A vector version of Witsenhausen's counterexample : Towards convergence of control, communication and computation

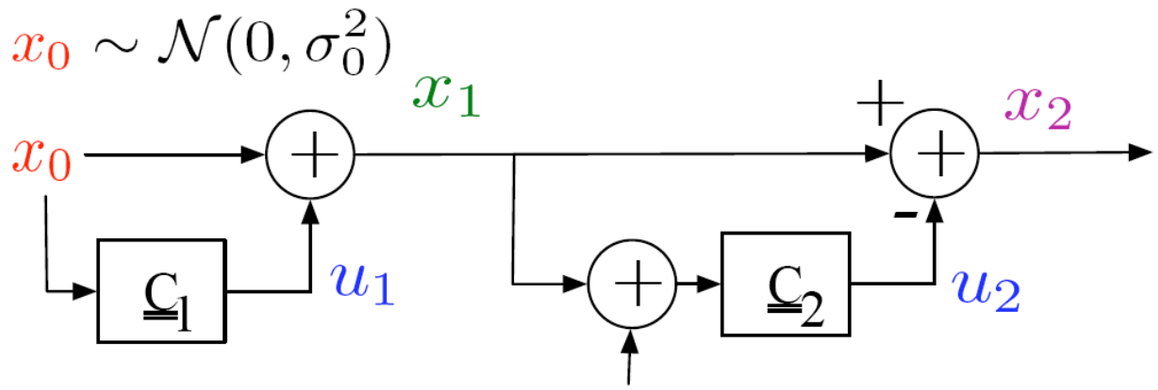
Pulkit Grover

UC Berkeley

Joint work with Prof. Anant Sahai

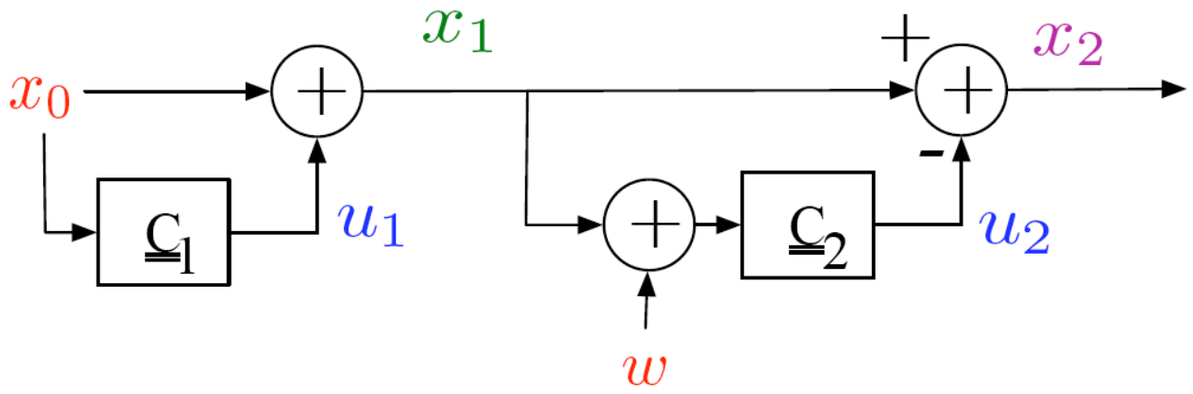
There are handouts for this talk, please take one!

Witsenhausen's counterexample



$$\mathcal{C} = k^2 {u_1}^2 + {x_2}^2$$

Quantization based signaling strategies



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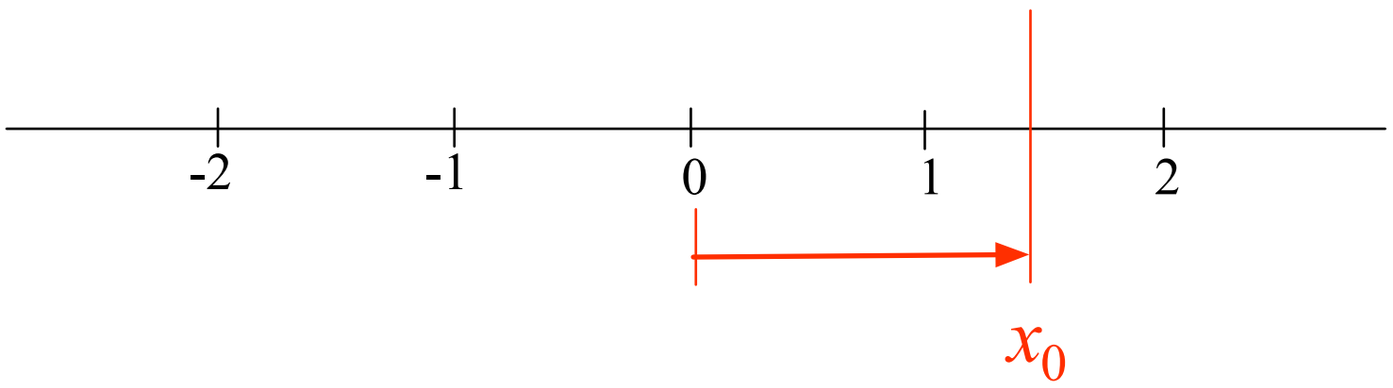
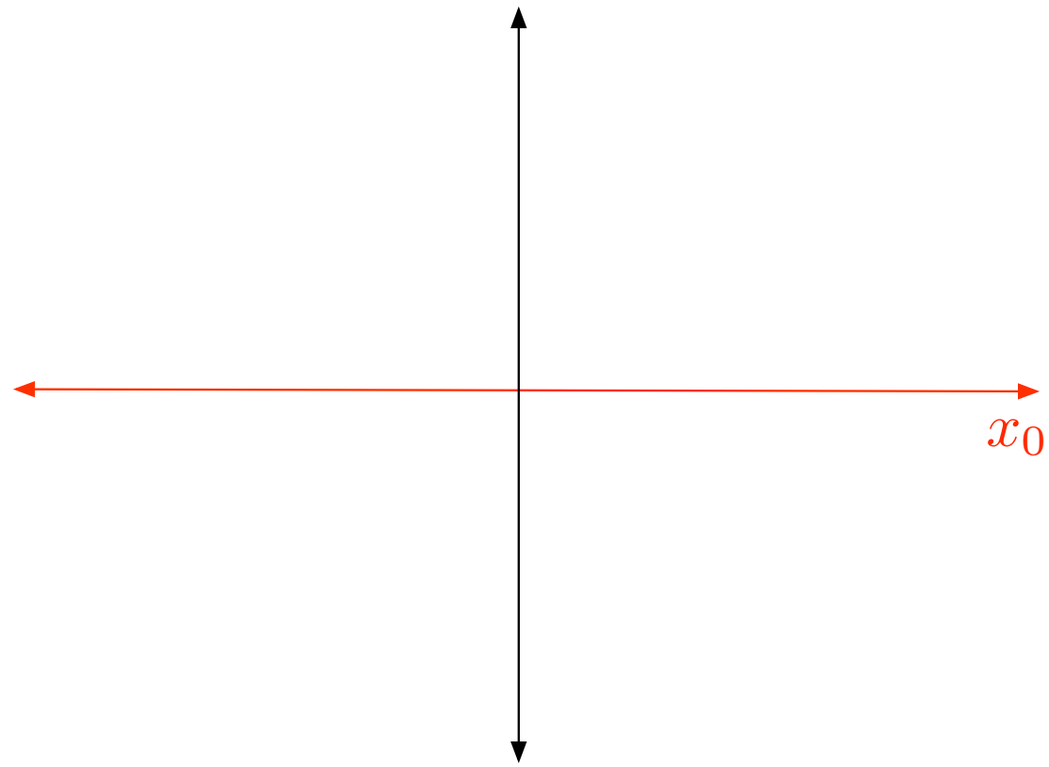
[Mitter and Sahai, '99]





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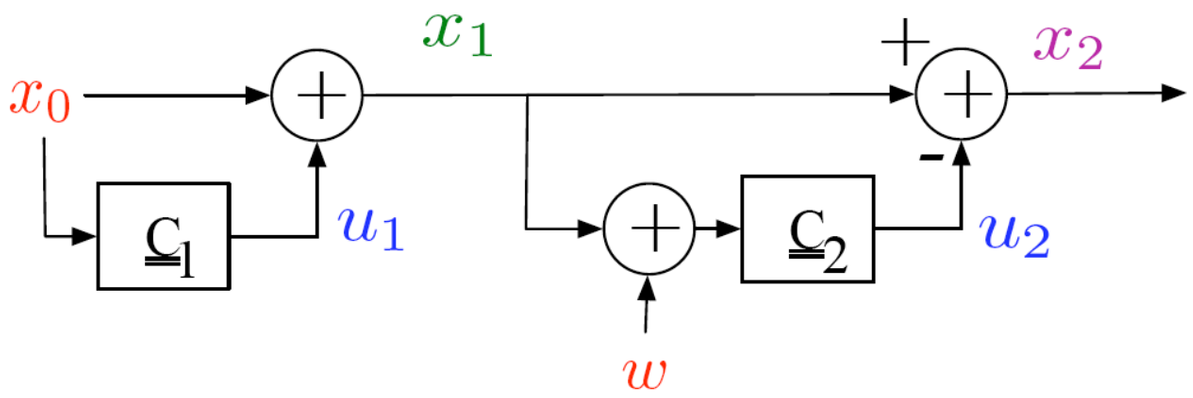

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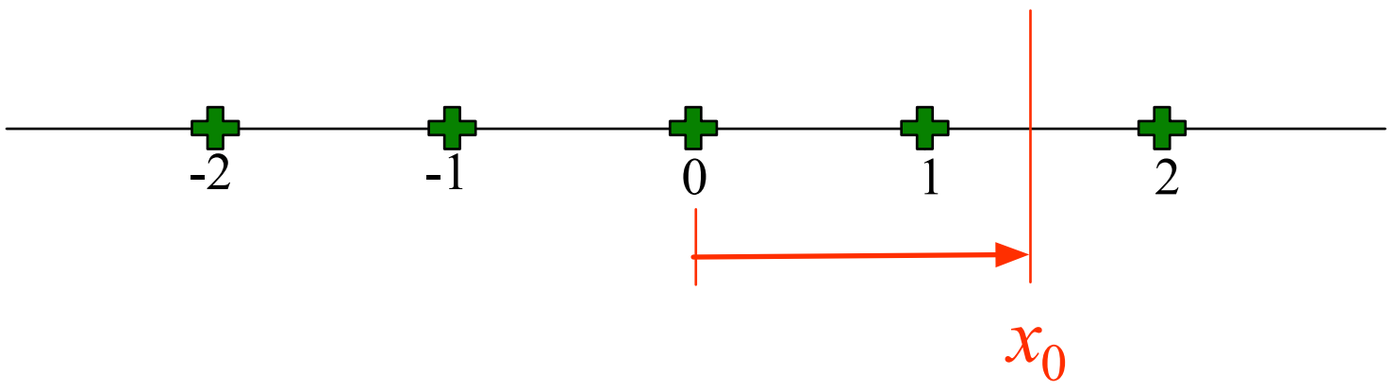

