Green Codes :
Energy-efficient short-range communication

Pulkit Grover

Department of Electrical Engineering and Computer Sciences
University of California at Berkeley

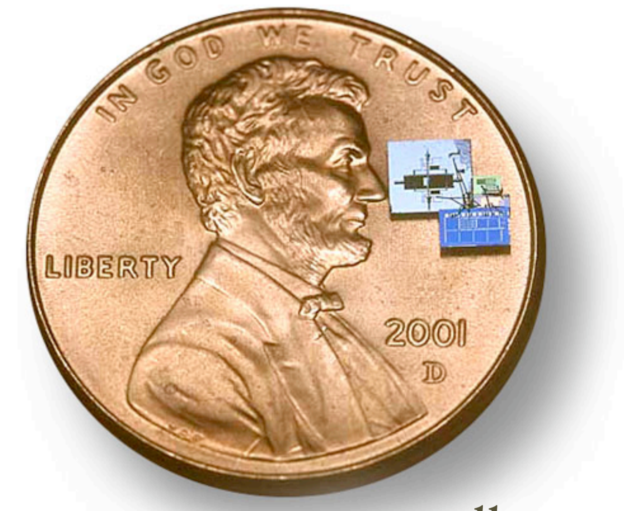
Joint work with Prof. Anant Sahai

Motivation : Understand processing power consumed in communicating

Fixed Rate

processor with heat sink

Fixed message size



small sensors

Motivation : Understand processing power consumed in communicating

Fixed Rate

processor with heat sink

Fixed message size

small sensors

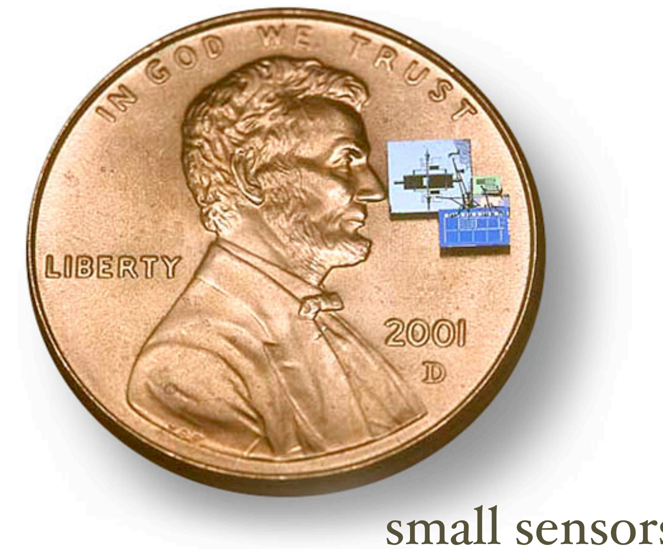
- Moore's law : decreasing implementation complexity

Motivation : Understand processing power consumed in communicating

Fixed Rate

processor with heat sink

Fixed message size



small sensors

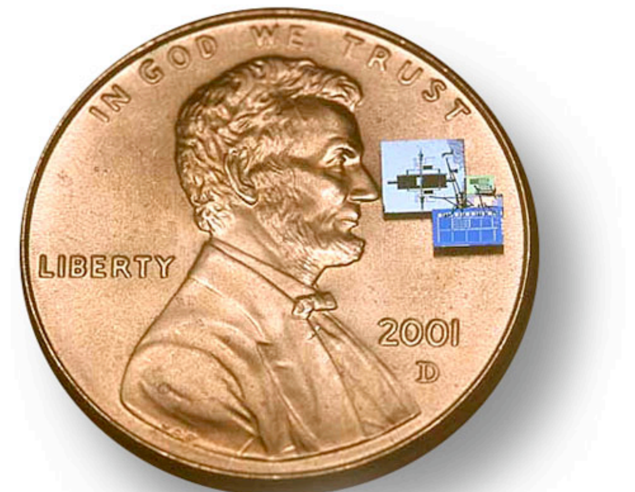
- Moore's law : decreasing implementation complexity
  - significant **power consumed in computations**

Motivation : Understand processing power consumed in communicating

Fixed Rate

processor with heat sink

Fixed message size



small sensors

- Moore's law : decreasing implementation complexity
  - significant **power consumed in computations**
- **total** power for communicating

Motivation : Understand processing power consumed in communicating

Fixed Rate

processor with heat sink

- Moore's law : decreasing implementation complexity
  - significant **power consumed in computations**
- **total** power for communicating

Fixed message size

small sensors

- Small battery operated wireless sensors

Motivation : Understand processing power consumed in communicating

Fixed Rate

processor with heat sink

Fixed message size

small sensors

- Moore's law : decreasing implementation complexity
  - significant **power consumed in computations**
- **total** power for communicating

- Small battery operated wireless sensors
  - energy at a premium.

Motivation : Understand processing power consumed in communicating

Fixed Rate

processor with heat sink

Fixed message size

small sensors

- Moore's law : decreasing implementation complexity
  - significant **power consumed in computations**
- **total** power for communicating

- Small battery operated wireless sensors
  - energy at a premium.
  - **flexibility in rate**.

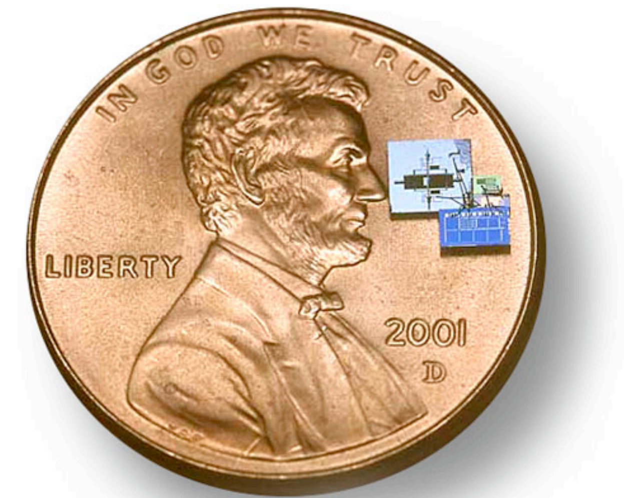
Motivation : Understand processing power consumed in communicating

Fixed Rate

processor with heat sink

- Moore's law : decreasing implementation complexity
  - significant **power consumed in computations**
- **total** power for communicating

Fixed message size

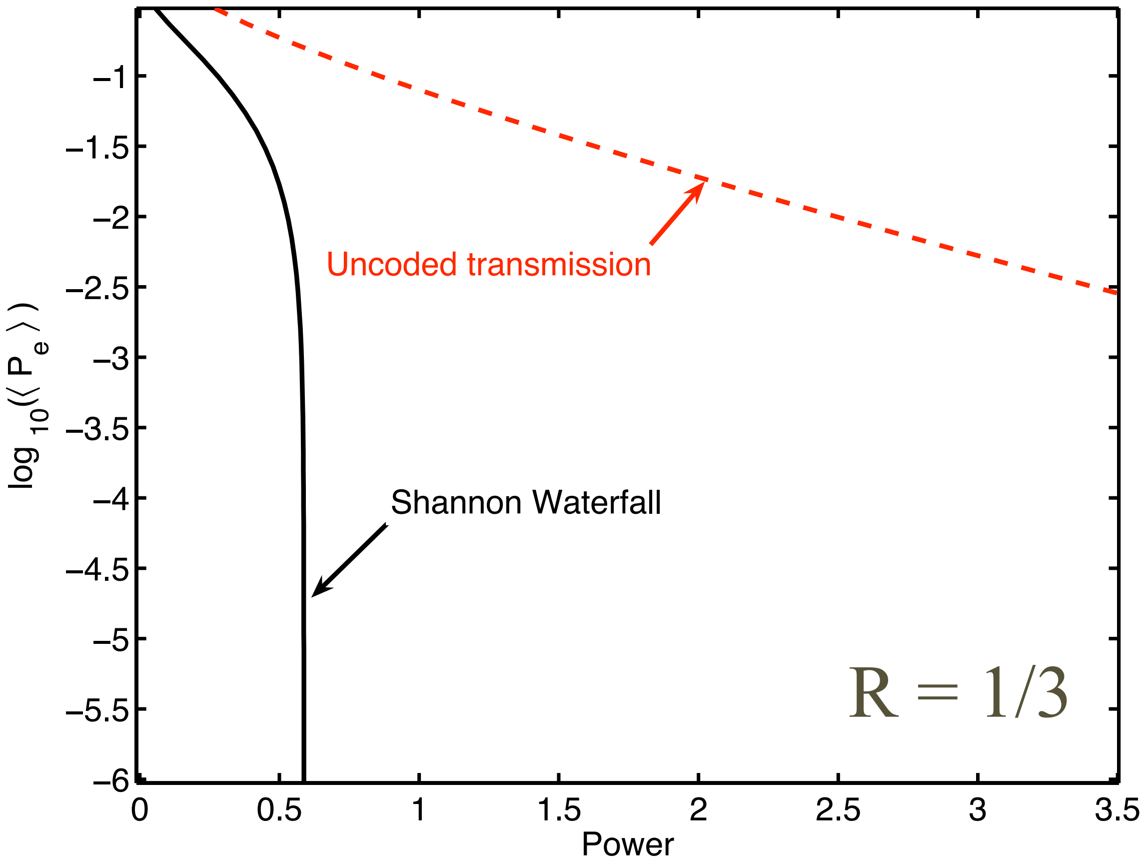


small sensors

- Small battery operated wireless sensors
  - energy at a premium.
  - **flexibility in rate**.
- **total** energy per bit

