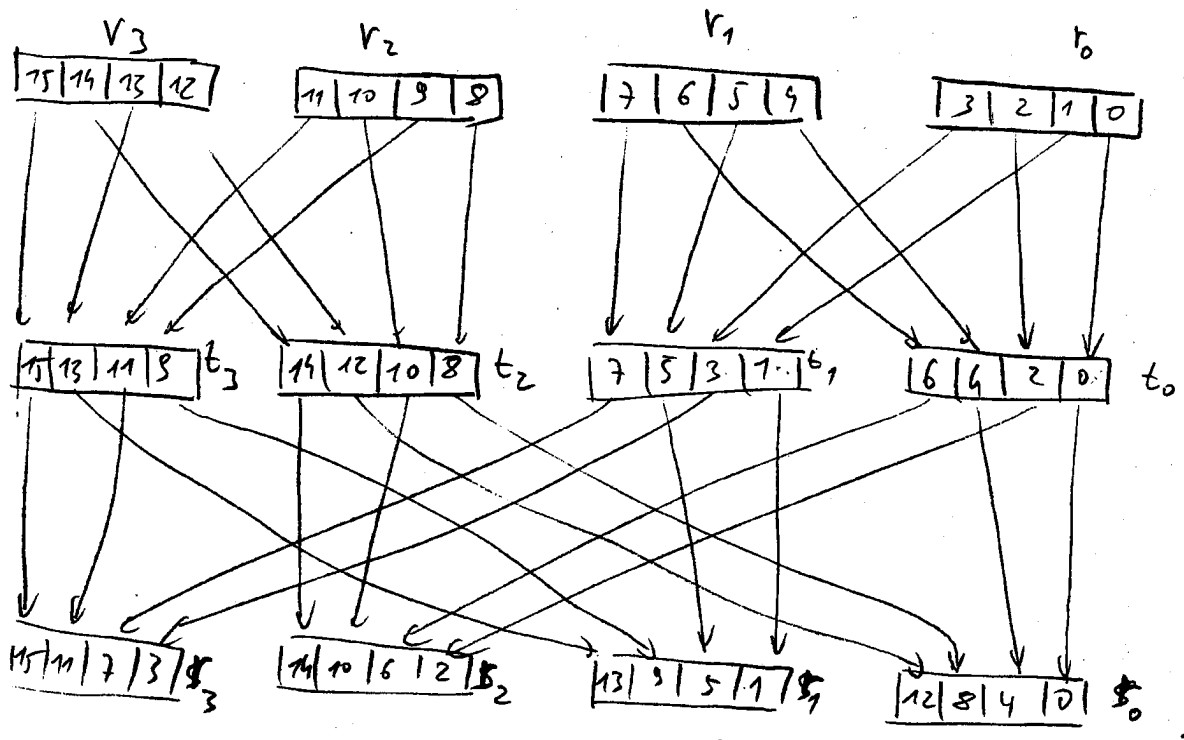
# Reorder Instructions - Examples



may need fixity

vectors are rows

Transpose 4x4:  $\frac{16}{4} = \frac{4 \oplus 4}{2} \oplus \frac{4 \oplus 4}{2} \oplus \frac{4 \oplus 4}{2} \oplus \frac{4 \oplus 4}{2}$



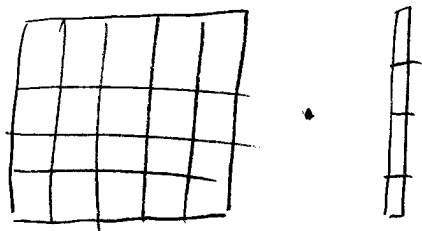
defined macro (SSE and later)

```
#define MM_TRANSPOSE4_PS(v0, v1, v2, v3) {
    __m128 t0, t1, t2, t3;
    t0 = _mm_shuffle_ps(v0, v1, _MM_SHUFFLE(2, 0, 2, 0));
    t1 = _mm_shuffle_ps(v0, v1, _MM_SHUFFLE(3, 1, 3, 1));
    t2 = _mm_shuffle_ps(v2, v3, _MM_SHUFFLE(2, 0, 2, 0));
    t3 = _mm_shuffle_ps(v2, v3, _MM_SHUFFLE(3, 1, 3, 1));
    v0 = _mm_shuffle_ps(t0, t2, _MM_SHUFFLE(2, 0, 2, 0));
    v1 = _mm_shuffle_ps(t1, t3, _MM_SHUFFLE(2, 0, 2, 0));
    v2 = _mm_shuffle_ps(t0, t2, _MM_SHUFFLE(3, 1, 3, 1));
    v3 = _mm_shuffle_ps(t1, t3, _MM_SHUFFLE(3, 1, 3, 1));
}
```