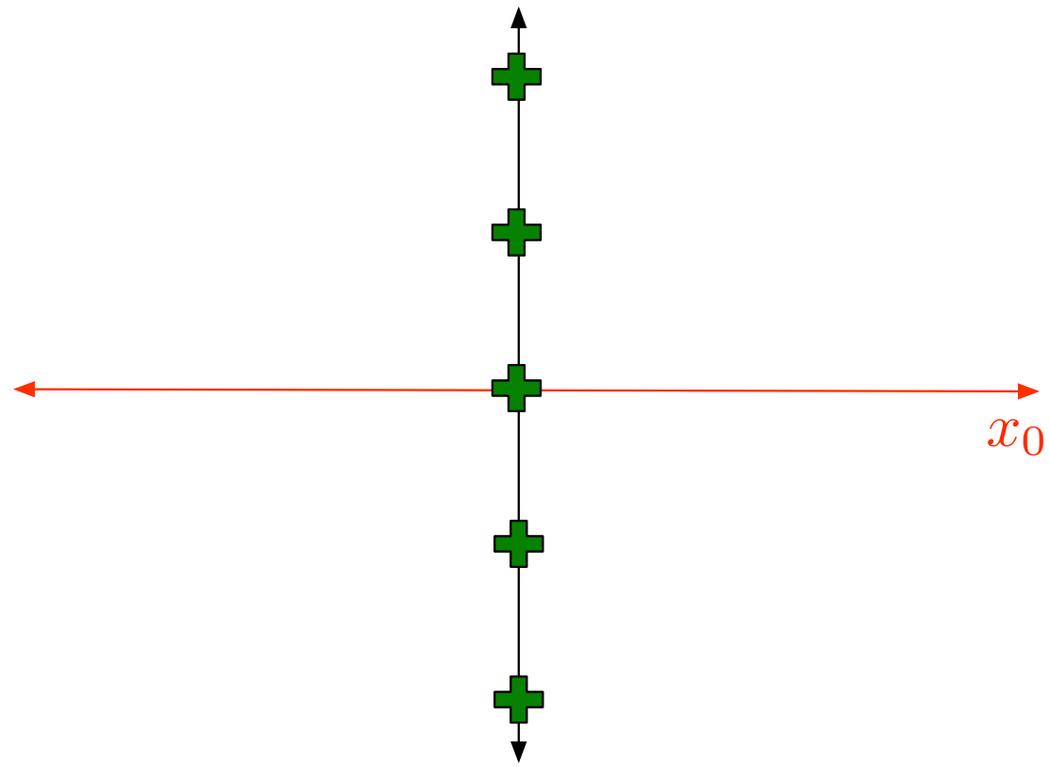

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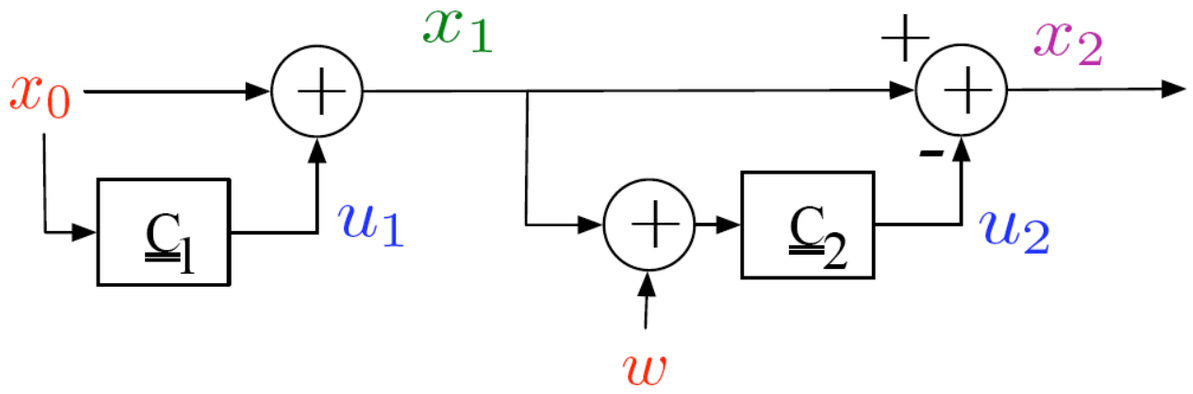

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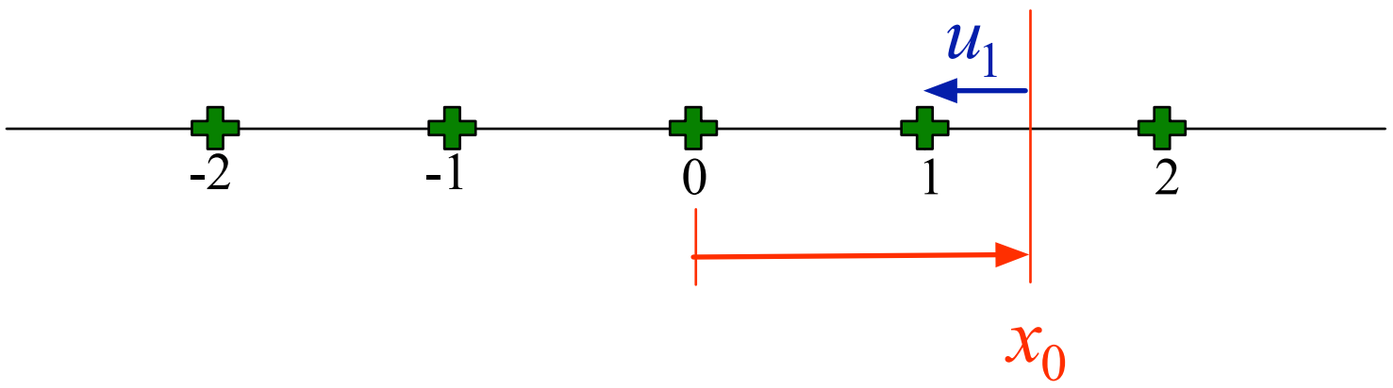

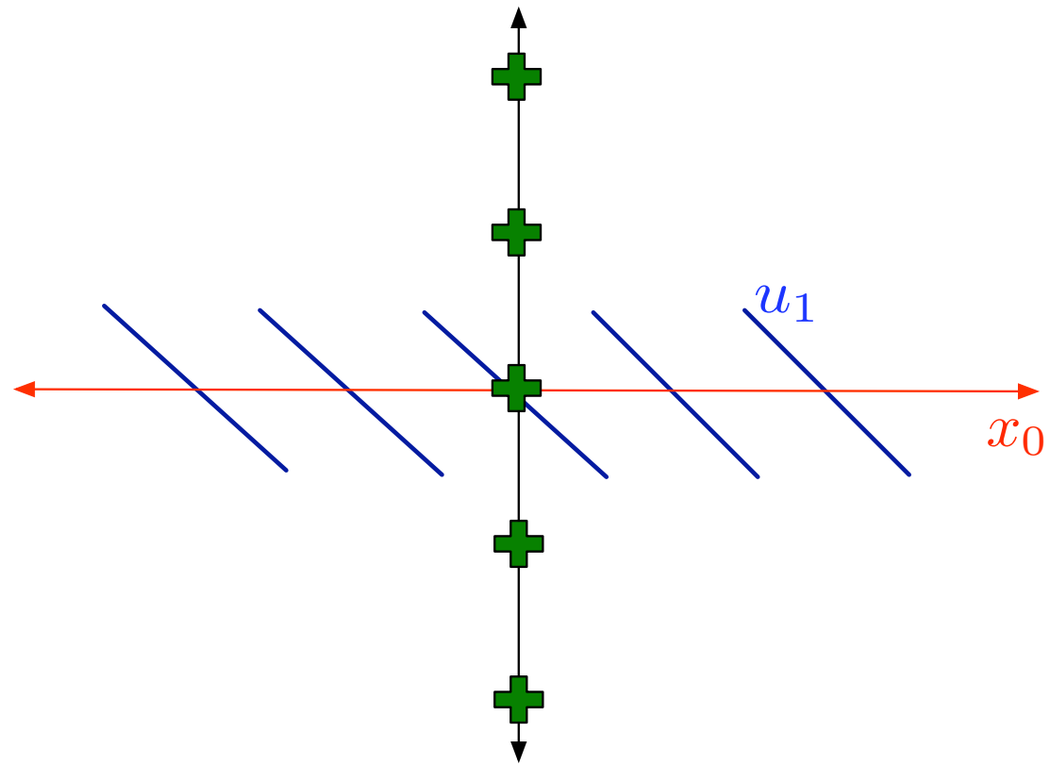

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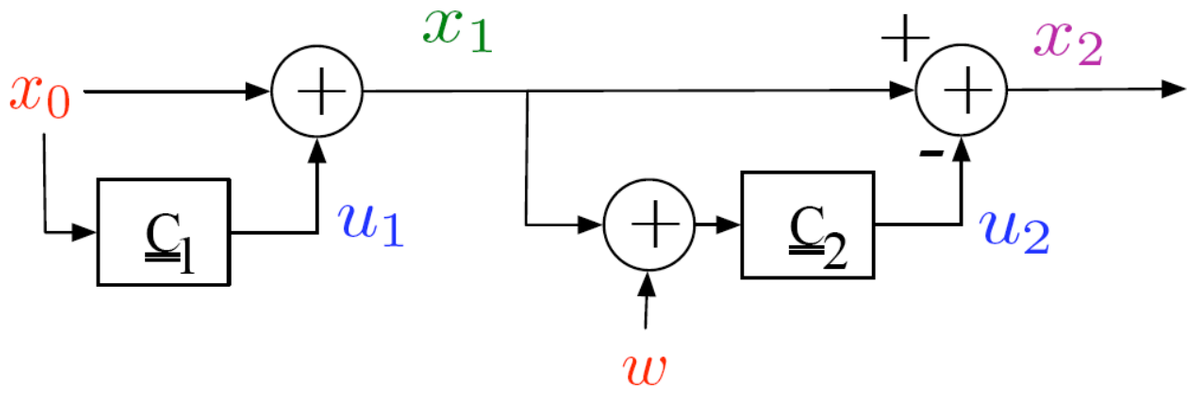