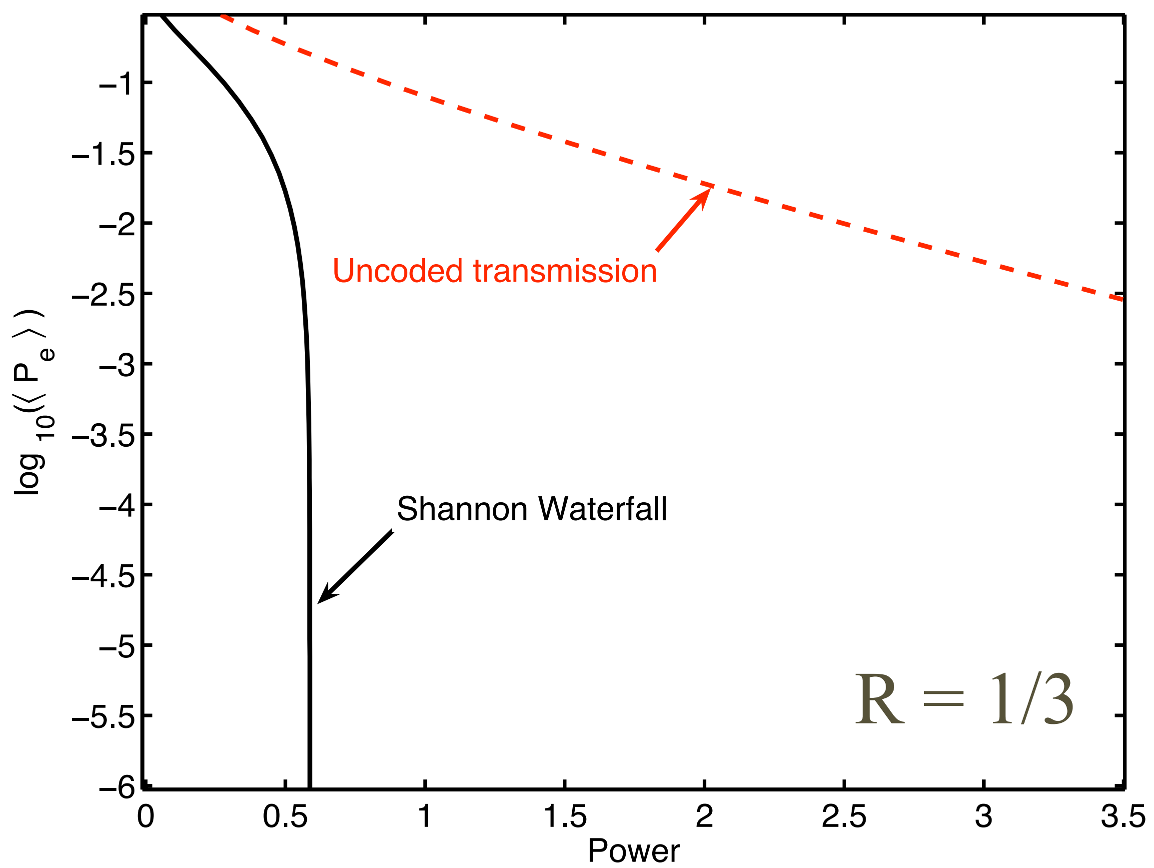
Promise of Shannon Theory

Fixed Rate: Shannon waterfall

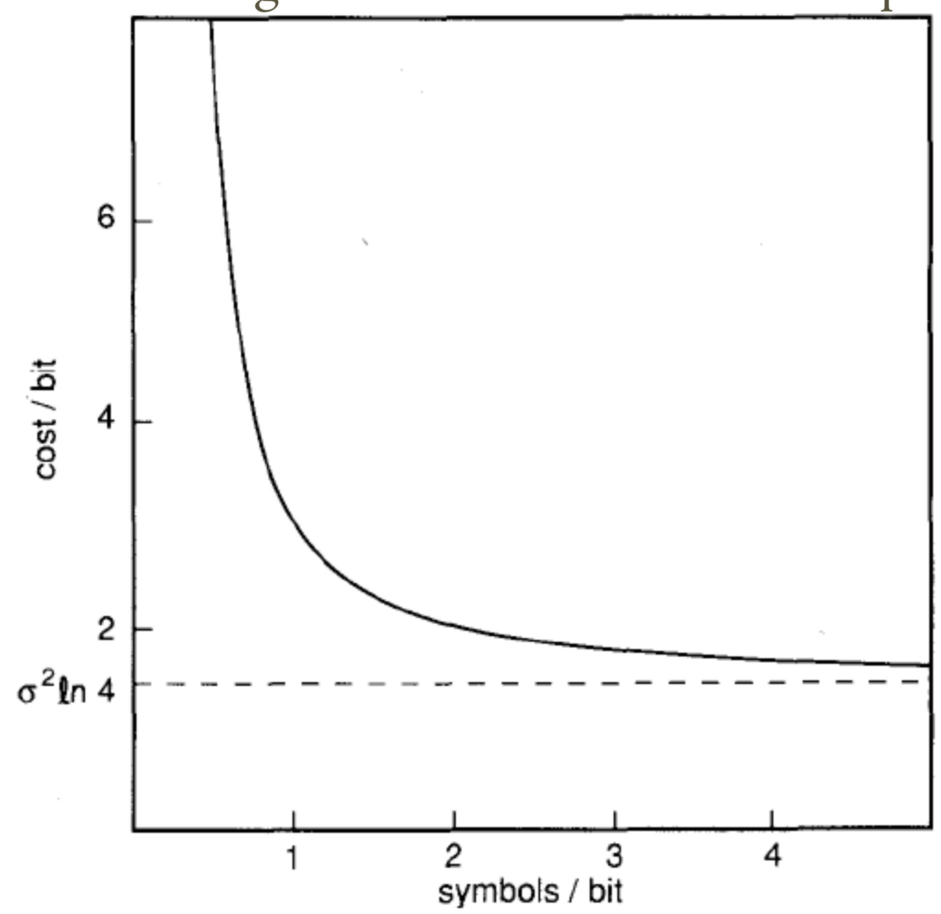


Promise of Shannon Theory

Fixed Rate: Shannon waterfall

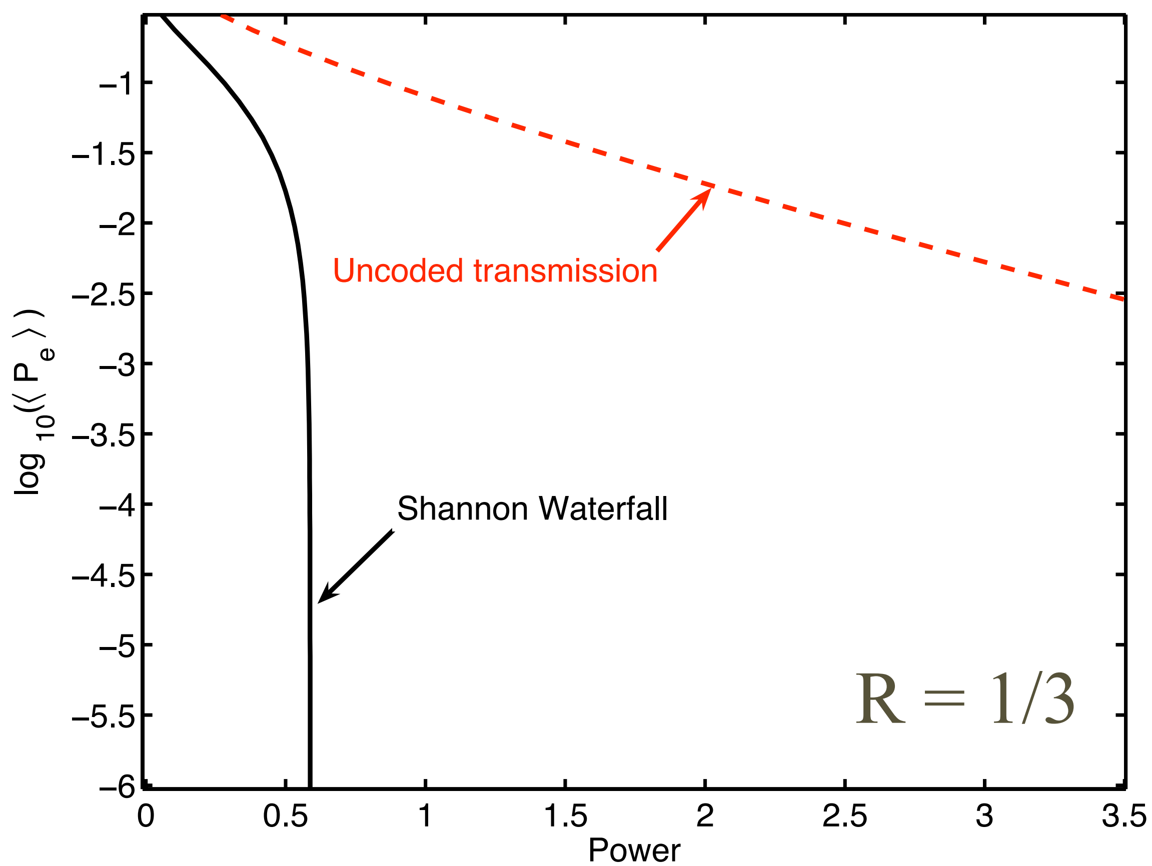


Fixed message size : **Verdu** "On channel capacity per unit cost"

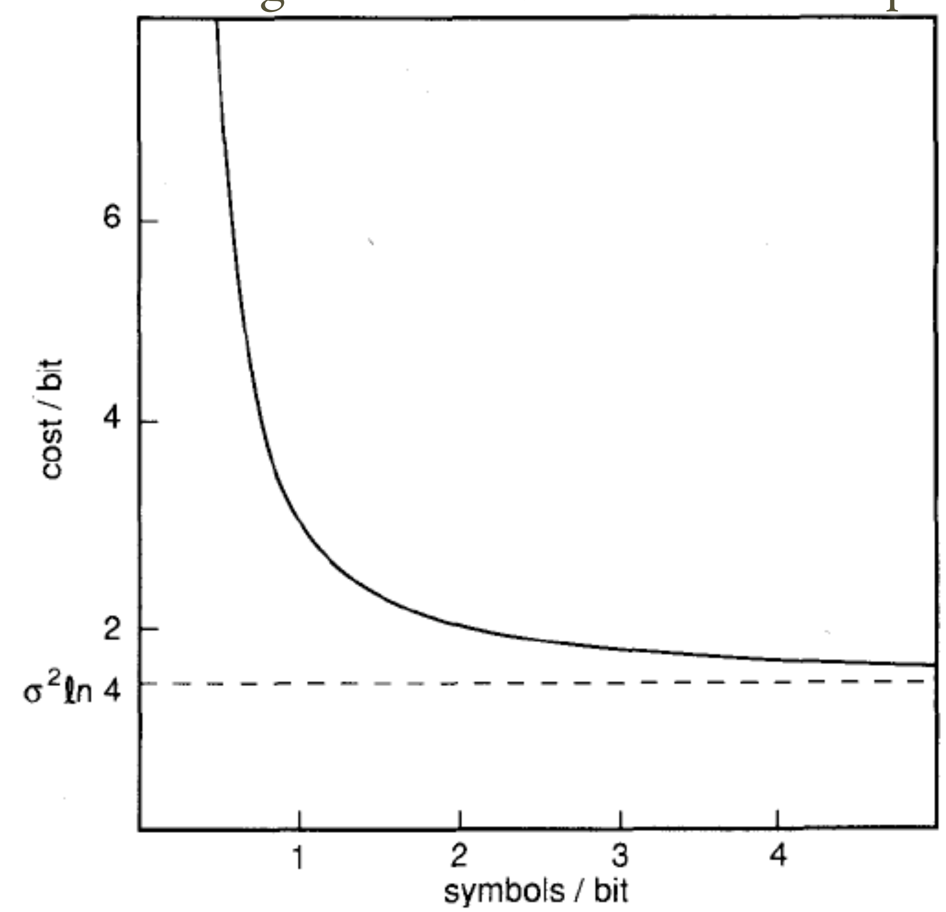


Promise of Shannon Theory

Fixed Rate: Shannon waterfall



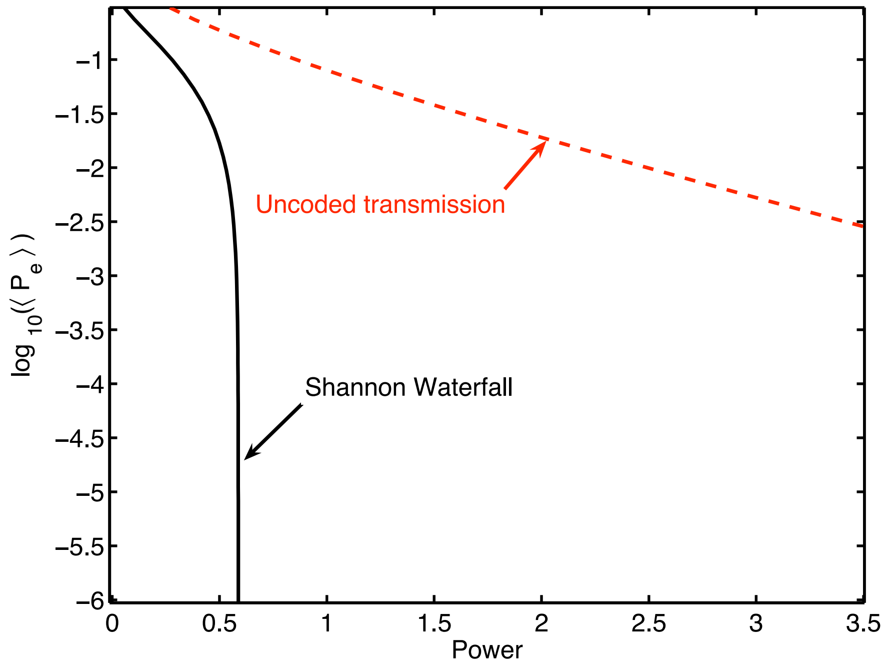
Fixed message size : **Verdu** “On channel capacity per unit cost”



- Long distance communication
  - processing power $\ll$ transmit power -- Shannon theory works!
- Short distance communication
  - Processing power can be substantial [Agarwal 98, Kravertz et al ’98, Goldsmith et al ’02, Cui et al ’05]

Information theory + processing power = ?

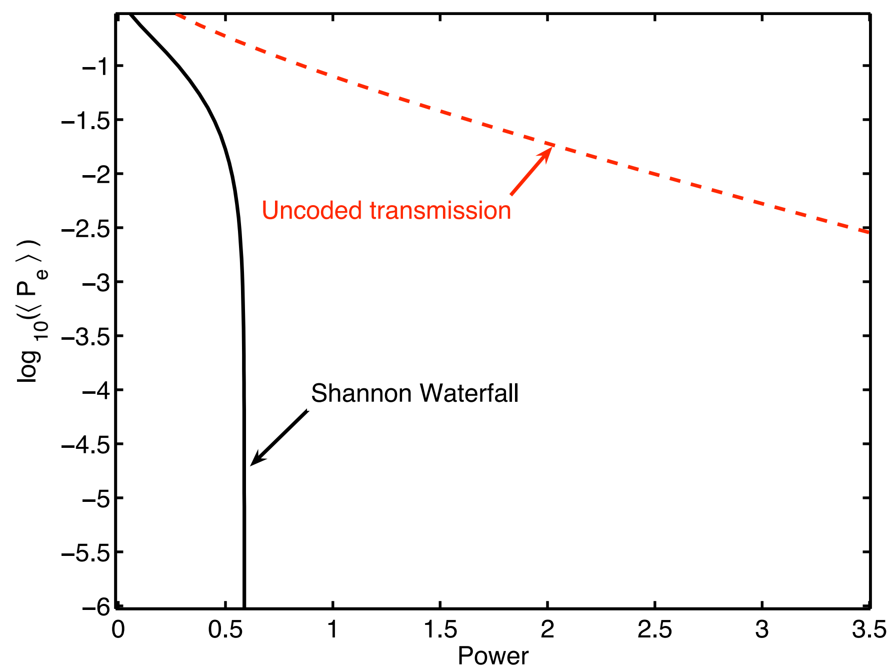
Fixed rate



processor with heat sink

Information theory + processing power = ?