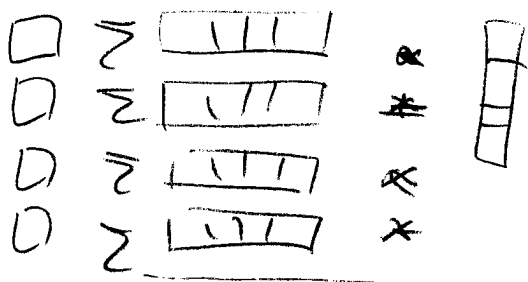
3

MM\_TRANSPOSE2\_PD also exists (for doubles)

# Matrix-Vector Product (SSE and later)



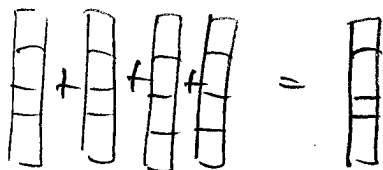
needs redrawing



5 vector loads  
4 vector multiplies  
but how to do the sums?

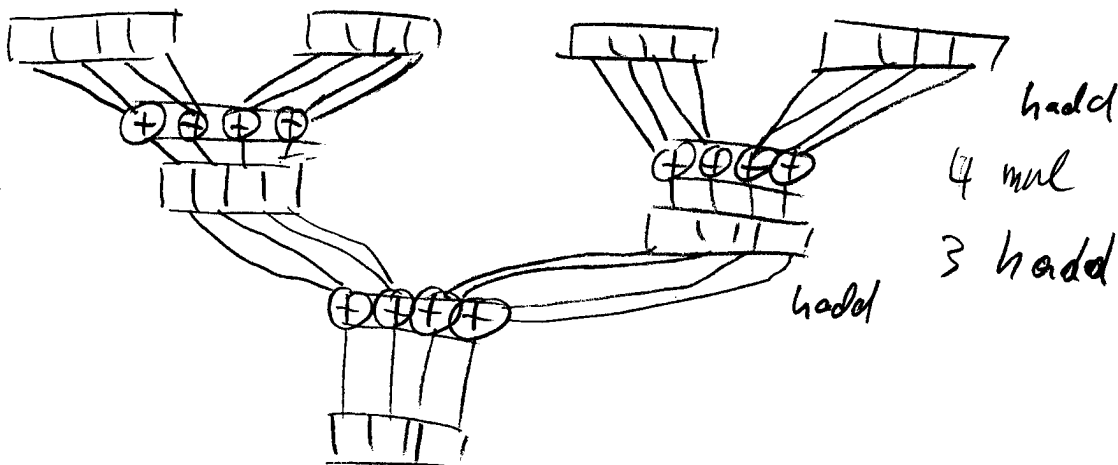
transpose

MV



3 adds, 8 shuffles

## other solutions (SSE3 and higher)



# Matrix-Vector Product (SSE4)

needs  
rednary

$$\begin{array}{l}
 \boxed{0|0|0|x} = \sum \boxed{\quad|\quad|\quad|\quad} \cdot \begin{array}{c} \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \end{array} \\
 \boxed{0|0|x|0} = \sum \boxed{\quad|\quad|\quad|\quad} \cdot \begin{array}{c} \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \end{array} \\
 \boxed{0|x|0|0} = \sum \boxed{\quad|\quad|\quad|\quad} \cdot \begin{array}{c} \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \end{array} \\
 \boxed{x|0|0|0} = \sum \boxed{\quad|\quad|\quad|\quad} \cdot \begin{array}{c} \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \end{array}
 \end{array}$$

---


$$\sum \boxed{x|x|x|x}$$

4 dot product instructions  
3 add instructions

---

or 4 dot products

+ scalar store ops :