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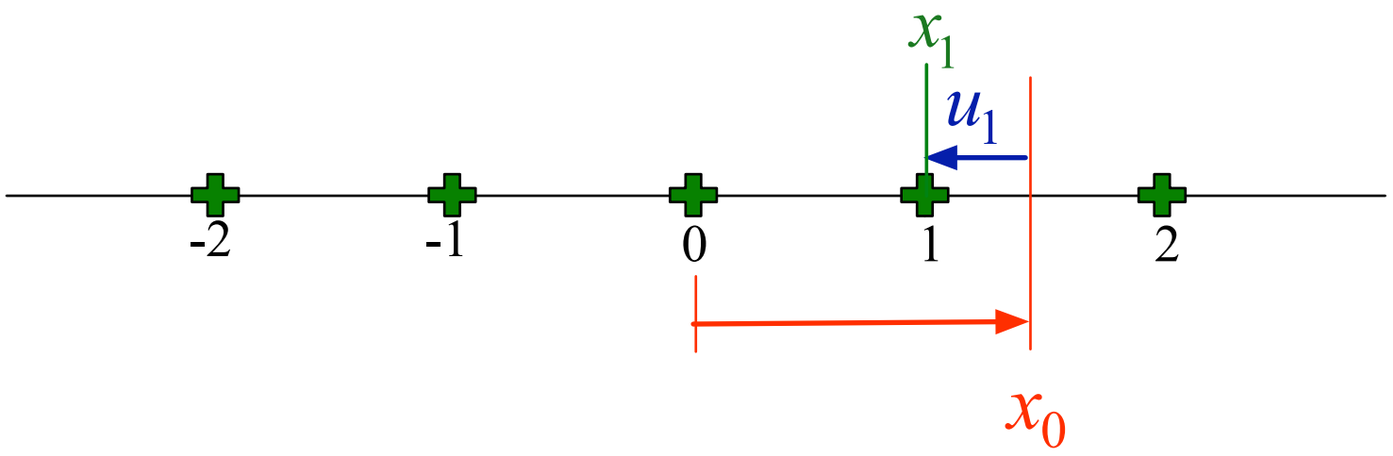
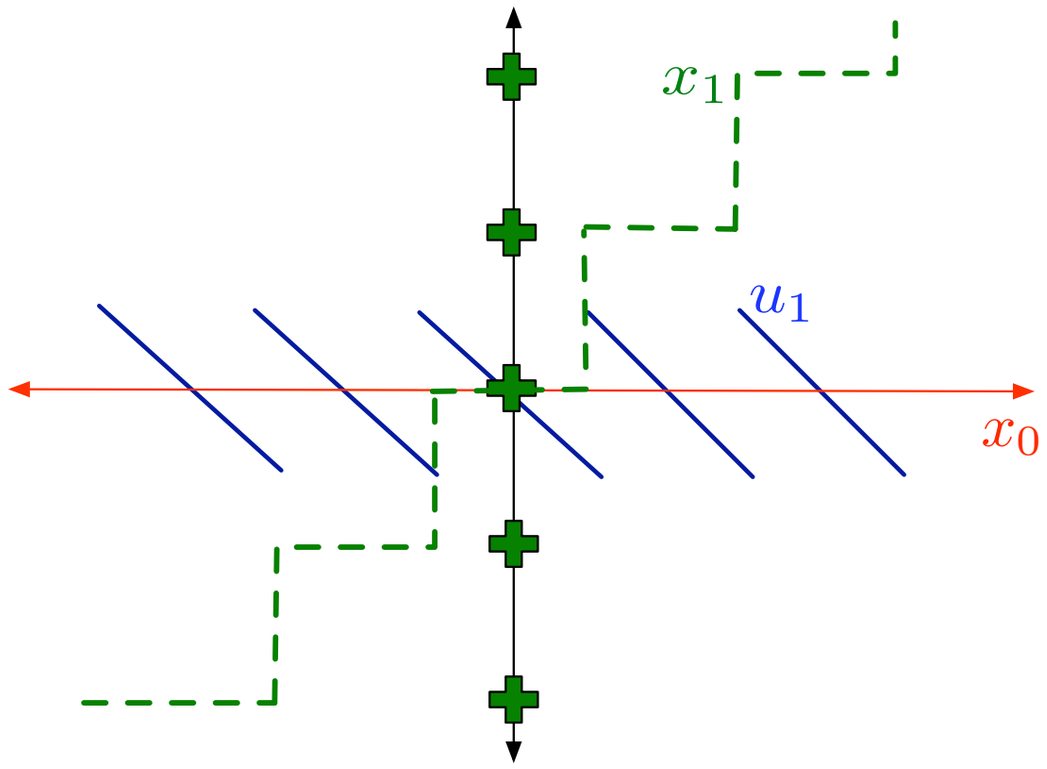
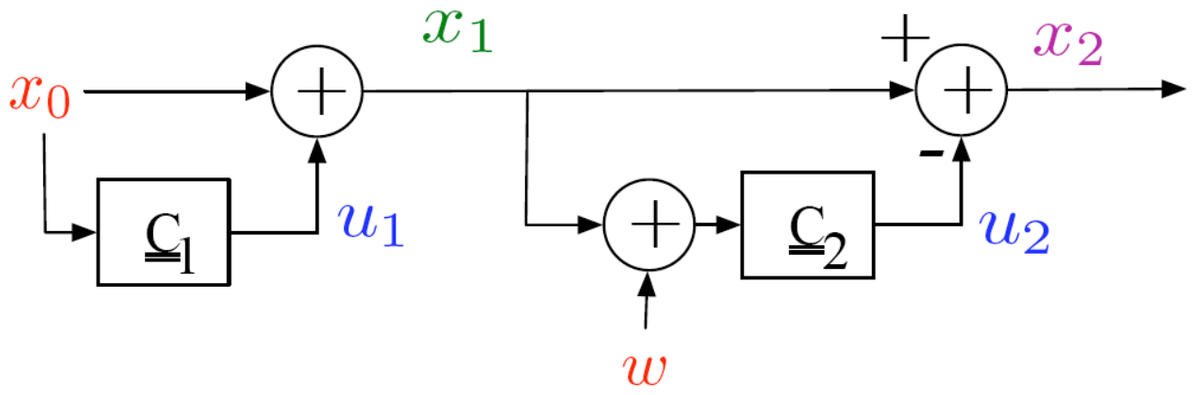




Quantization based signaling strategies



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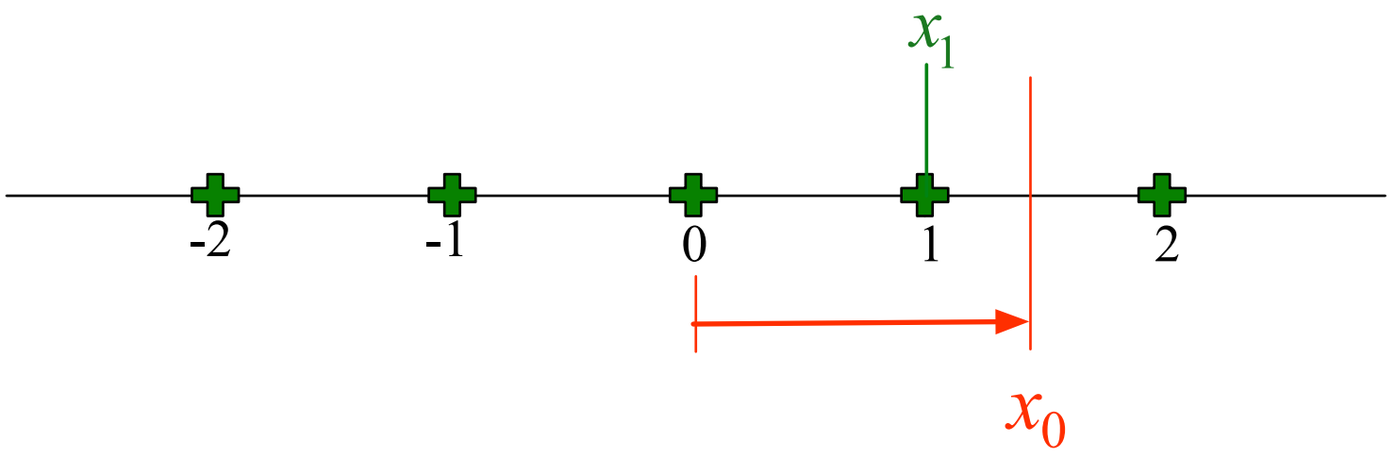

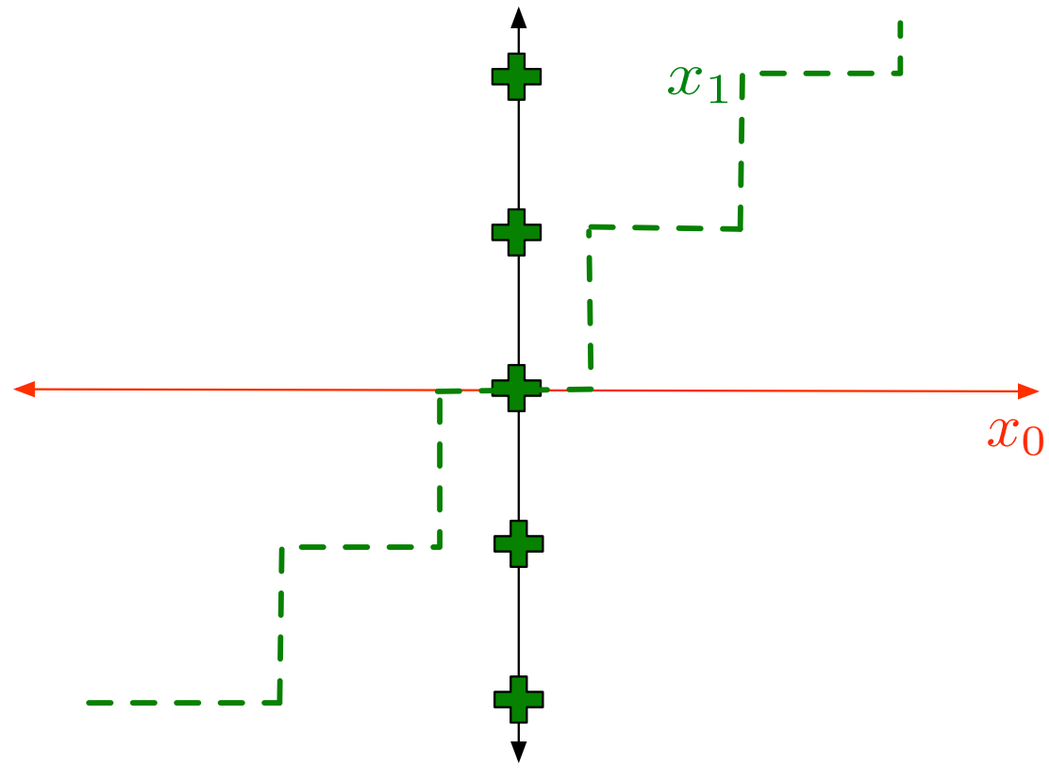

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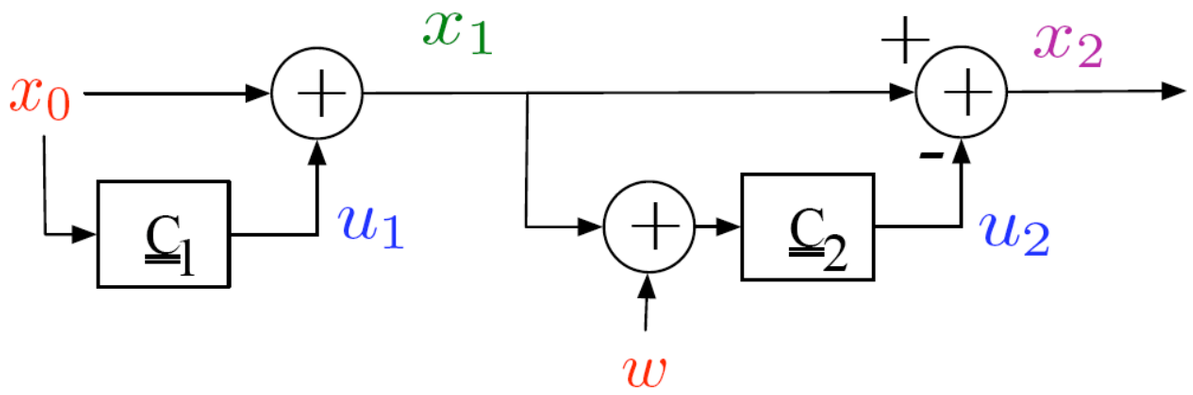