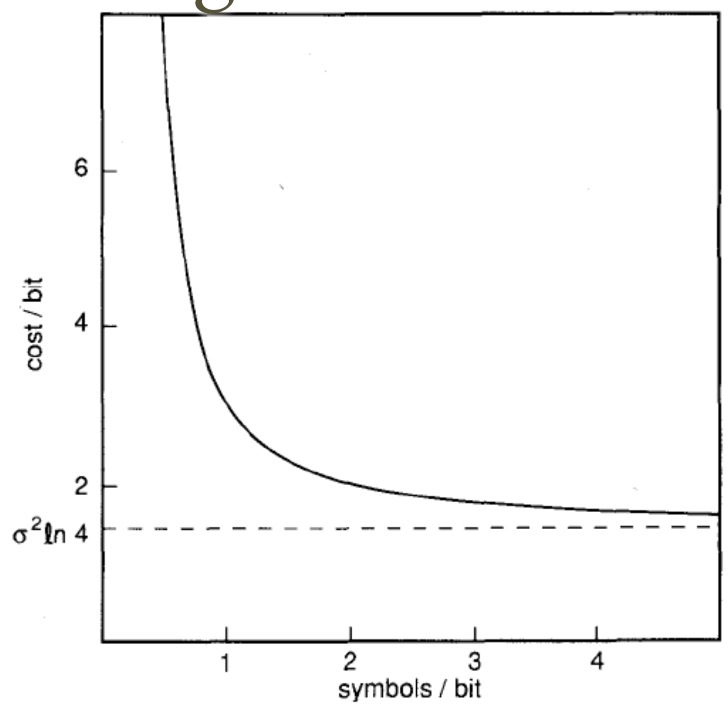
Fixed rate

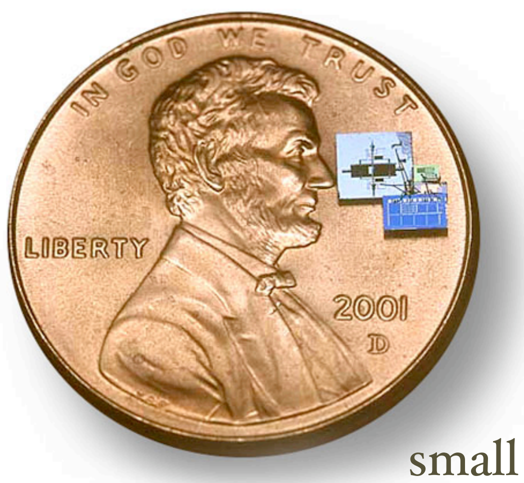


+  = ?

processor with heat sink

Fixed message size



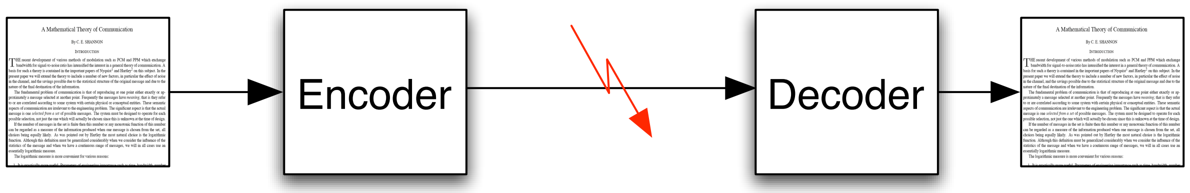
+  = ?

small sensors

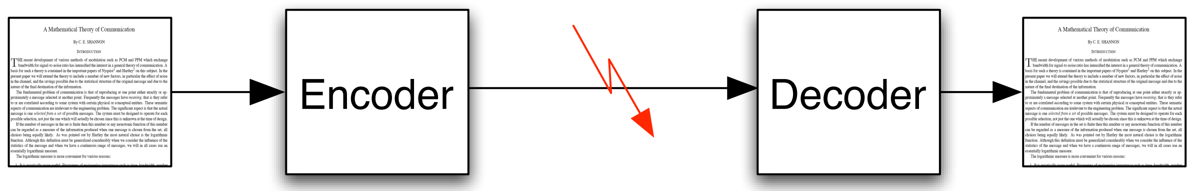
Talk Outline

- Motivation: Power consumption
  - Fixed rate and fixed message size problems.
- **Decoding power using decoding complexity.**
- Complexity-performance tradeoffs.
  - our bounds for iterative decoding.
- Fixed rate -- lower bounds on total power.
- Fixed message size (Green codes) -- lower bounds on min energy.
- How tight are our bounds : Related coding-theoretic literature

Modeling processing power through decoding complexity

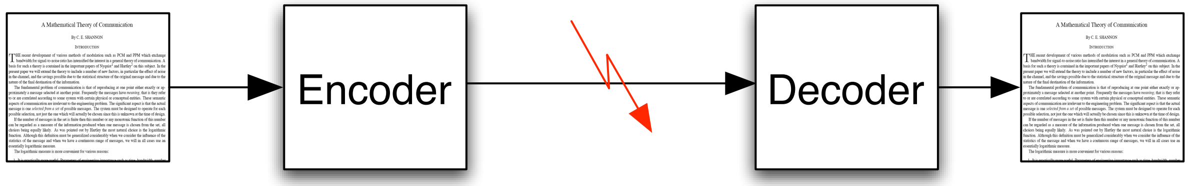


Modeling processing power through decoding complexity



- power consumed in **decoding**: model using the decoding complexity
  - decoding complexity : **number of operations** performed at the decoder
  - **constant amount of energy** per operation.

Modeling processing power through decoding complexity



- power consumed in **decoding**: model using the decoding complexity
  - decoding complexity : **number of operations** performed at the decoder
  - **constant amount of energy** per operation**.**
- **the common currency:** power

Talk Outline

- Motivation: Power consumption
  - Fixed rate and fixed message size problems.
- Decoding power using decoding complexity.
- **Complexity-performance tradeoffs.**
  - **our bounds for iterative decoding.**
- Fixed rate -- lower bounds on total power.
- Fixed message size (Green codes) -- lower bounds on min energy.
- How tight are our bounds : Related coding-theoretic literature

Understanding decoding complexity : complexity - performance tradeoffs

- complexity-performance tradeoffs :
  - Required complexity to attain error probability P_e and rate R.
  - Lower bounds : **Abstract away from details of code structure**.
  - Upper bounds : code constructions.
- e.g. block codes :

$$P_e \approx \exp(-mE_r(R))$$

- e.g. convolution codes :
  - error exponents with constraint length [Viterbi 67]
  - cut-off rate for sequential decoding [Jacobs and Berlekamp 67]

Understanding decoding complexity : complexity - performance tradeoffs

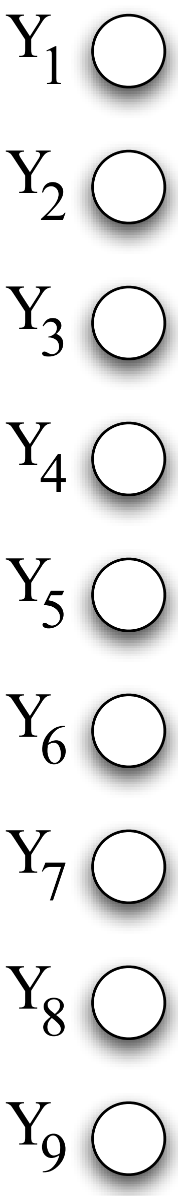
- complexity-performance tradeoffs :
  - Required complexity to attain error probability P_e and rate R.
  - Lower bounds : **Abstract away from details of code structure**.
  - Upper bounds : code constructions.
- e.g. block codes :

$$P_e \approx \exp(-mE_r(R))$$

- e.g. convolution codes :
  - error exponents with constraint length [Viterbi 67]
  - cut-off rate for sequential decoding [Jacobs and Berlekamp 67]
- Want a similar analysis for iterative decoding.

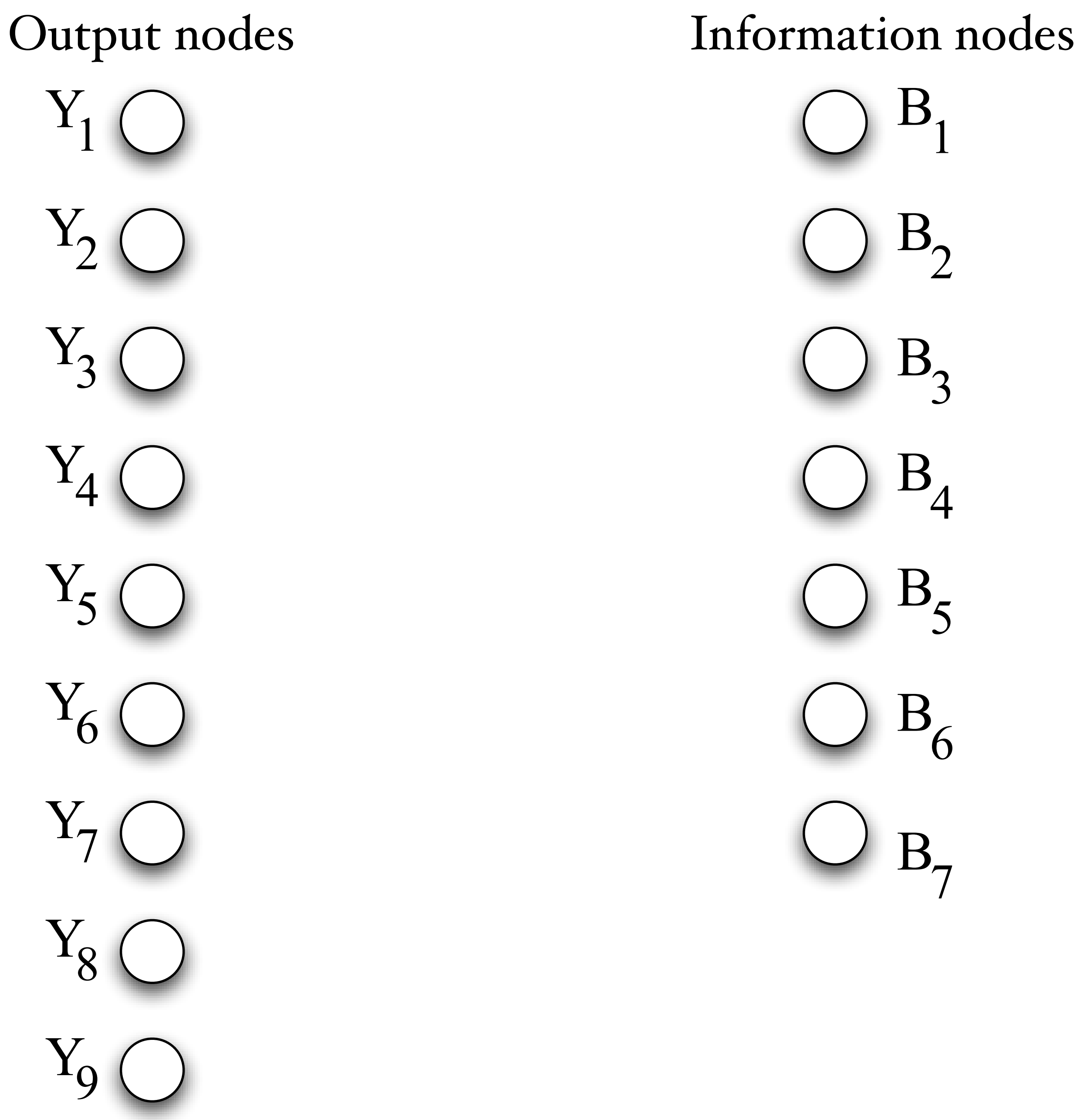
Iterative decoding : Decoding by passing messages