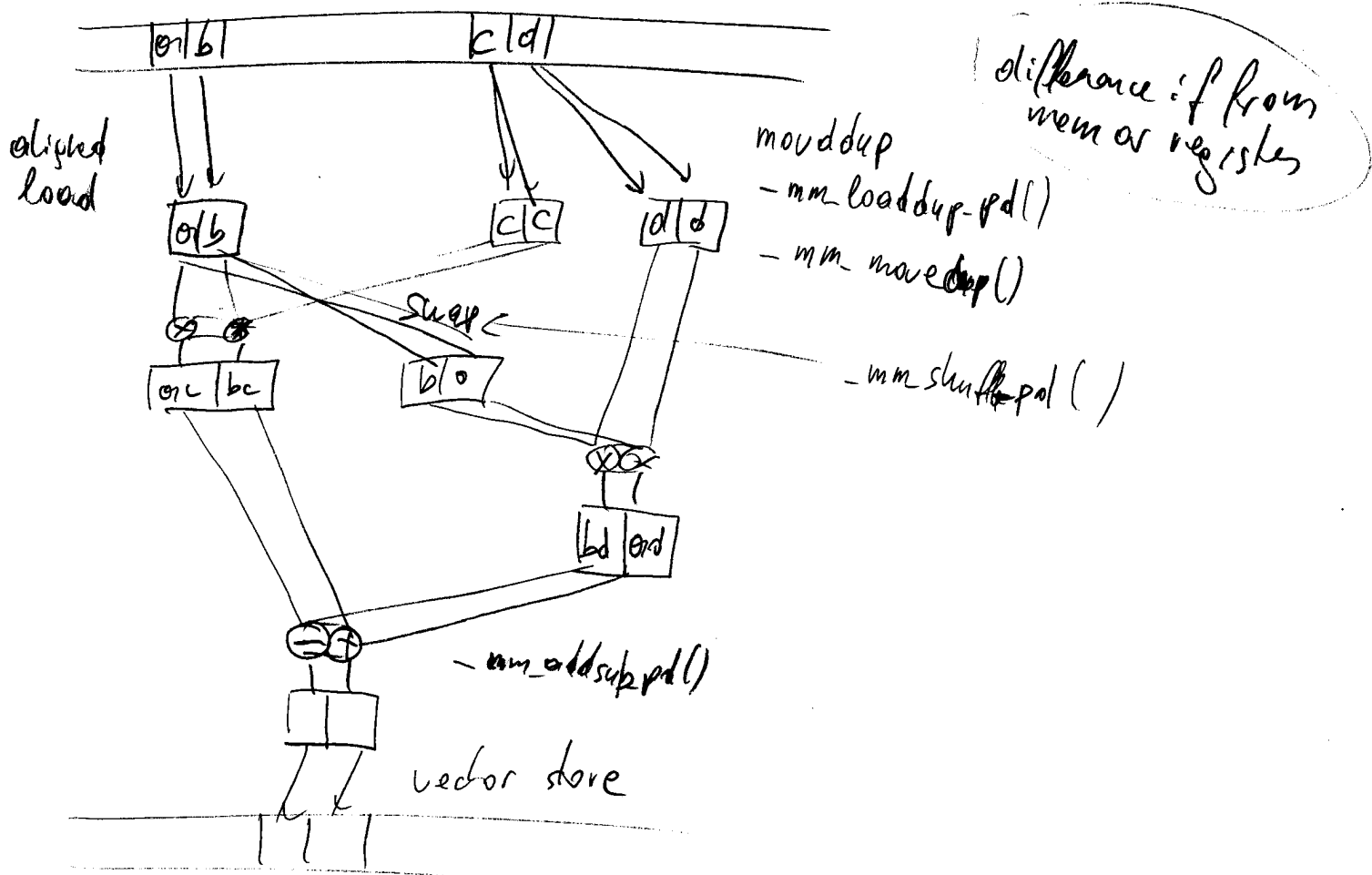
$$\boxed{\quad|\quad|\quad|x} = \sum \boxed{\quad|\quad|\quad|\quad} \cdot \begin{array}{c} \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \\ \boxed{\quad} \end{array}$$

performance implications?



# Complex Multiplication double SSE3

$$(a+ib)(c+id) = (ac-bd) + i(ad+bc)$$



SSE, SSE2: see slides