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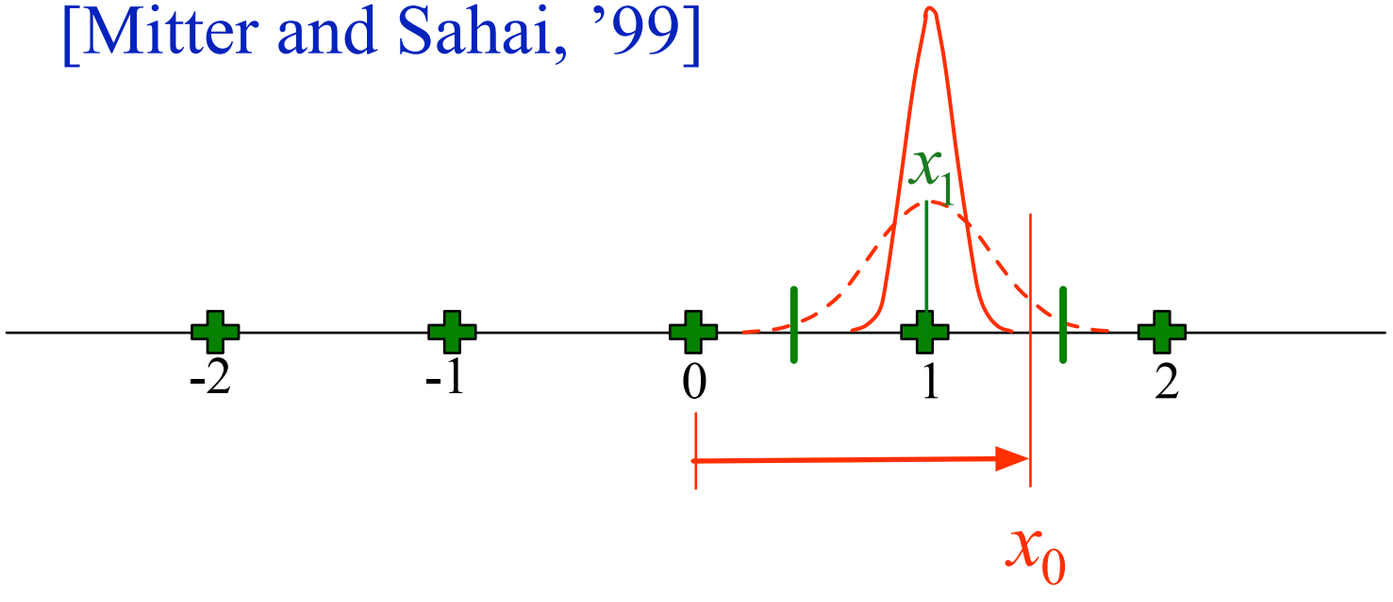
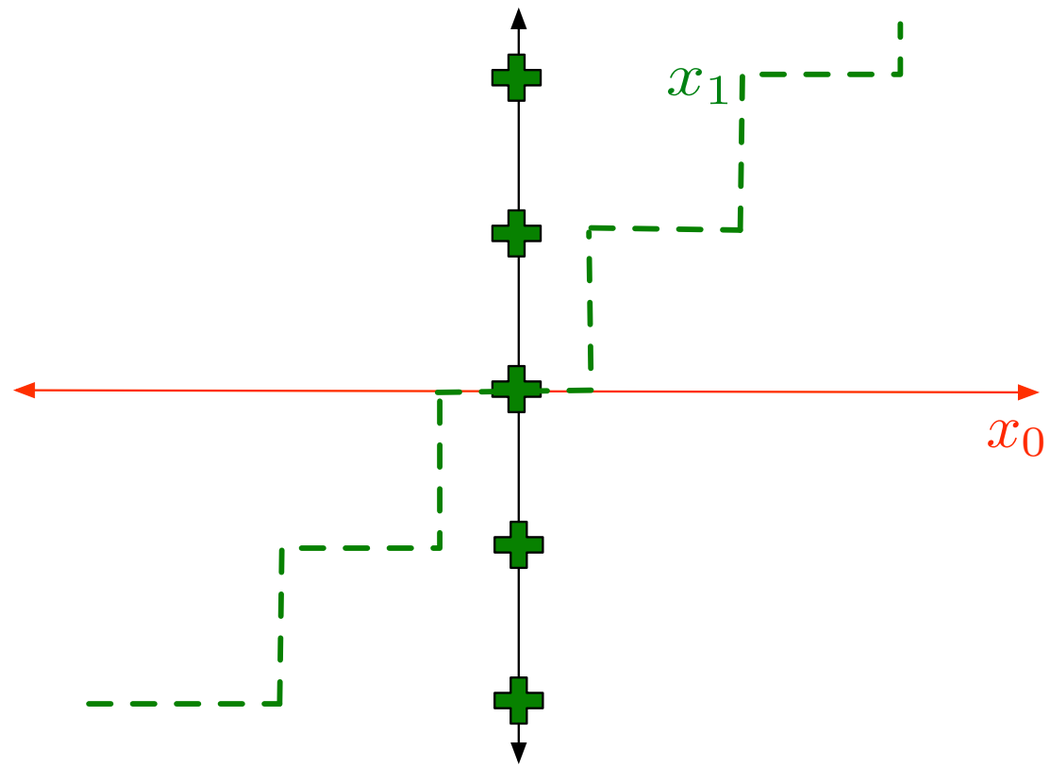
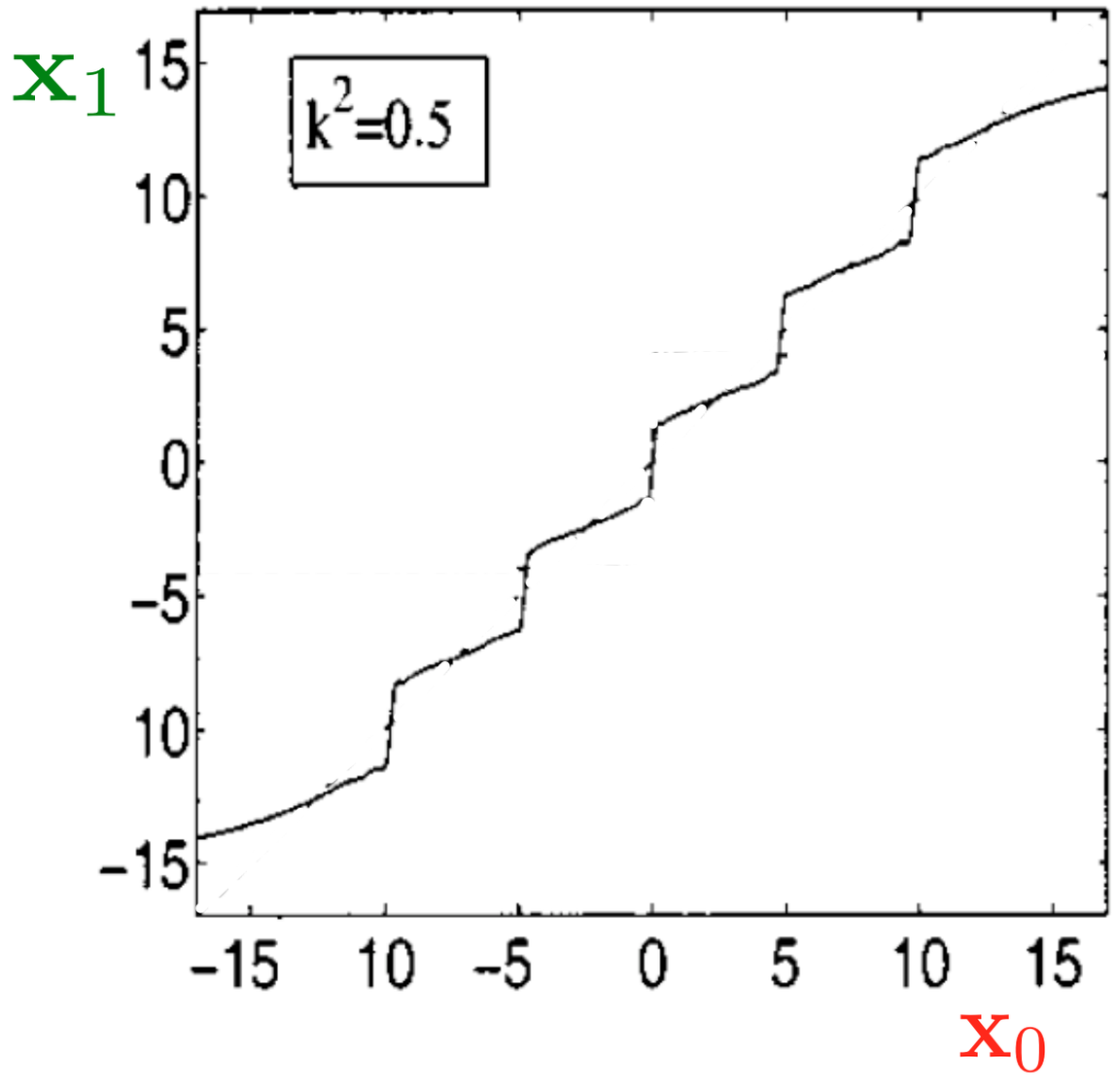




“Quantization” to disjoint intervals

[Baglietto et al, 2001][Lee, Lau and Ho]



* Figure taken from [Baglietto et al, 2001]

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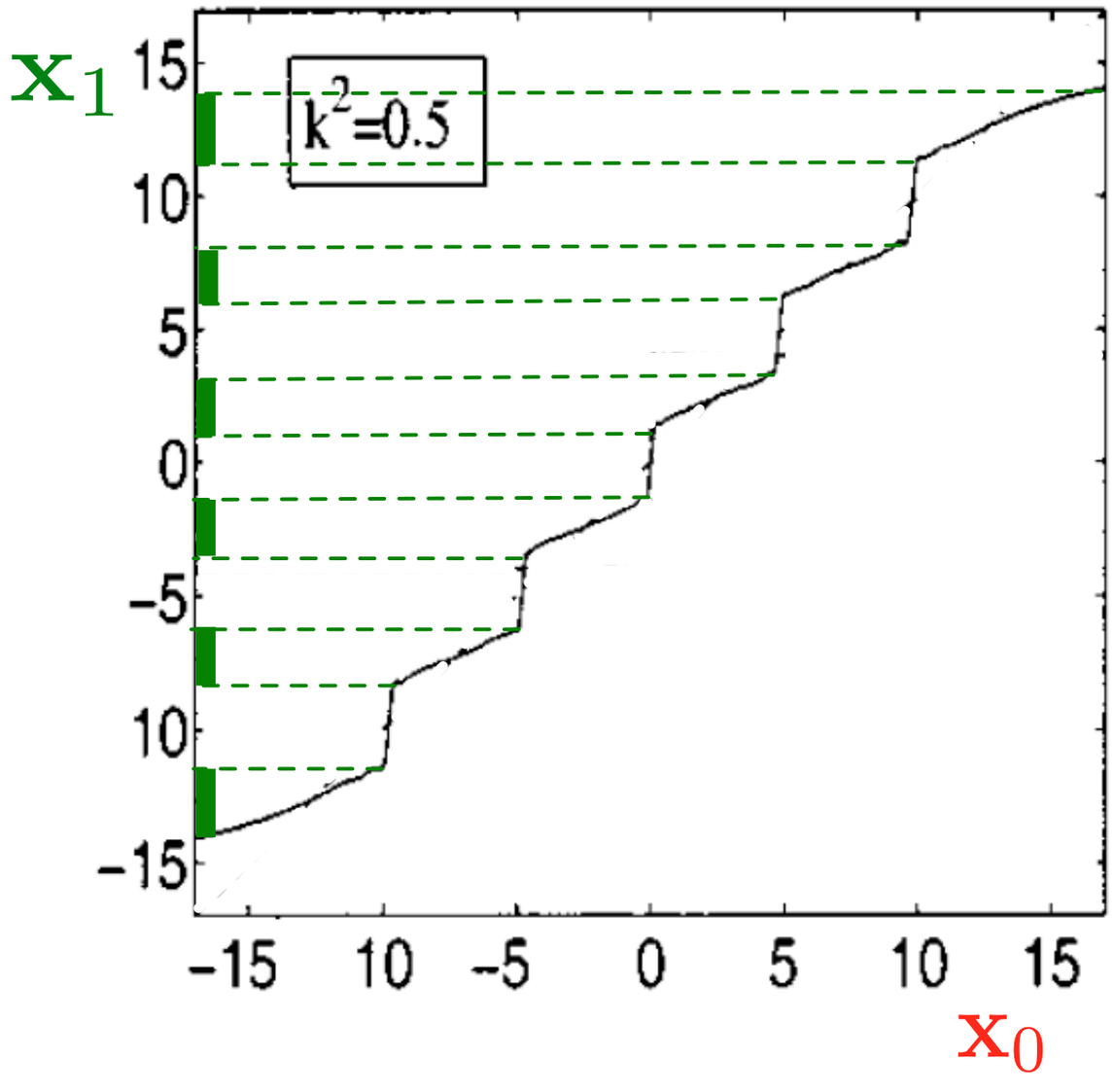