Output nodes



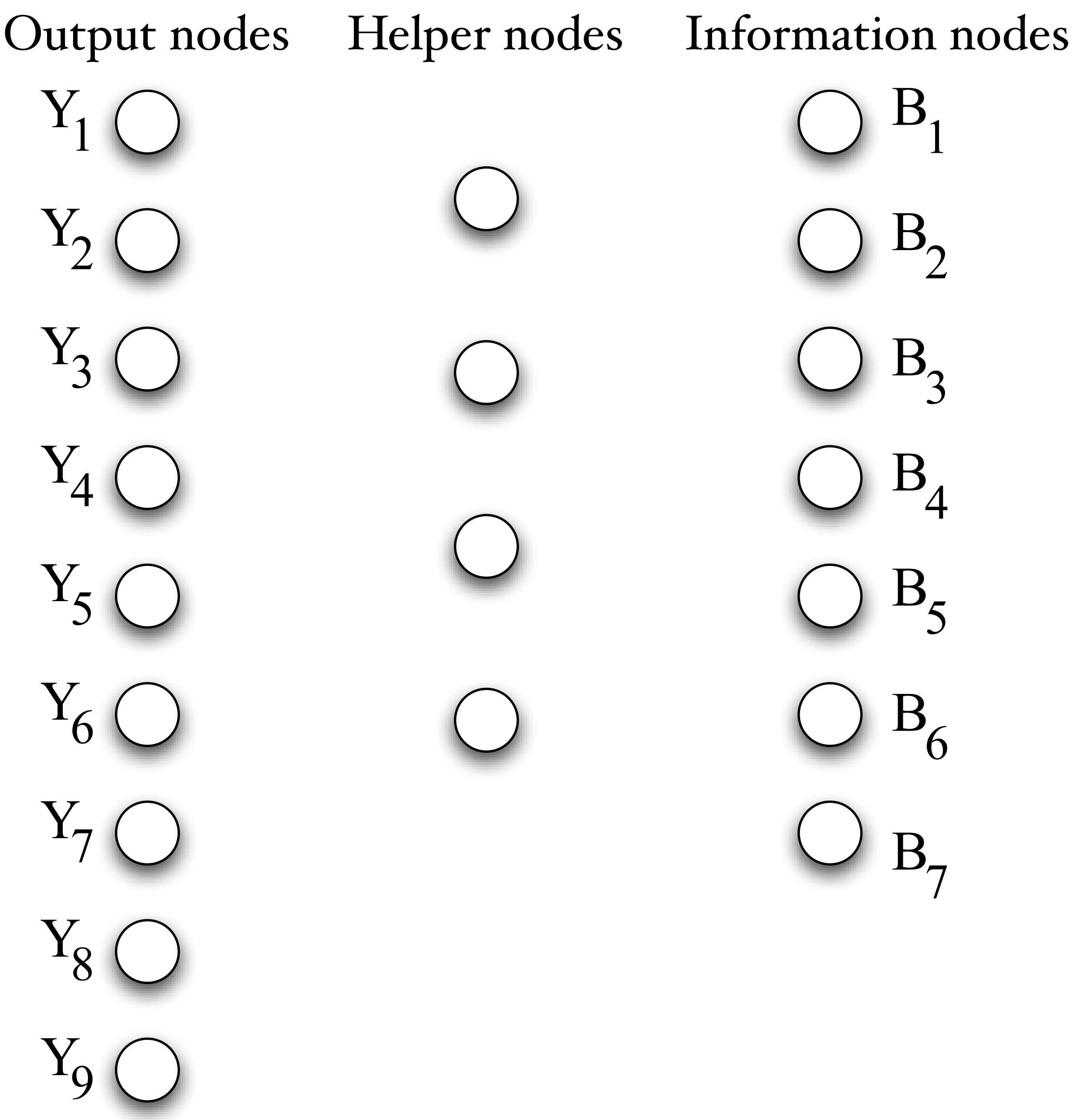
Decoder implementation graph

Iterative decoding : Decoding by passing messages



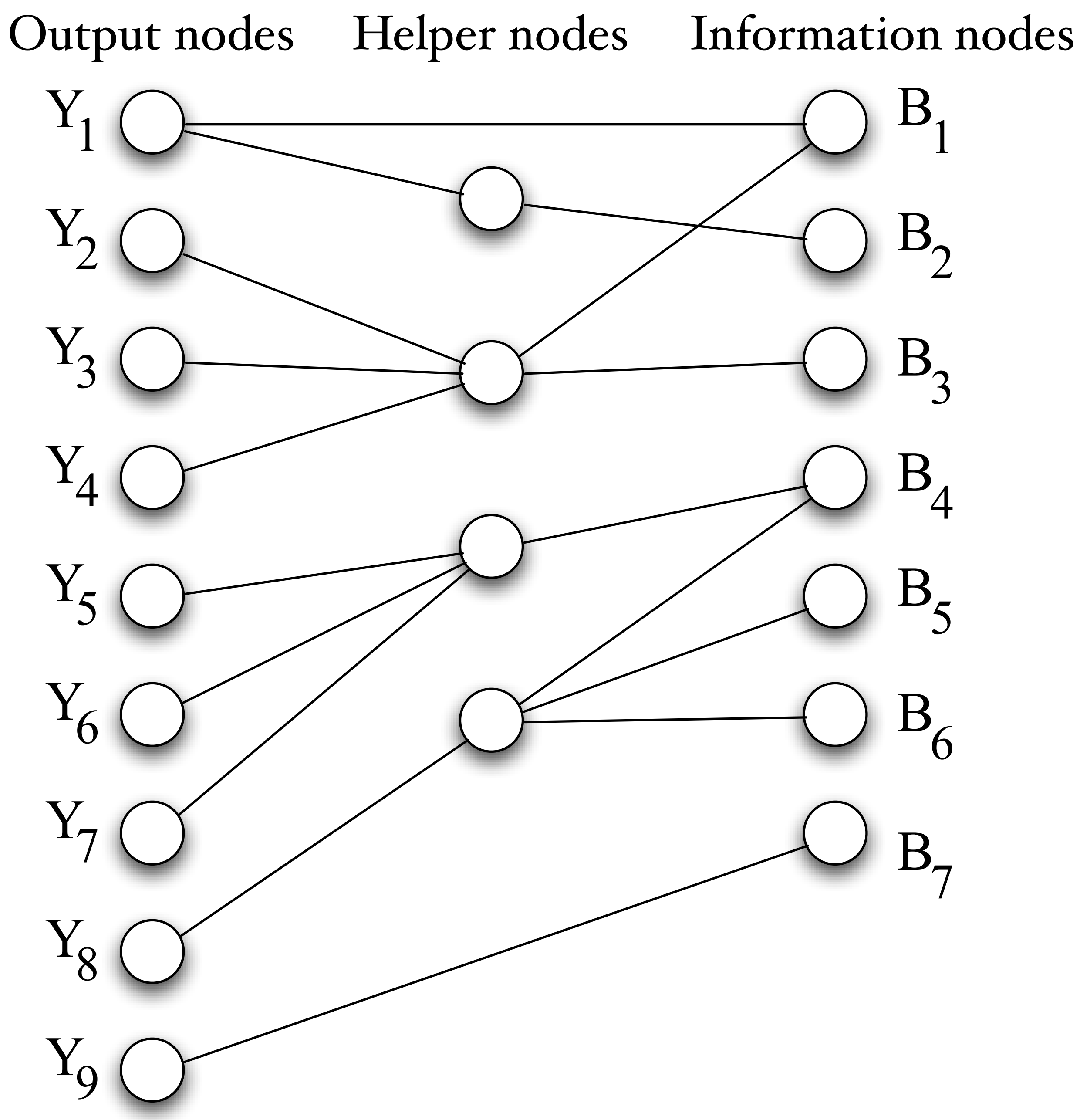
Decoder implementation graph

Iterative decoding : Decoding by passing messages



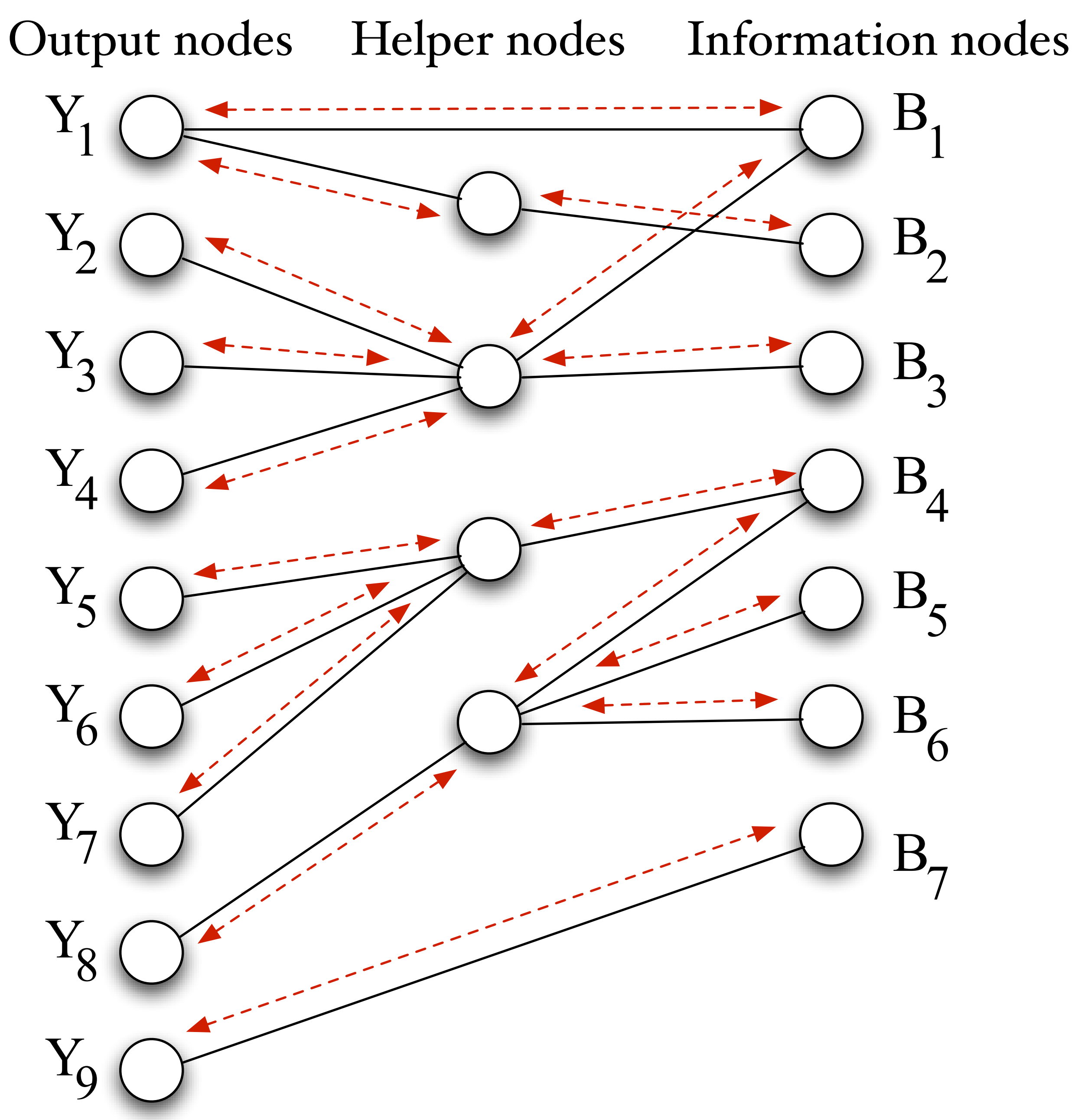
Decoder implementation graph

Iterative decoding : Decoding by passing messages



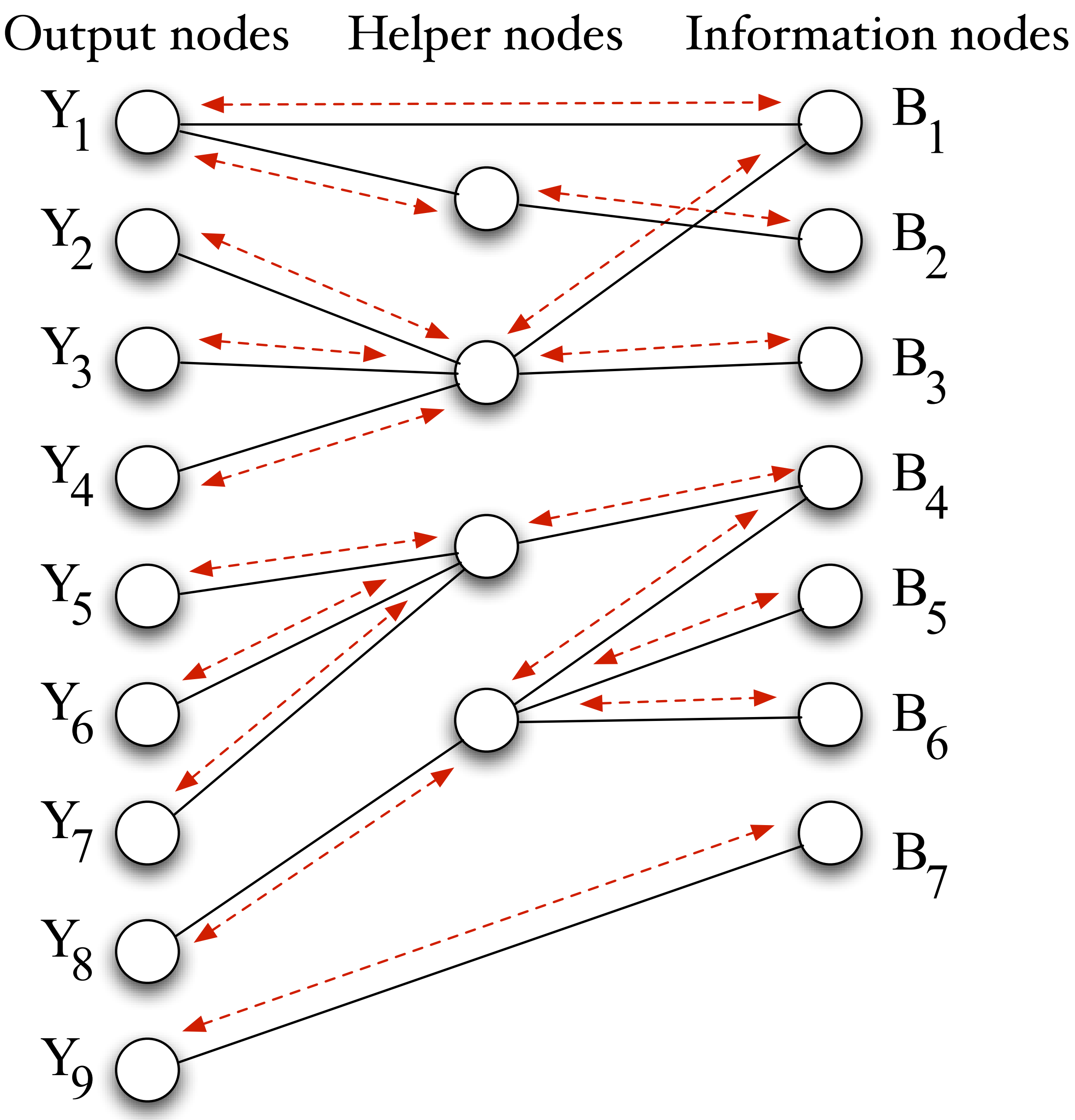
Decoder implementation graph

Iterative decoding : Decoding by passing messages



Decoder implementation graph

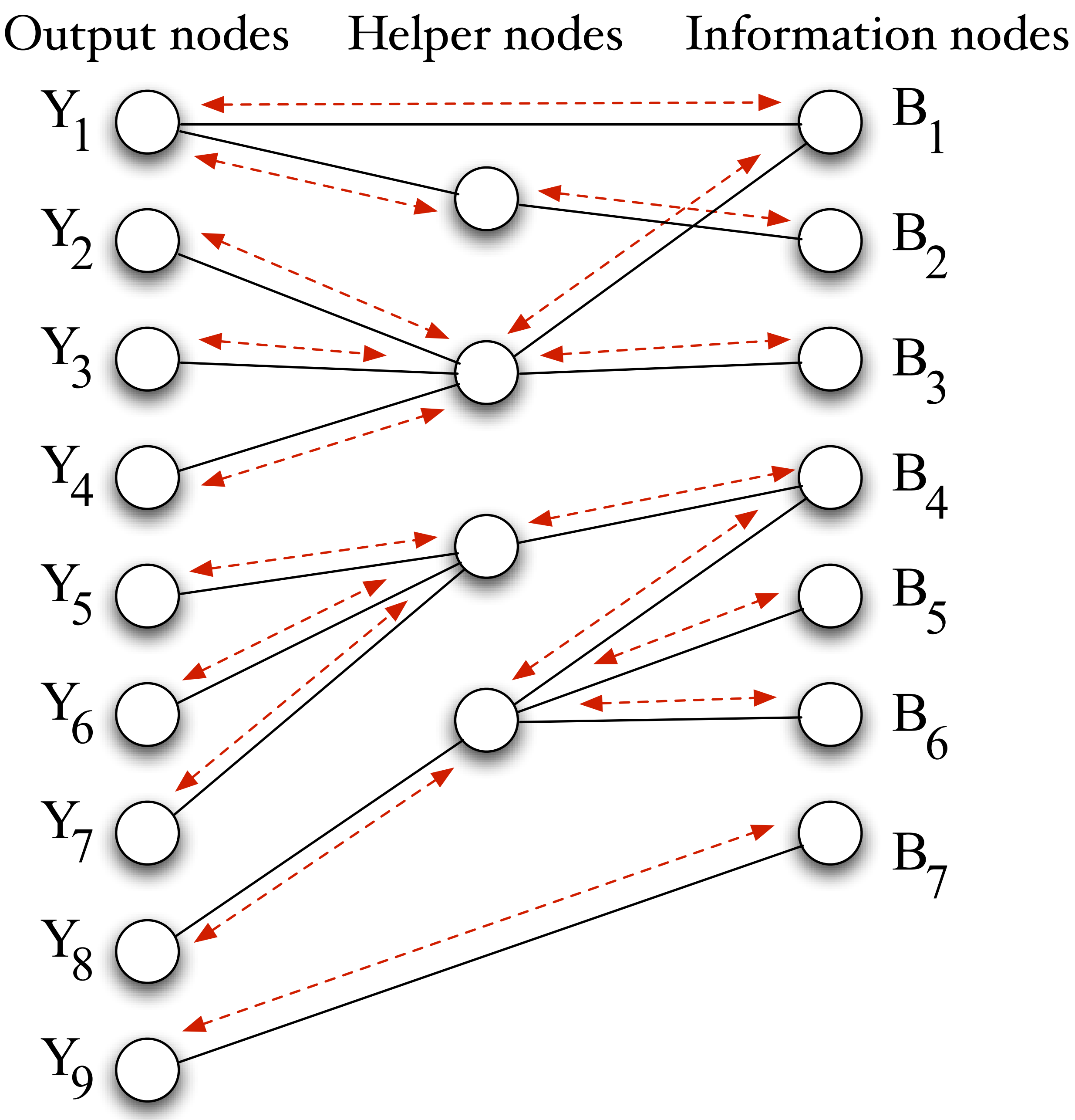
Iterative decoding : Decoding by passing messages



Decoder implementation graph

- Each node consumes γ Joules of energy per iteration.
- After l iterations, the energy consumed is $\gamma \times l \times \#\ of\ nodes$