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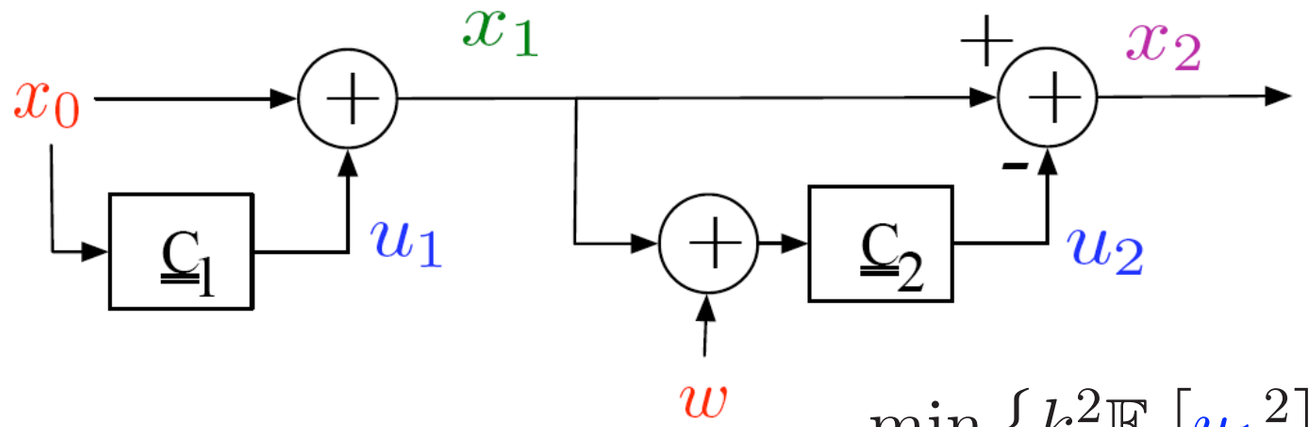
An information-theoretic interpretation of the counterexample

Original counterexample



$$\min \left\{ k^2 \mathbb{E}\left[{u_1}^2\right] + \mathbb{E}\left[{x_2}^2\right] \right\}$$

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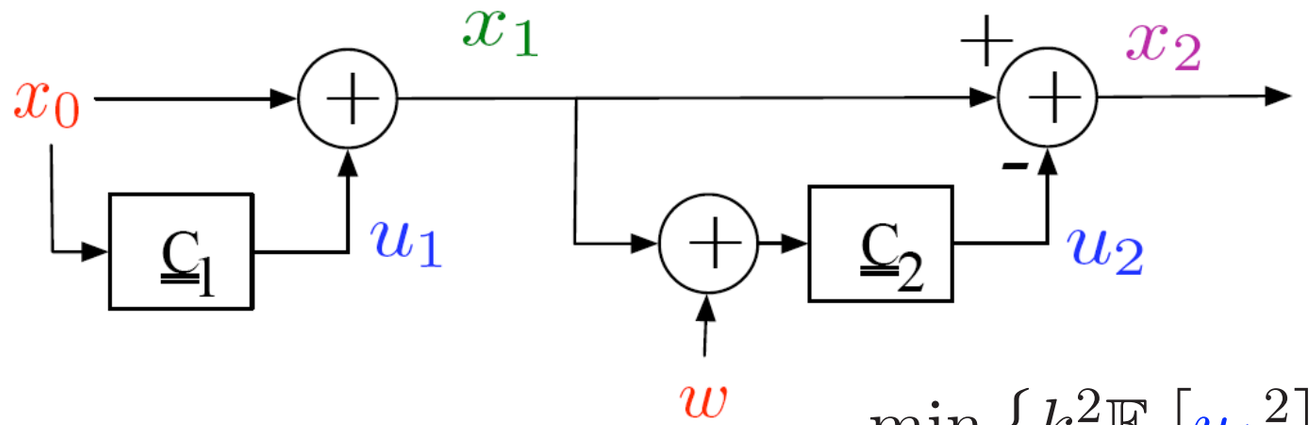

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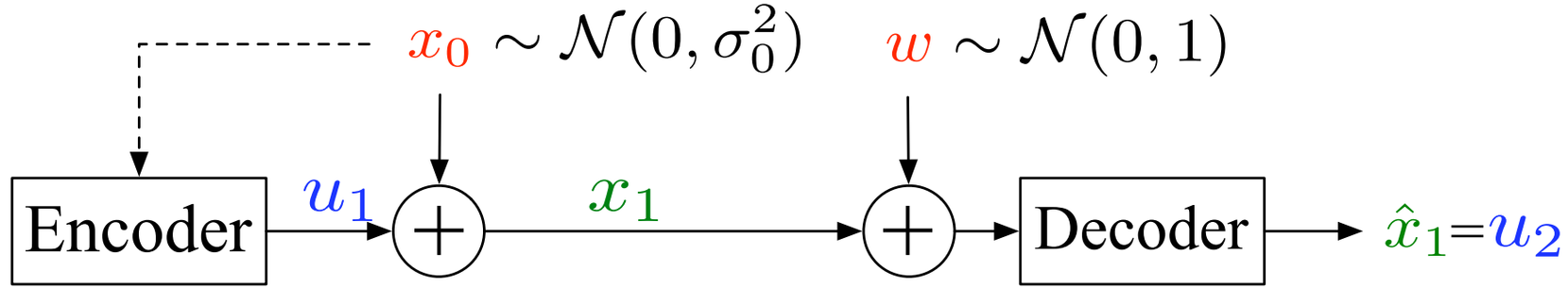
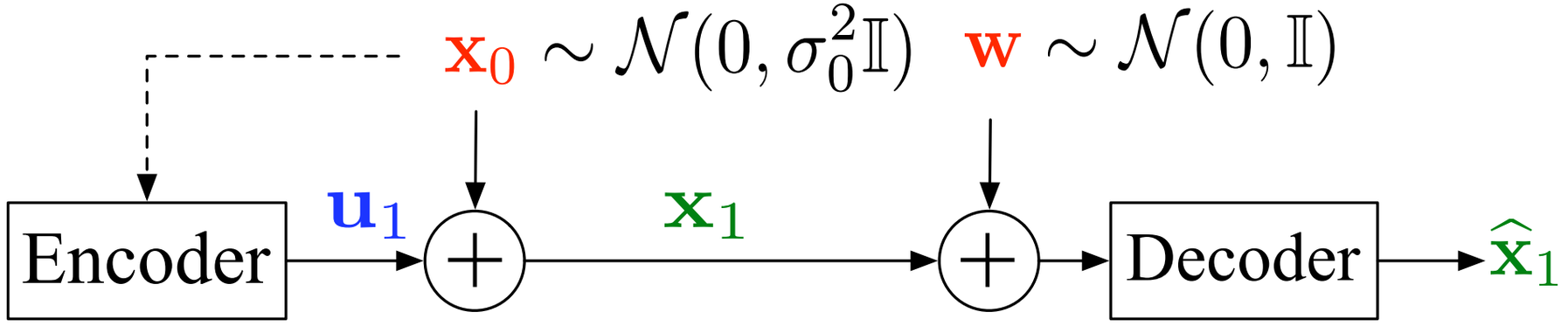


$$\mathbb{E}\left[{u_1}^2\right] \leq P \qquad \min \mathbb{E}\left[(x_1 - \hat{x}_1)^2\right]$$

Vector extension : an information-theory inspired simplification



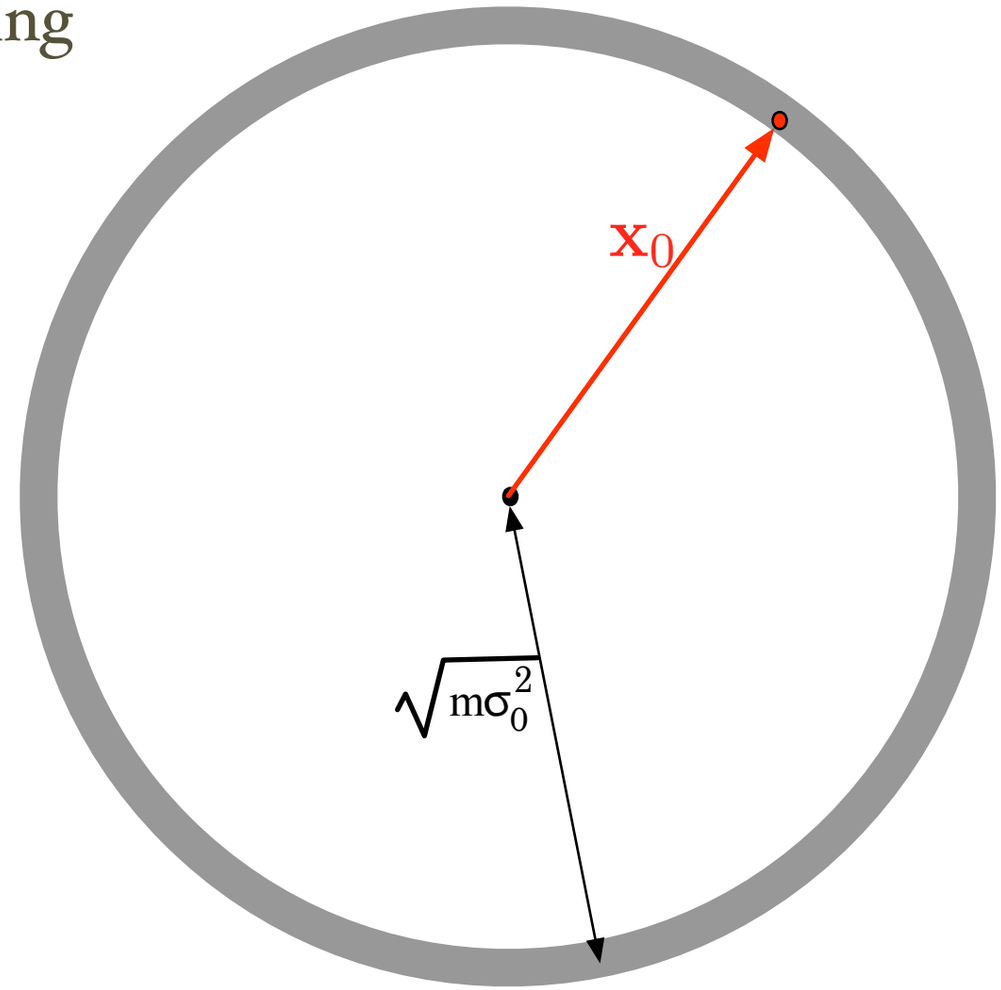
$$\frac{1}{m}\mathbb{E}\left[\|\mathbf{u}_1\|^2\right] \leq P \qquad \min \frac{1}{m}\mathbb{E}\left[\|\mathbf{x}_1 - \widehat{\mathbf{x}}_1\|^2\right]$$

$$\mathbb{E}[\mathcal{C}] = k^2 P + \frac{1}{m}\mathbb{E}\left[\|\mathbf{x}_1 - \widehat{\mathbf{x}}_1\|^2\right]$$

[Ho, Kastner, Wong '78]

A nonlinear vector quantization scheme

Joint source-channel coding (JSCC)