- Each node is connected to at most α other nodes -- an implementation constraint.

Iterative decoding : Decoding by passing messages

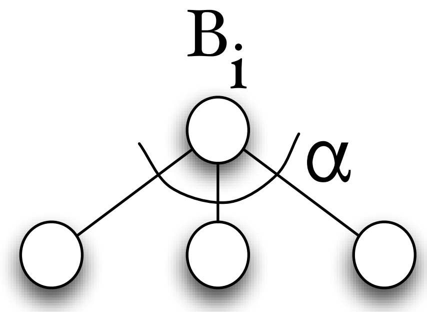


Decoder implementation graph

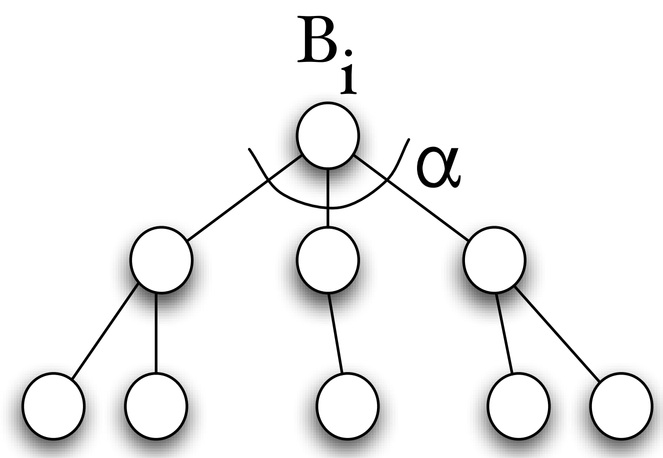
- Each node consumes γ Joules of energy per iteration.
- After l iterations, the energy consumed is $\gamma \times l \times \#\ of\ nodes$
- Each node is connected to at most α other nodes -- an implementation constraint.

Suffices now to find l

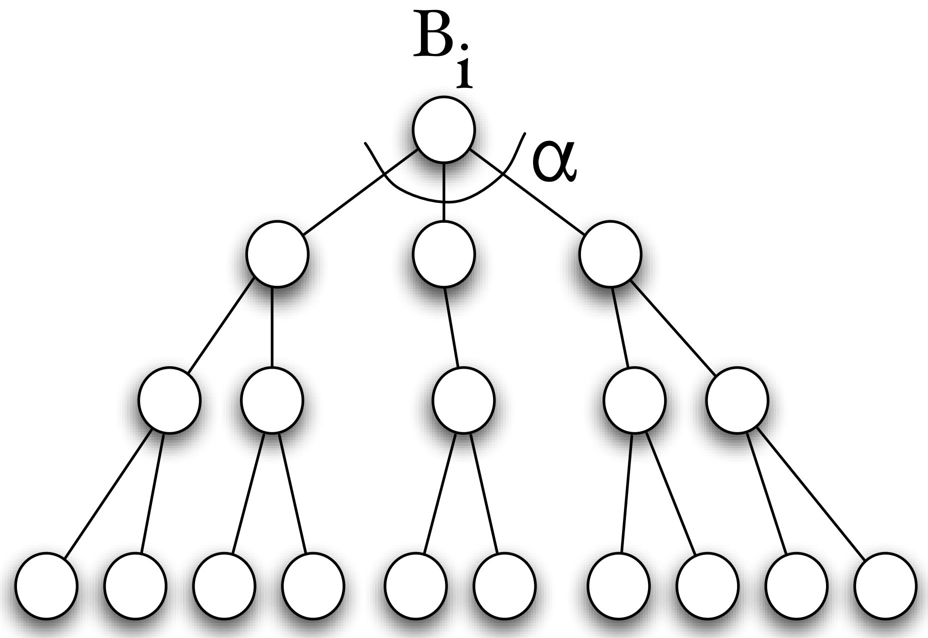
Lower bound on *l* : Key Idea



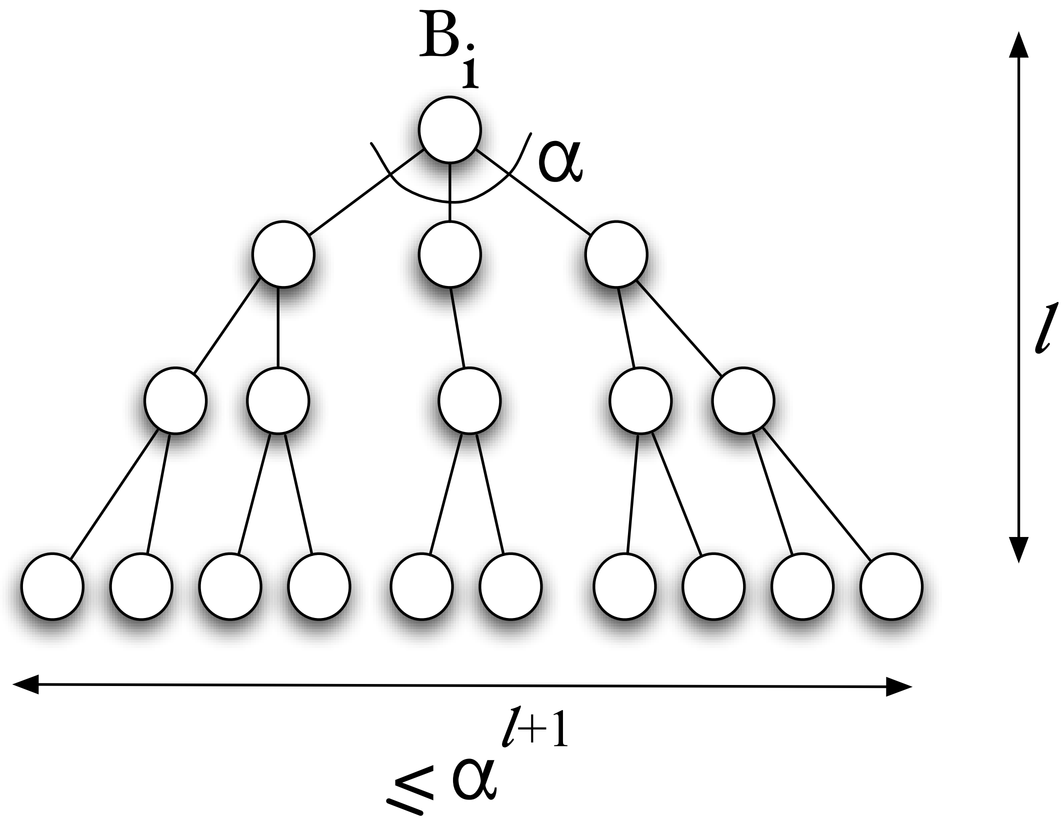
Lower bound on *l* : Key Idea



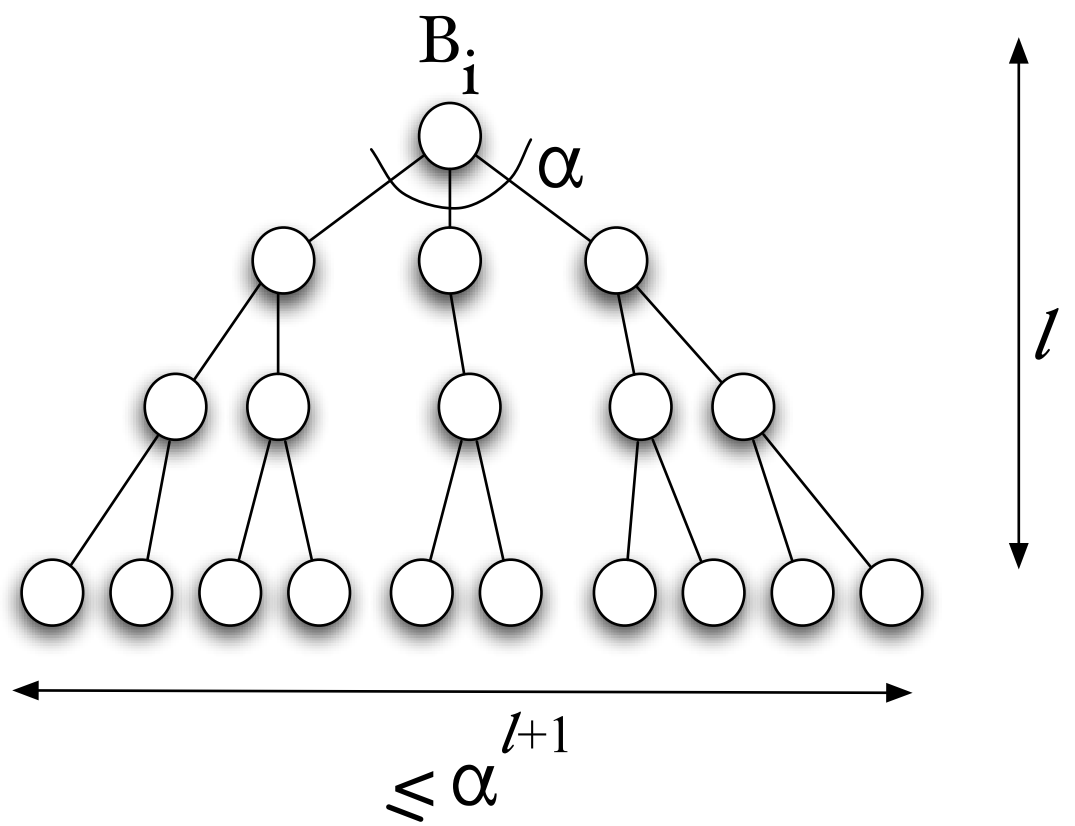
Lower bound on *l* : Key Idea



Lower bound on *l* : Key Idea



Lower bound on l : Key Idea



Channel needs to **behave atypically** only in the decoding neighborhood to cause an error