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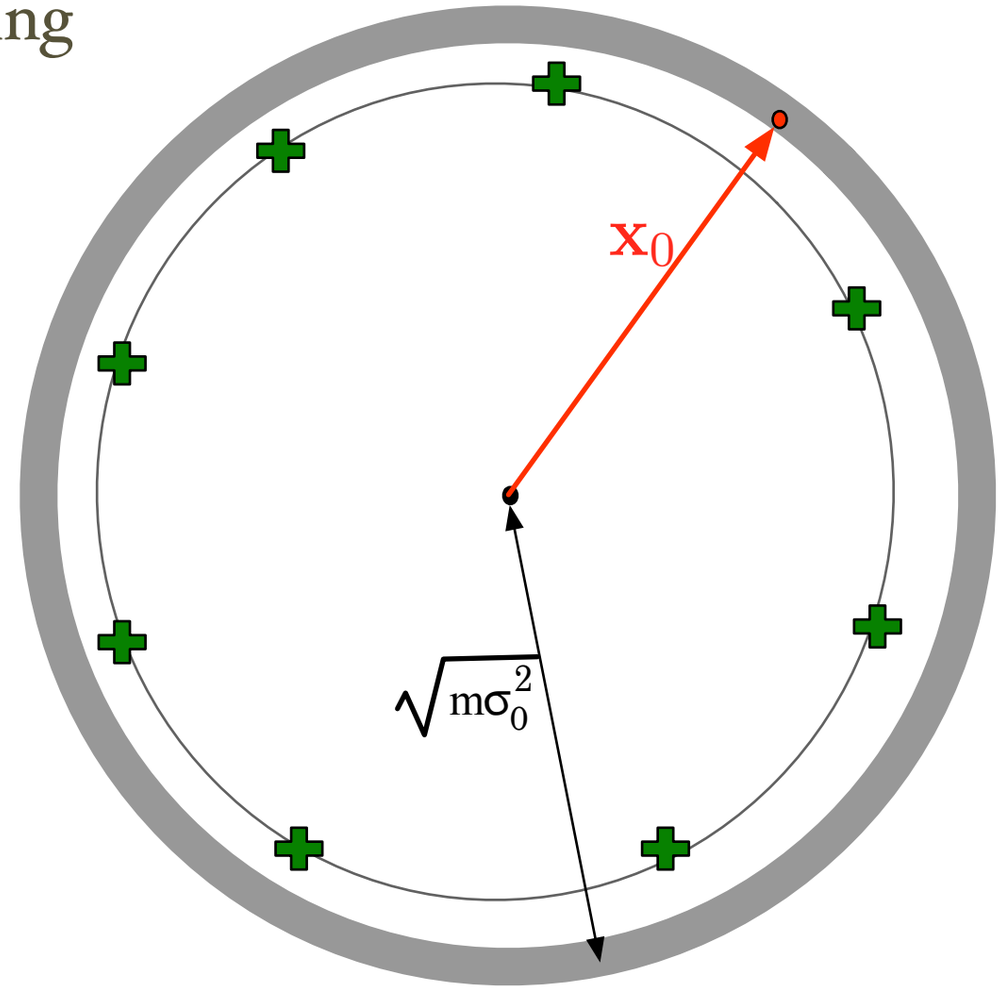

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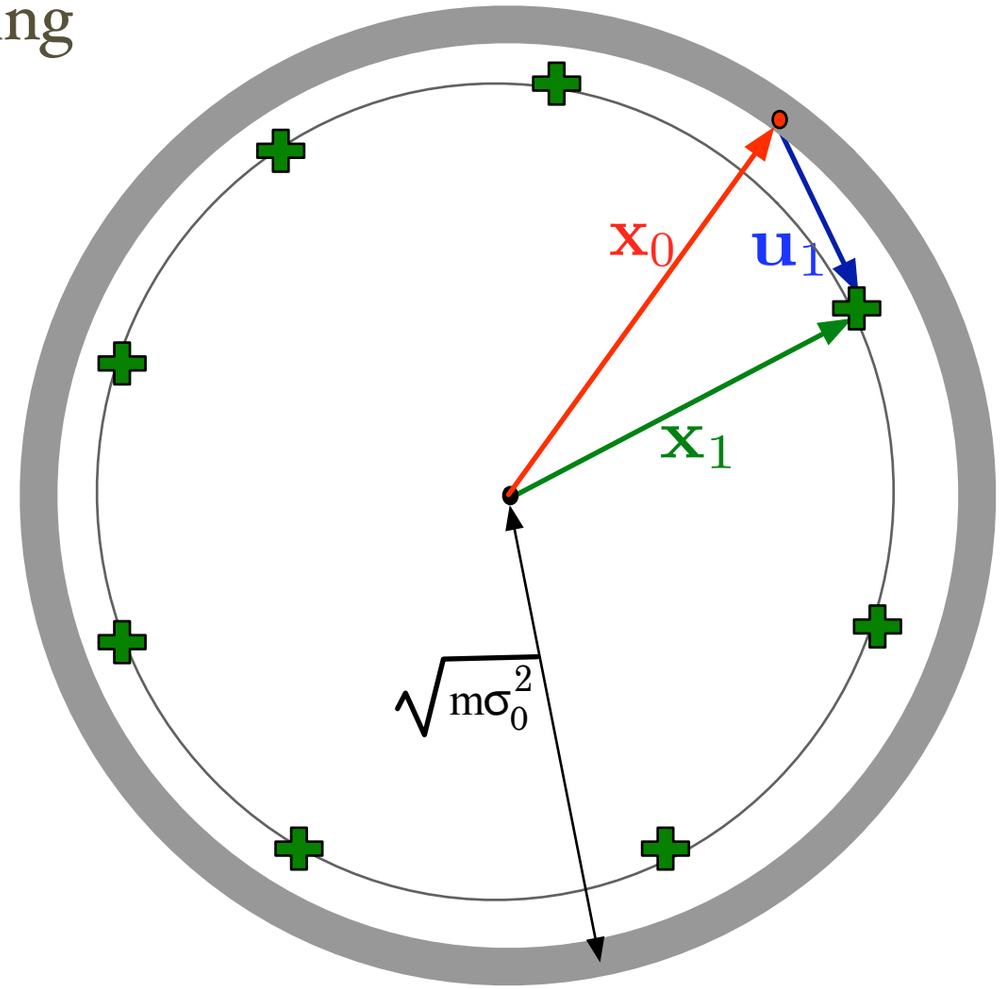

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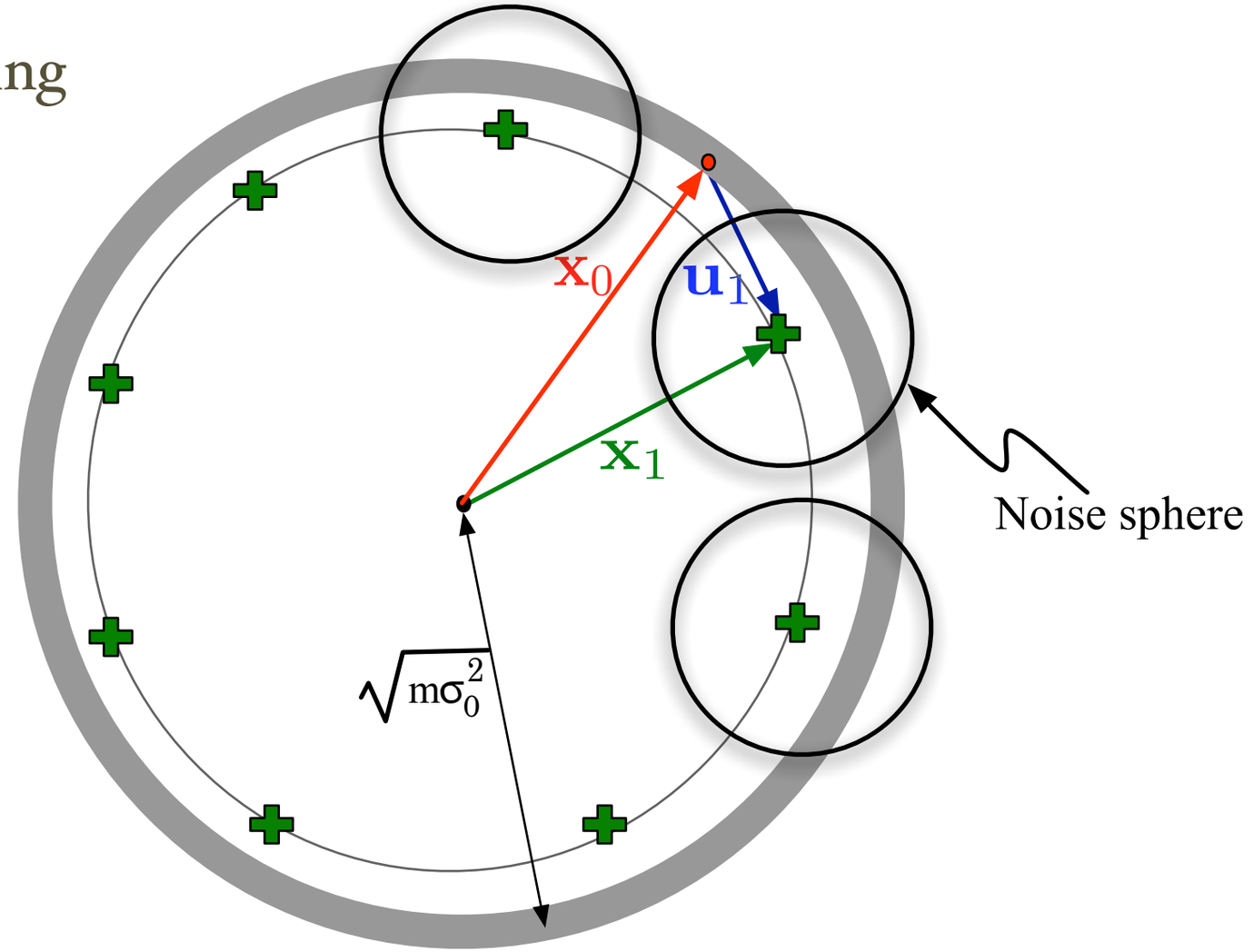