Lower bound on decoding complexity

Result [Sahai, Grover, *Submitted to IT Trans. 07*]

In the limit of small P_e

$$l \gtrsim \frac{1}{\log(\alpha)} \log\left(\frac{\log \frac{1}{P_e}}{(C-R)^2}\right)$$

- C = Channel capacity
- R = Rate
- P_e = error probability
- α = maximum node degree

Lower bound on decoding complexity

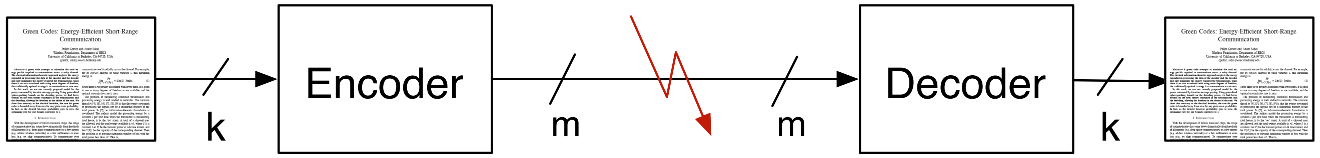
$$l \gtrsim \frac{1}{\log(\alpha)} \log\left(\frac{\log\frac{1}{P_e}}{(C-R)^2}\right)$$

- A general lower bound
  - applies to **all** (possible) codes with decoding based on passing messages.
  - applies regardless of the **presence of cycles**.
  - applies to **all decoding algorithms** based on passing messages.

Talk Outline

- Motivation: Power consumption
  - Fixed rate and fixed message size problems.
- Decoding power using decoding complexity.
- Complexity-performance tradeoffs.
  - our bounds for iterative decoding.
- **Fixed rate -- lower bounds on total power.**
- Fixed message size (Green codes) -- lower bounds on min energy.
- How tight are our bounds : Related coding-theoretic literature

Fixed Rate: Total power consumption



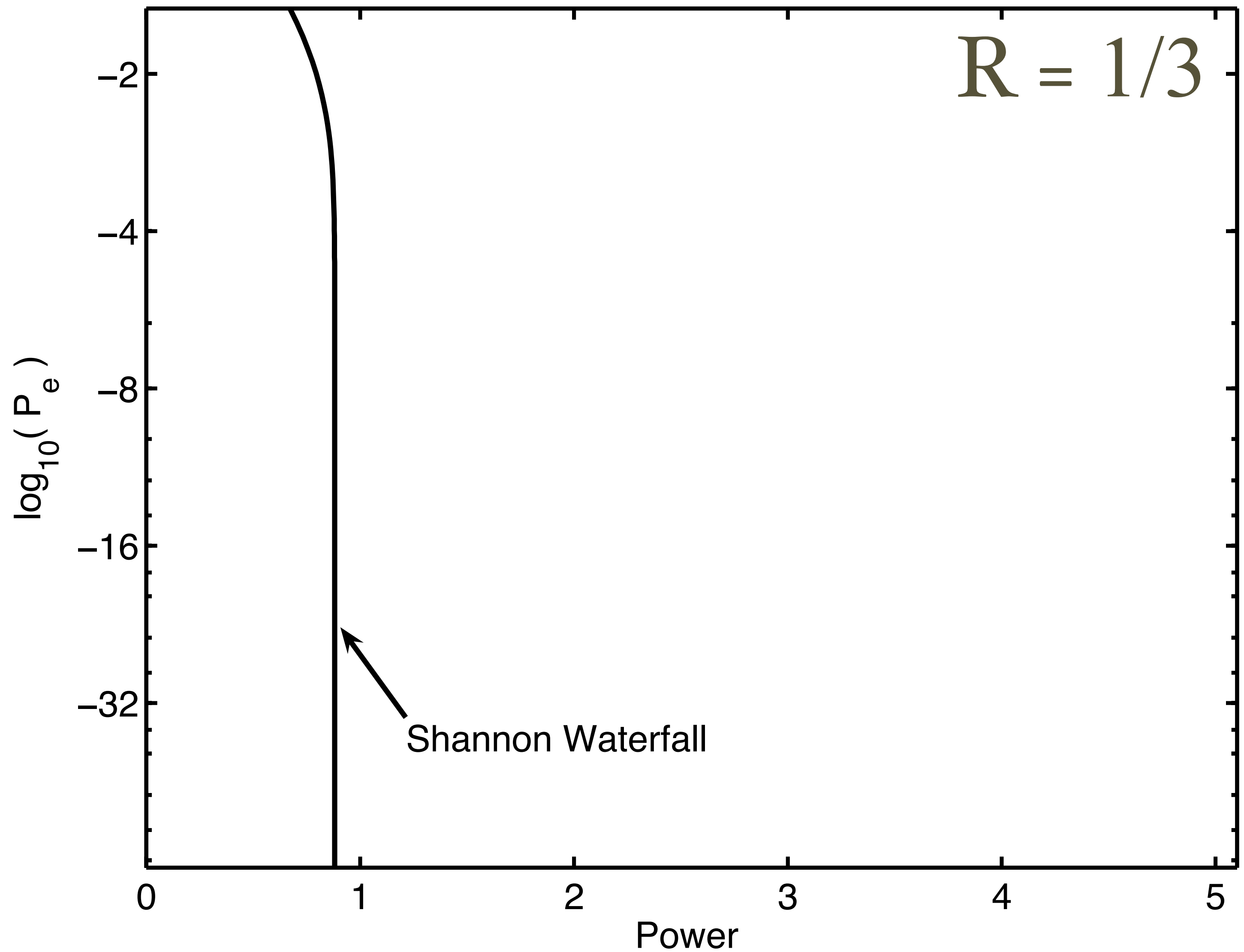
$$P_{\text{total}} = P_T + \gamma \times l \times \frac{\#\ of\ nodes}{m}$$
$$\geq P_T + \gamma \times l$$
$$\geq P_T + \frac{\gamma}{\log(\alpha)} \log\left(\frac{\log \frac{1}{P_e}}{(C(P_T) - R)^2}\right)$$

Minimize P_{total} by optimizing over P_T

- l = Number of iterations
- γ = Energy consumed per node per iteration
- P_T = Transmit power
- m = block-length

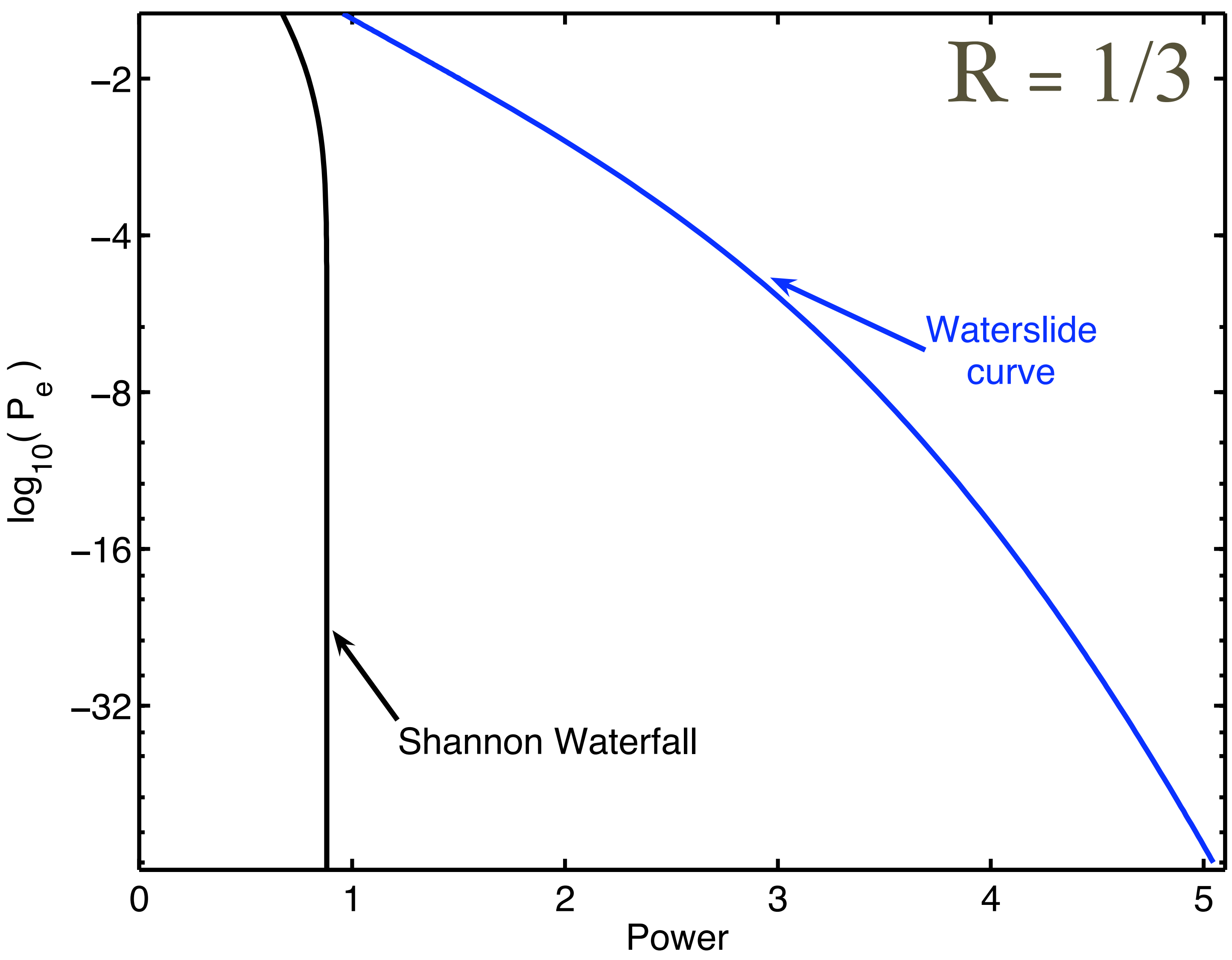
Fixed Rate: Total Power Curves

$$P_{\text{total}} \geq P_T + \frac{\gamma}{\log(\alpha)} \log\left(\frac{\log\frac{1}{P_e}}{(C(P_T) - R)^2}\right)$$



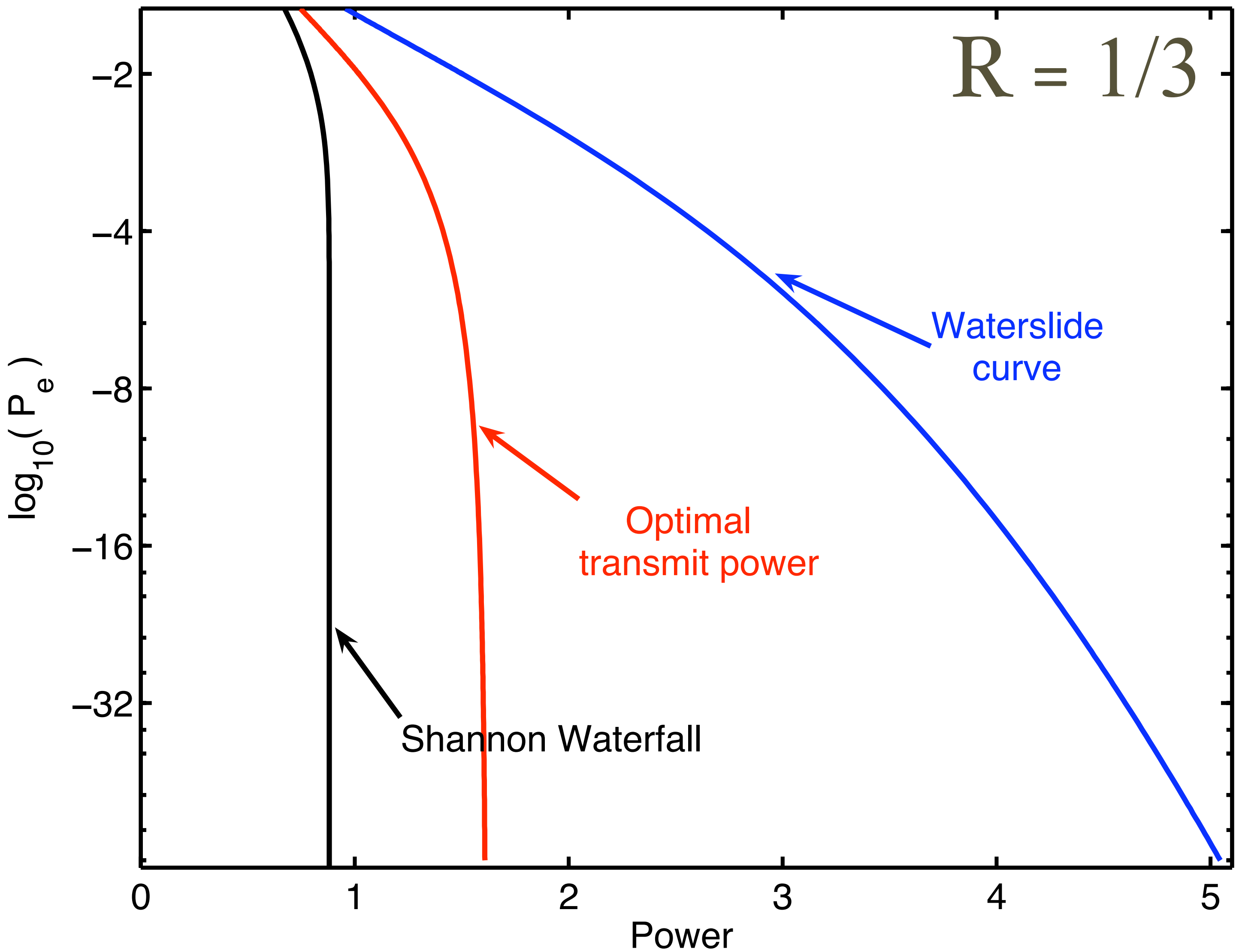
Fixed Rate: Total Power Curves

$$P_{\text{total}} \geq P_T + \frac{\gamma}{\log(\alpha)} \log\left(\frac{\log \frac{1}{P_e}}{(C(P_T) - R)^2}\right)$$



Fixed Rate:
Total Power Curves

$$P_{\text{total}} \geq P_T + \frac{\gamma}{\log(\alpha)} \log\left(\frac{\log\frac{1}{P_e}}{(C(P_T) - R)^2}\right)$$



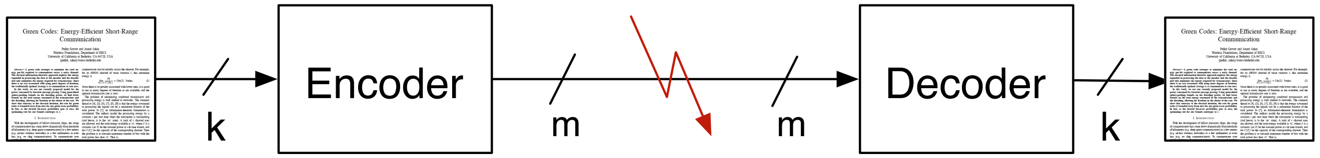
Fixed Rate: Summary

- Total power **increases unboundedly** as $P_e \rightarrow 0$
- Optimal transmit power **strictly larger than the Shannon limit** (transmit power - decoding power tradeoff)

Talk Outline

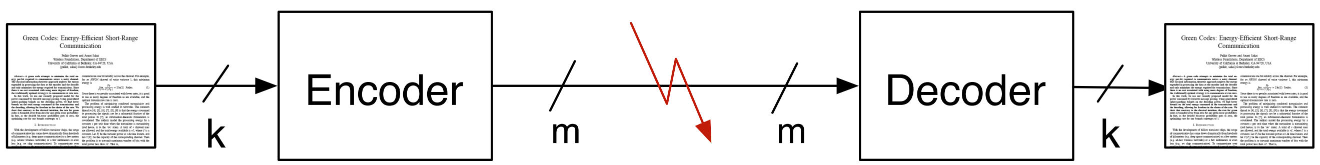
- Motivation: Power consumption
  - Fixed rate and fixed message size problems.
- Decoding power using decoding complexity.
- Complexity-performance tradeoffs.
  - our bounds for iterative decoding.
- Fixed rate -- lower bounds on total power.
- **Fixed message size (Green codes) -- lower bounds on min energy.**
- How tight are our bounds : Related coding-theoretic literature

Fixed message size : Green Codes Minimum energy per-bit



$$\begin{aligned} E_{\text{total}} &= mP_{\text{total}} \\ &= m\ P_T + \gamma \times l \times \#\ of\ nodes \end{aligned}$$

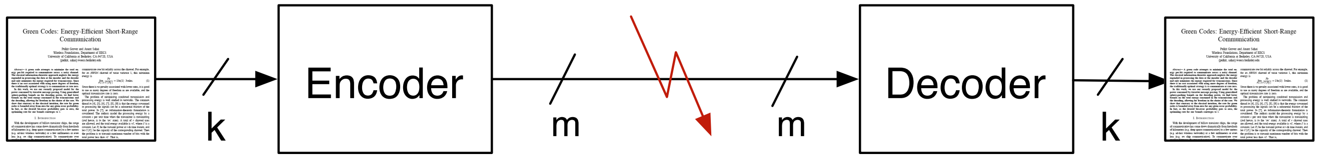
Fixed message size : Green Codes Minimum energy per-bit



$$\begin{aligned} E_{\text{total}} &= m P_{\text{total}} \\ &= m\ P_T + \gamma \times l \times \#\ of\ nodes \end{aligned}$$

$$\begin{aligned} E_{\text{per bit}} &= \frac{E_{\text{total}}}{k} \\ &= \frac{P_T}{R} + \gamma \times l \times \frac{\#\ of\ nodes}{k} \end{aligned}$$

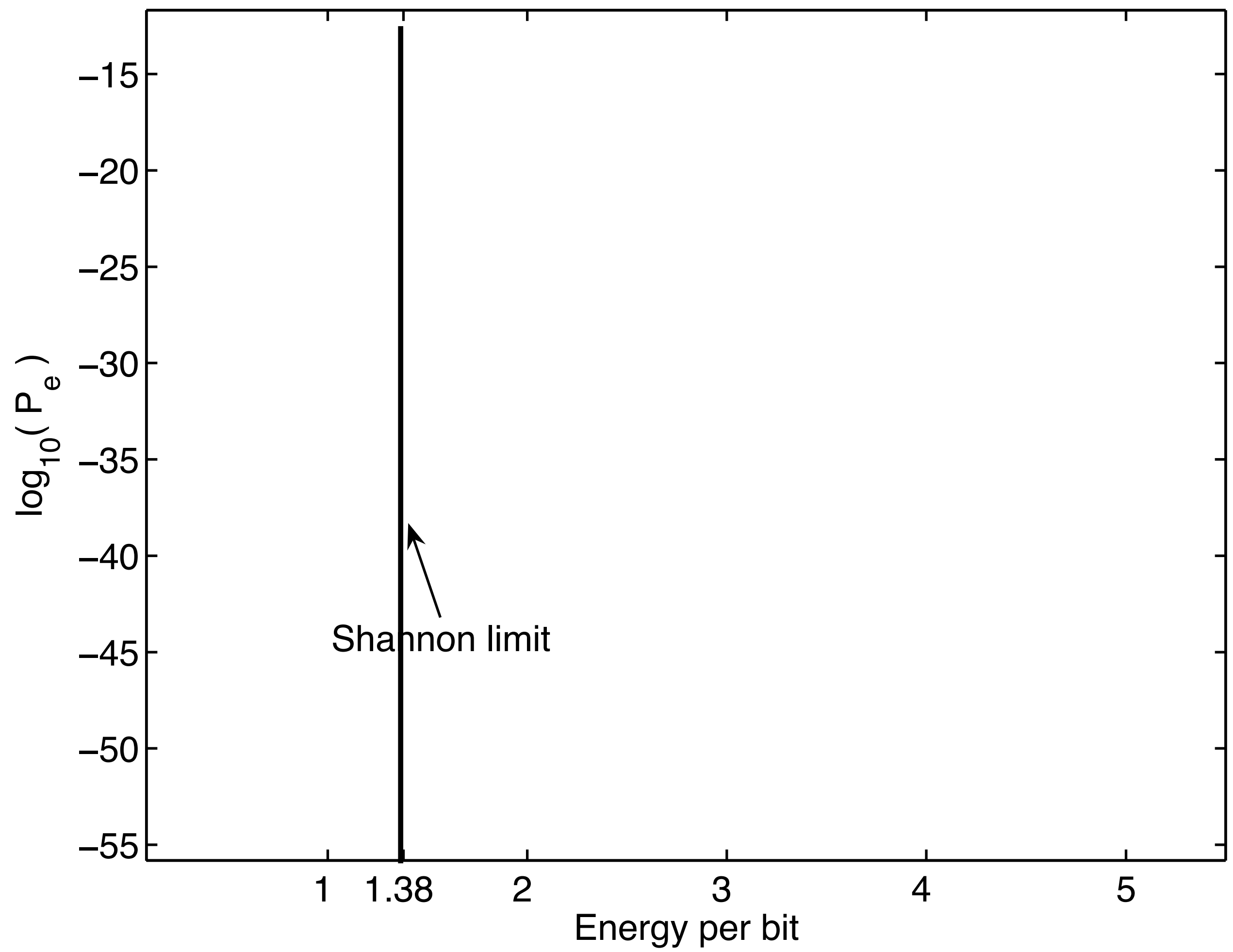
Fixed message size : Green Codes
Minimum energy per-bit



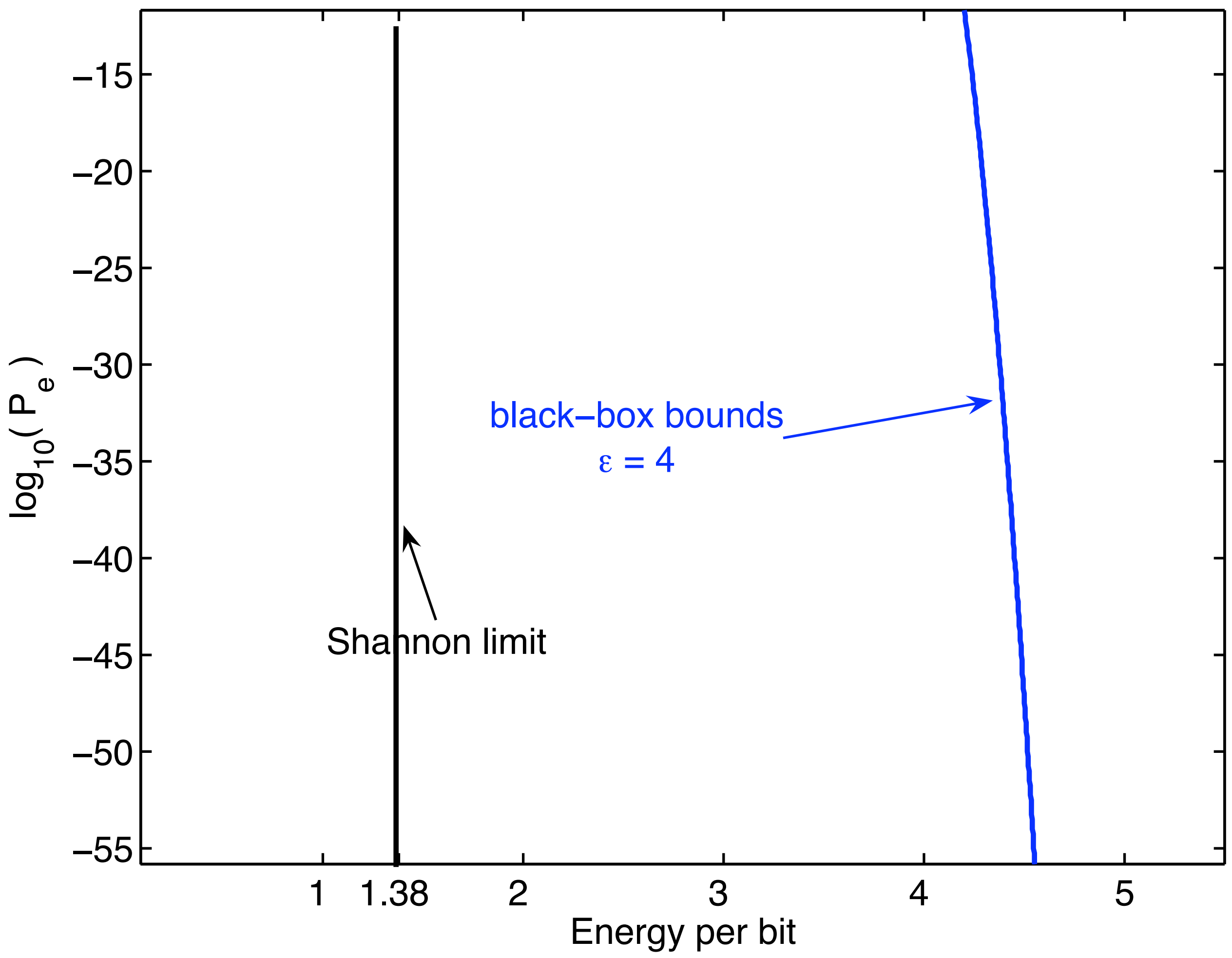
$$
\begin{aligned}
E_{\text{total}} &= mP_{\text{total}} \\
&= m\ P_T + \gamma \times l \times \#\ of\ nodes
\end{aligned}
$$

$$
\begin{aligned}
E_{\text{per bit}} &= \frac{E_{\text{total}}}{k} \\
&= \frac{P_T}{R} + \gamma \times l \times \frac{\#\ of\ nodes}{k} \\
&\geq \frac{P_T}{R} + \gamma \times l \times \frac{\max\{k, m\}}{k}
\end{aligned}
$$