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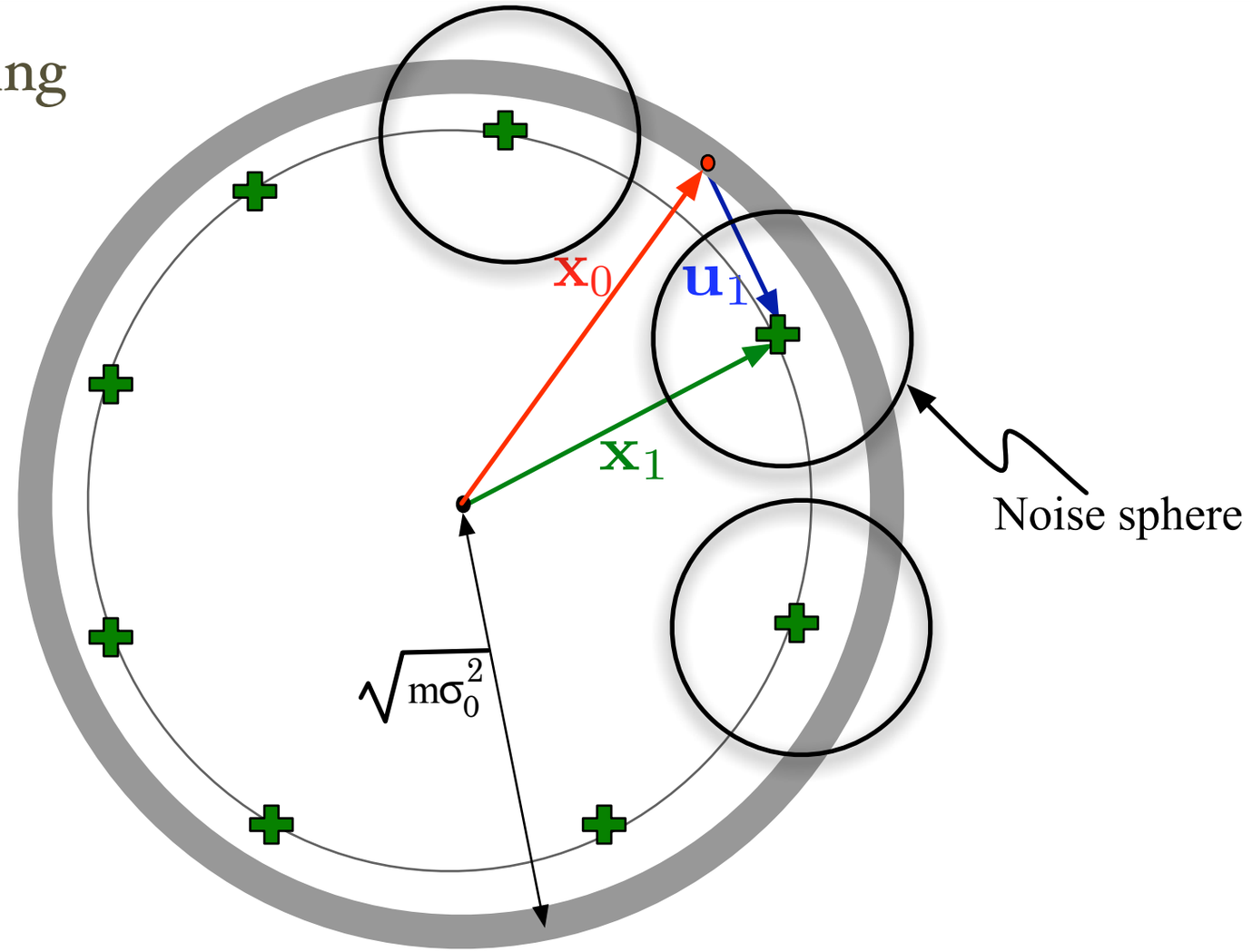
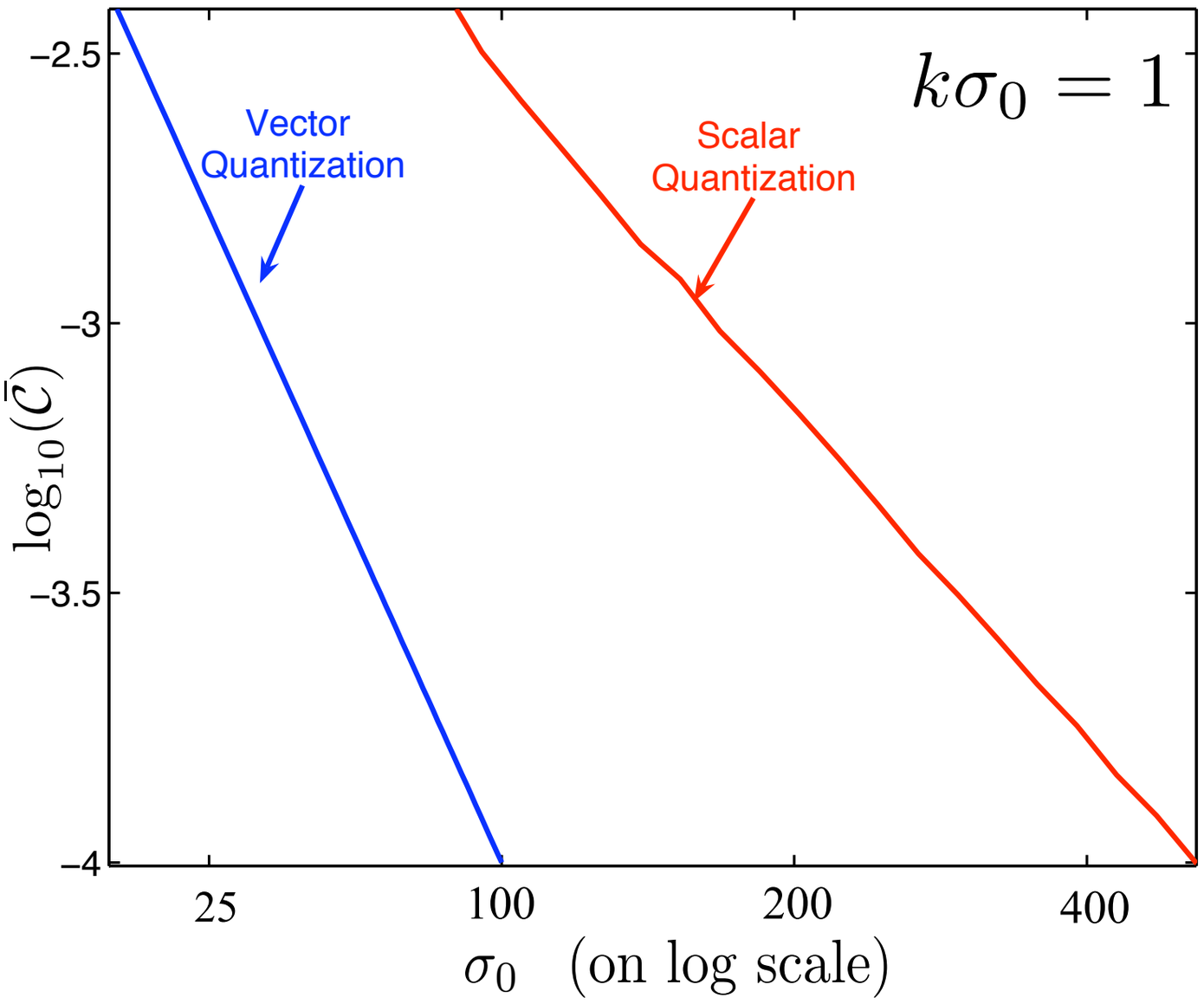
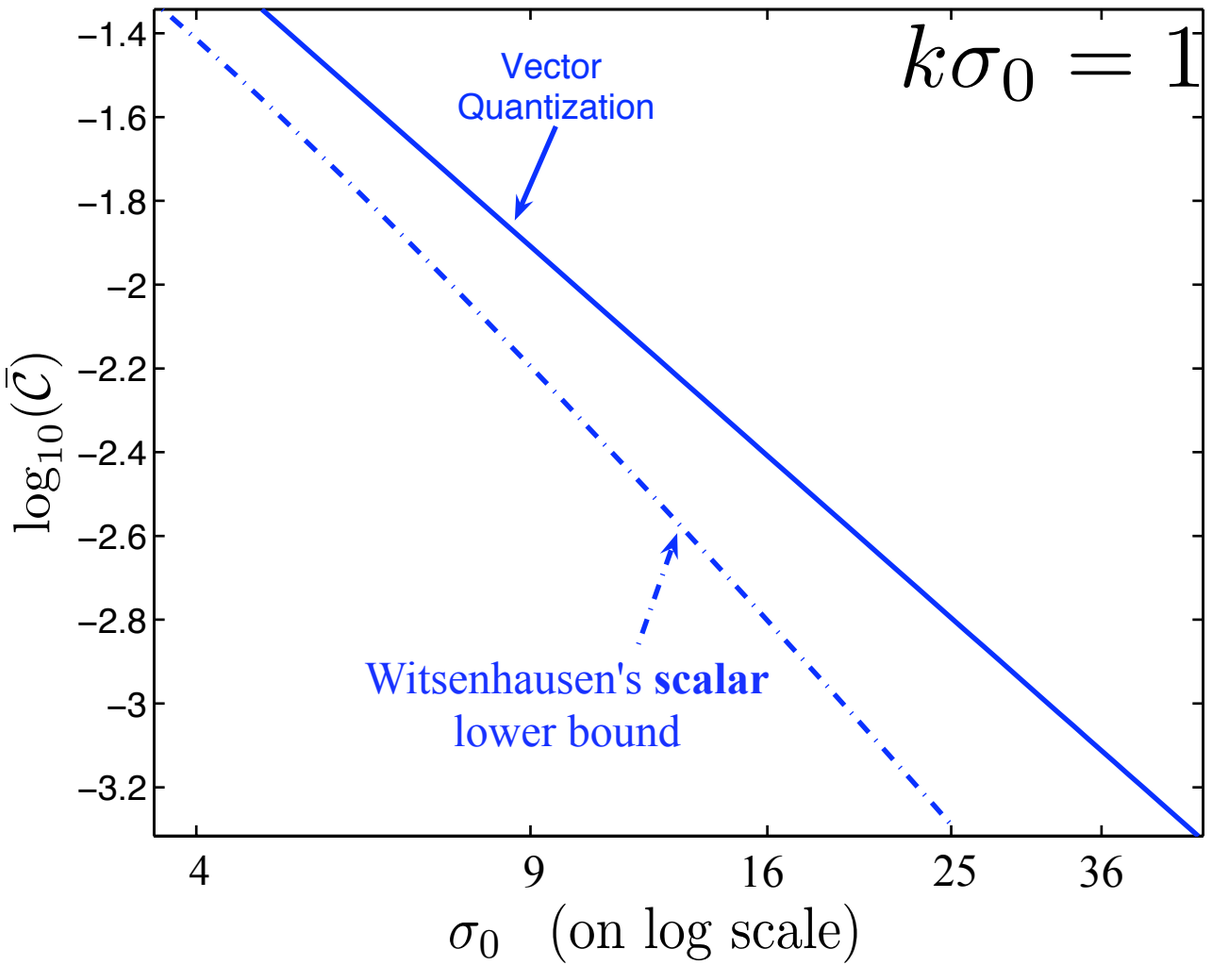


$$\bar{C} = k^2 + 0$$

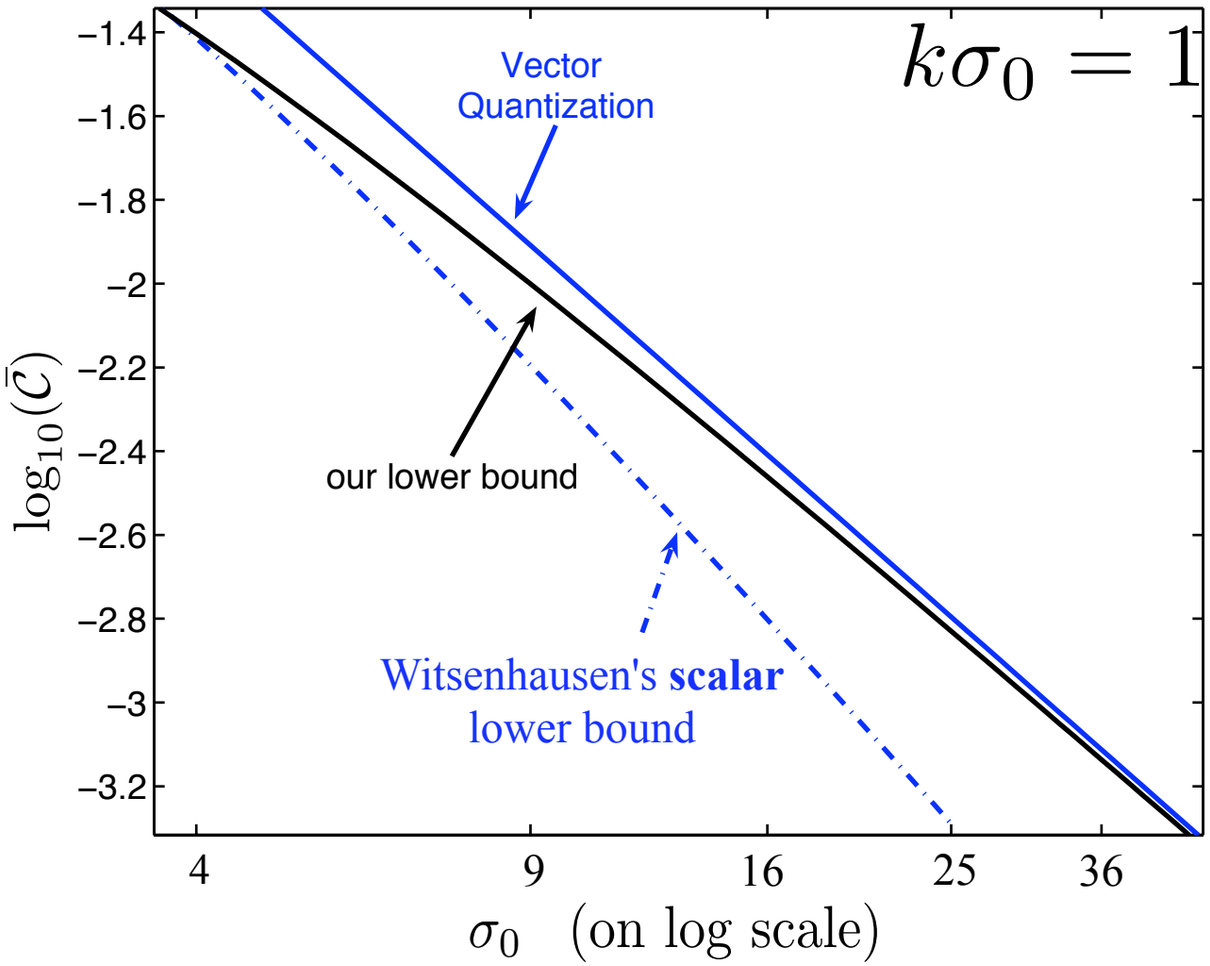
Vector quantization beats scalar quantization