Fixed message size: Minimum energy per bit curves

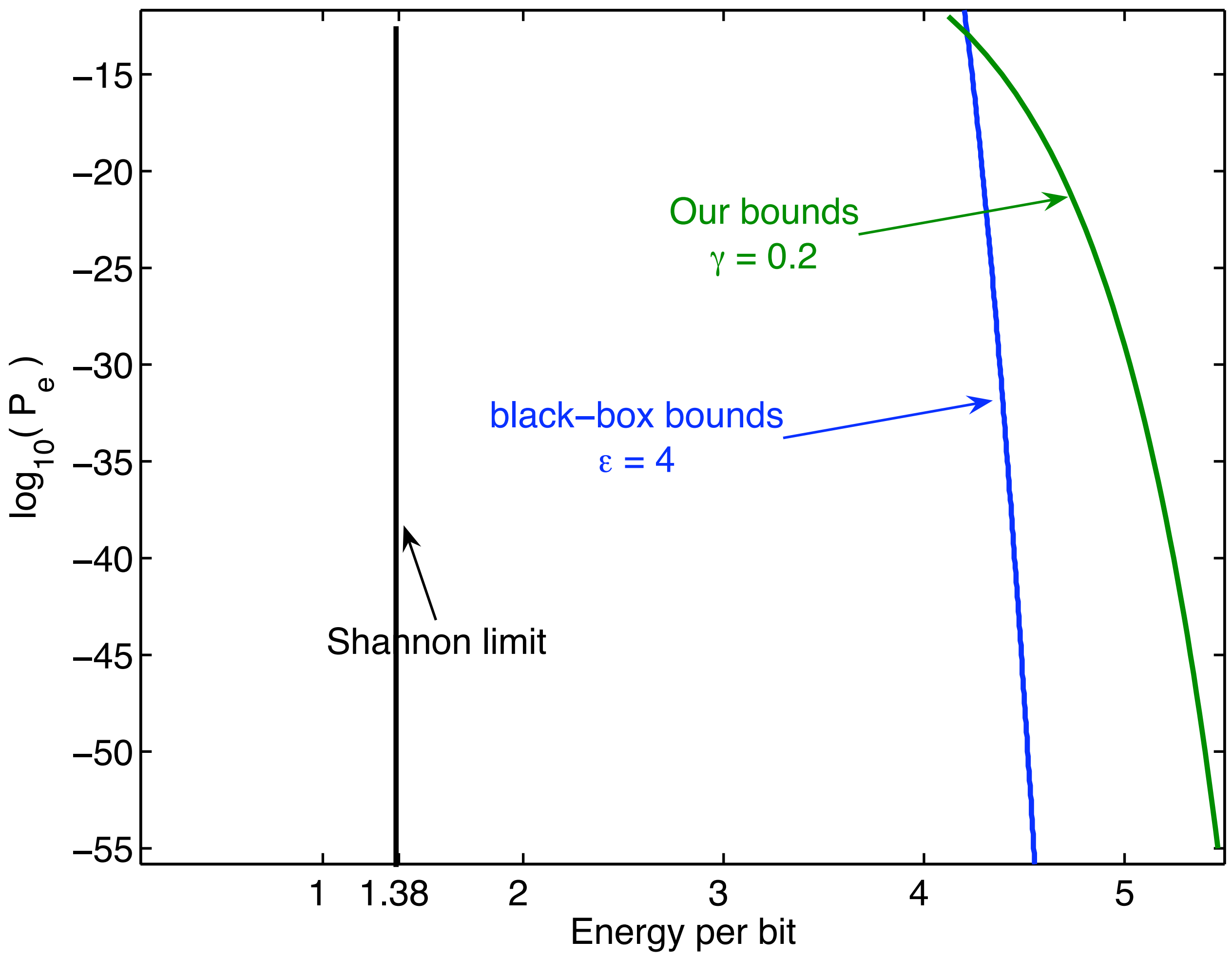


Fixed message size: Minimum energy per bit curves



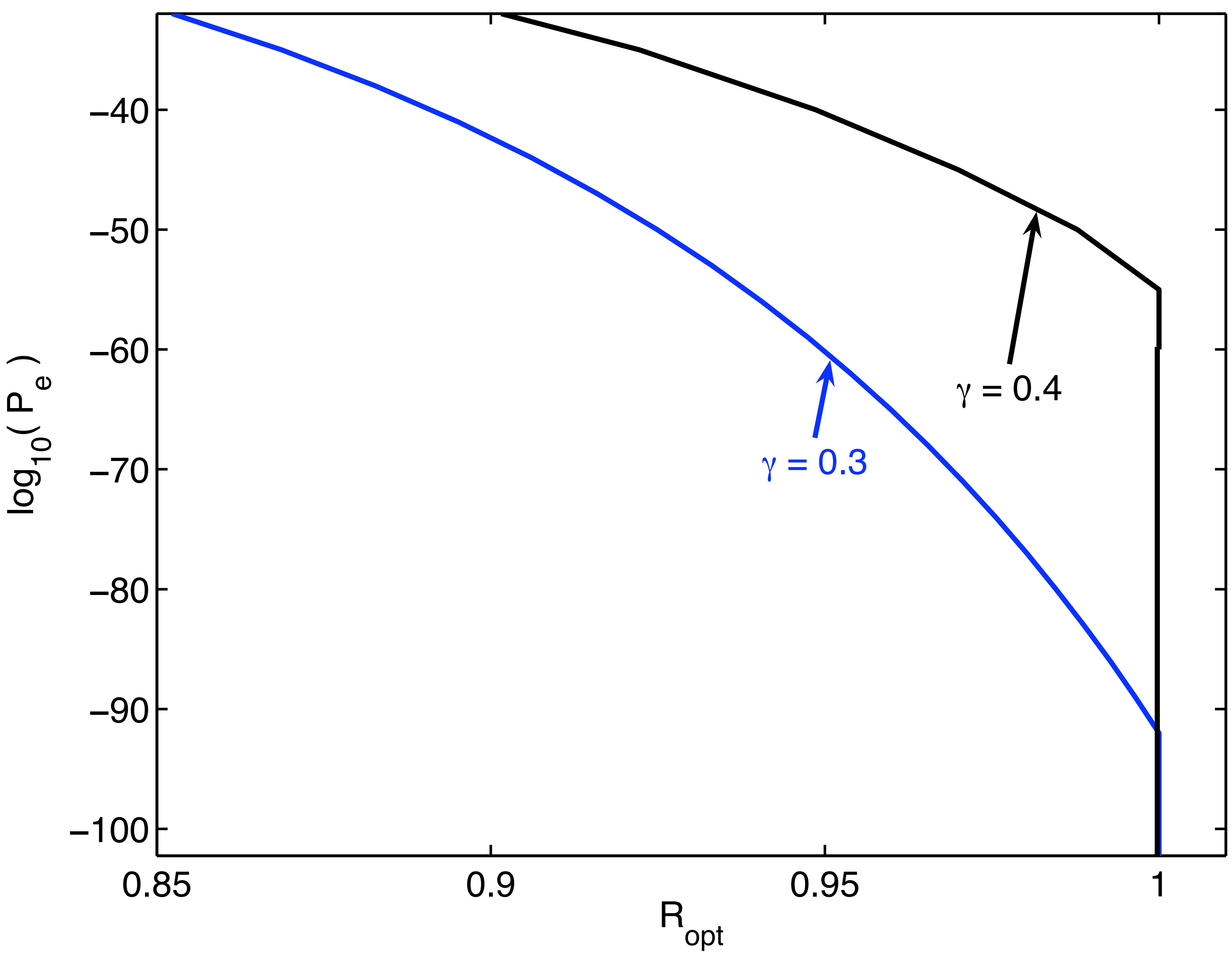
Black-box bounds : Based on [Massaad, Medard and Zheng]

Fixed message size:
Minimum energy per bit curves



Black-box bounds : Based on [Massaad, Medard and Zheng]

Fixed message size: Optimal rate curves



Fixed message size: Summary

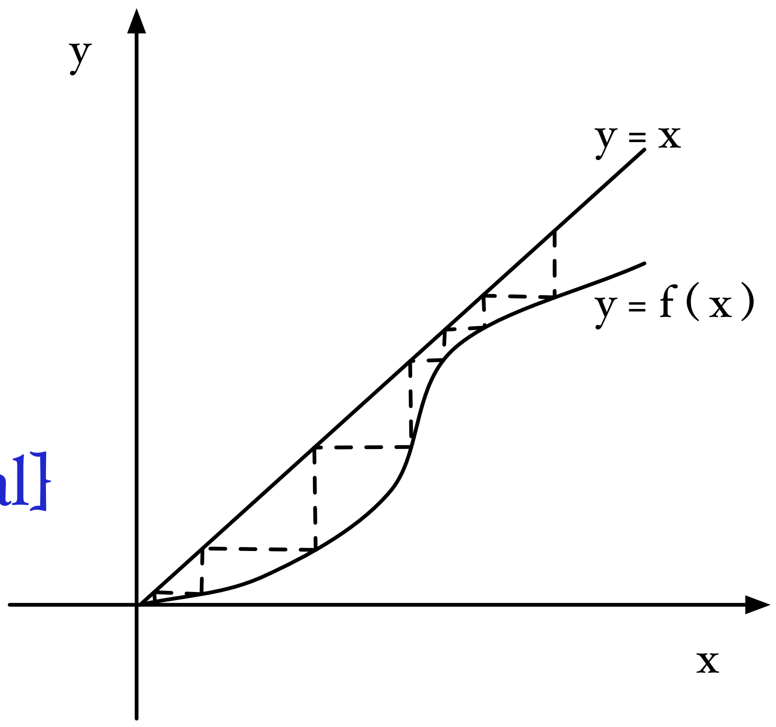
- Minimum energy per bit **increases to infinity** as $P_e \rightarrow 0$
  - compare with a constant, $\ln(4)$, in classical information theory.
- Optimizing rate **converges to 1**.
  - **zero** in classical information theory.

Talk Outline

- Motivation: Power consumption
  - Fixed rate and fixed message size problems.
- Decoding power using decoding complexity.
- Complexity-performance tradeoffs.
  - our bounds for iterative decoding.
- Fixed rate -- lower bounds on total power.
- Fixed message size (Green codes) -- lower bounds on min energy.
- **How tight are our bounds : Related coding-theoretic literature**

Lower bounds on complexity: how tight are they?

$$l \gtrsim \frac{1}{\log(\alpha)} \log\left(\frac{\log \frac{1}{P_e}}{(C-R)^2}\right)$$



- Optimal behavior with respect to P_e
  - regular LDPC's achieve this! [Lentmaier et al]
- what about behavior with $gap = C - R$?

Complexity behavior with $gap = C - R$

- [Gallager, Burshtein et al, Sason-Urbanke] Lower bounds on density for LDPCs.
- [Pfister-Sason, Hsu-Anastastopoulos] Upper bounds.
- Khandekar-McEliece conjecture: $l \geq \Omega\left(\frac{1}{C-R}\right)$
- [Sason, Weichman] For LDPCs, IRAs, ARAs, if there are a non-zero fraction of degree 2 nodes, and the graph is a tree, **the conjecture holds.**
  - but with degree-2 nodes, $l \approx \log\left(\frac{1}{P_e}\right)$
  - and it seems that degree-2 nodes are needed to approach capacity.
  - from energy perspective, is it worth approaching capacity?

Thank you

- Full paper on arxiv
  - ‘The price of certainty: “Waterslide curves” and the gap to capacity’. Anant Sahai and Pulkit Grover.