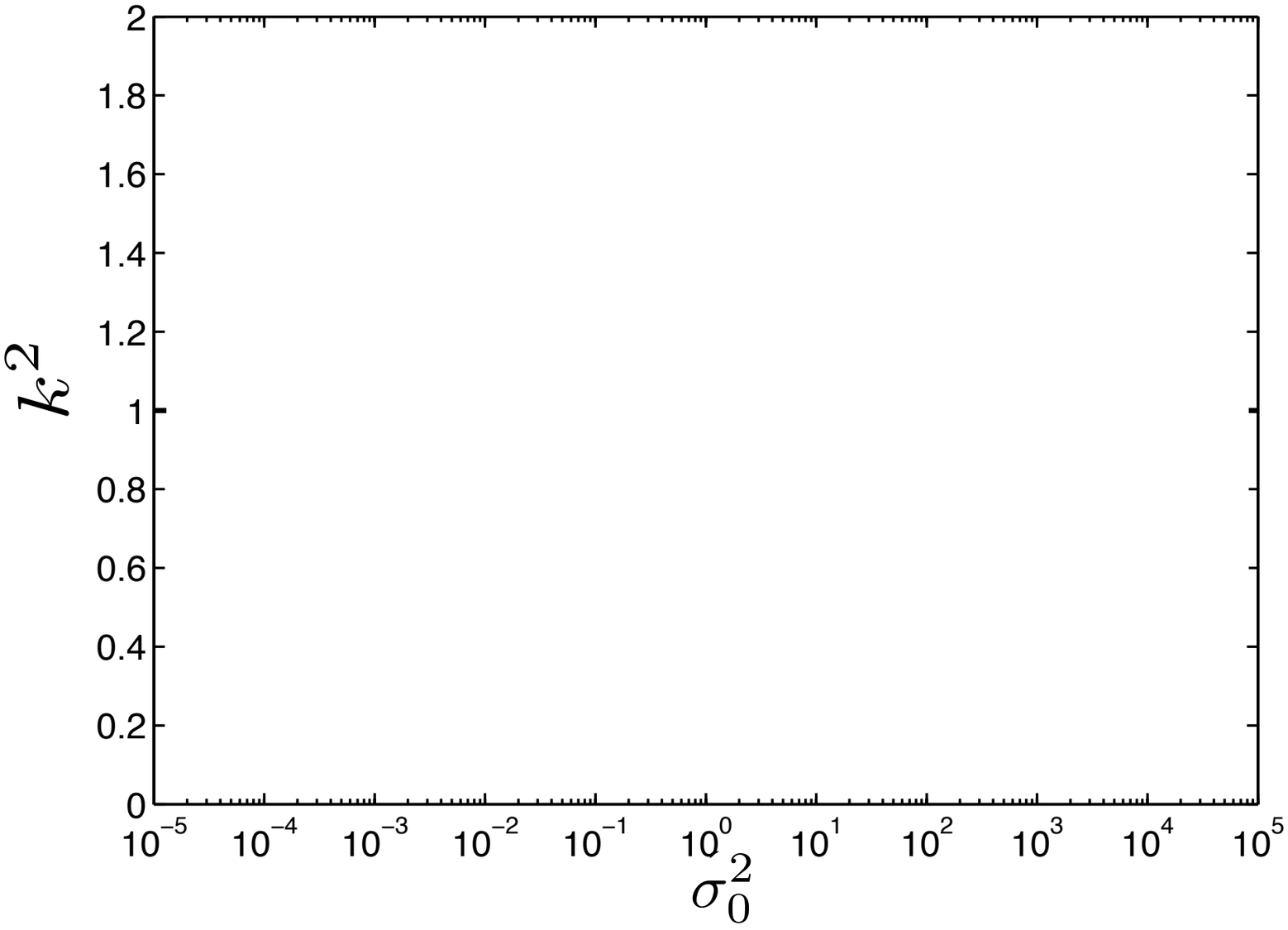
Even vector strategies might be far from optimal!



Closing the gap: our information-theoretic lower bound



When do we need nonlinear strategies ?



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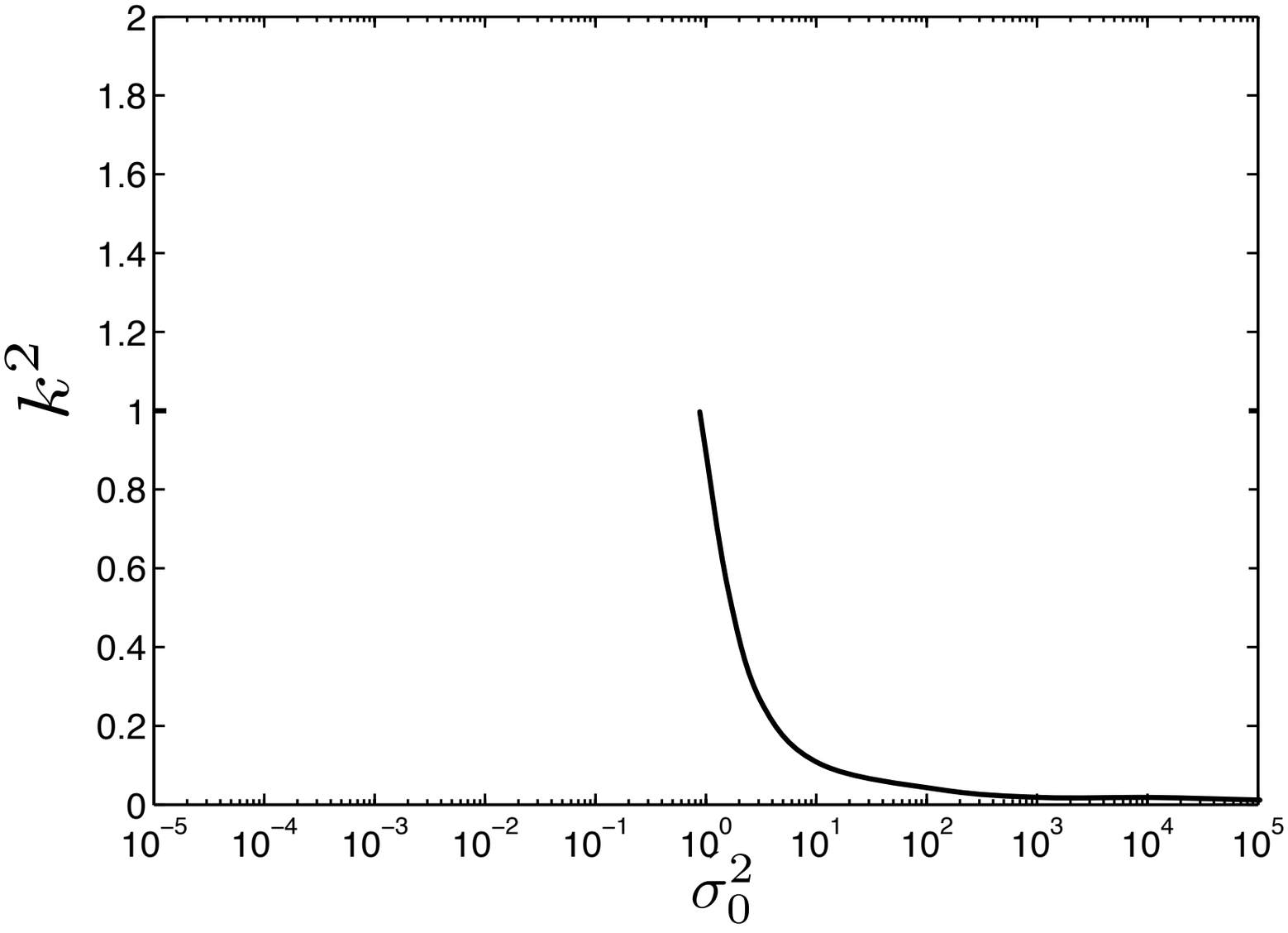

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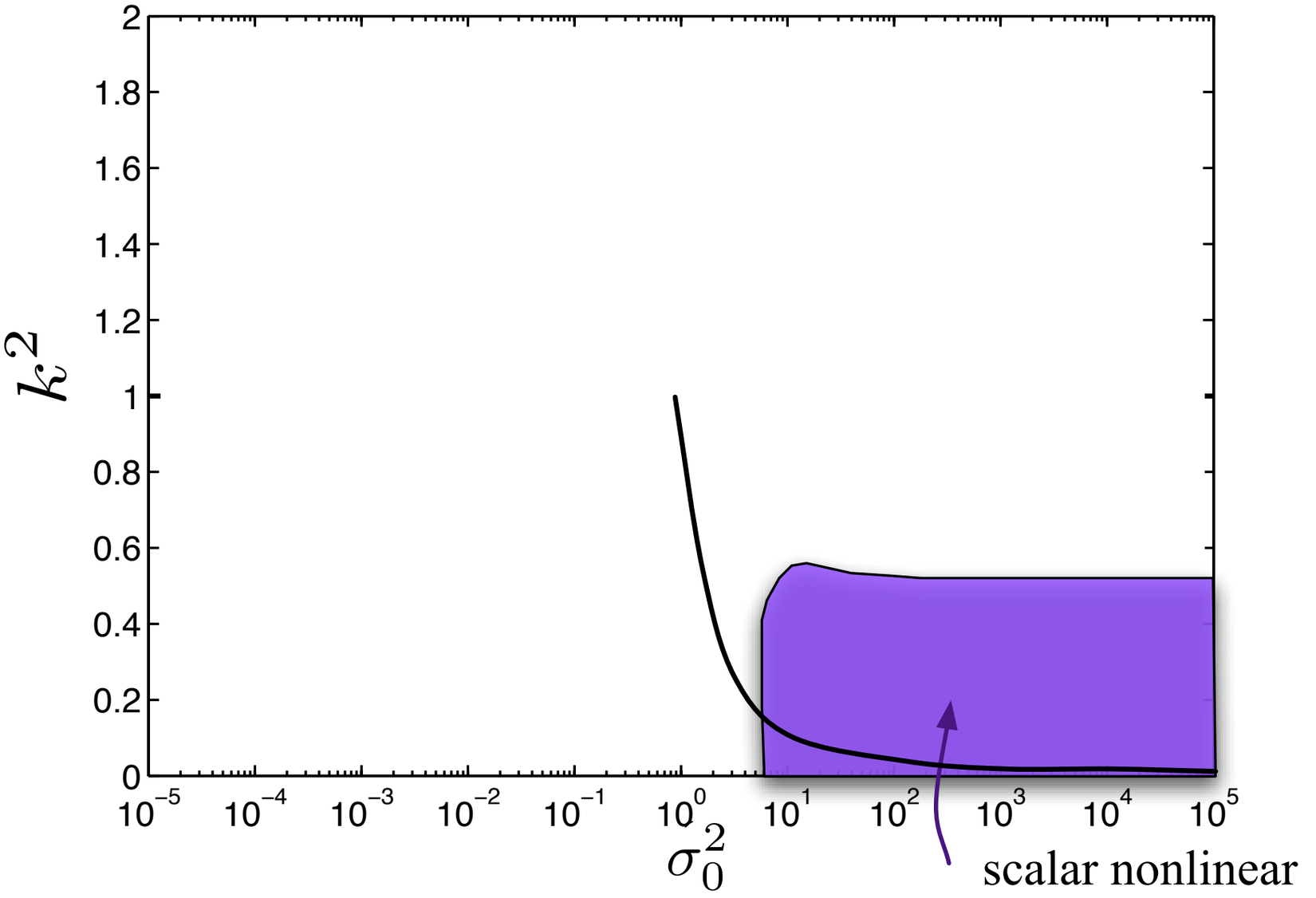

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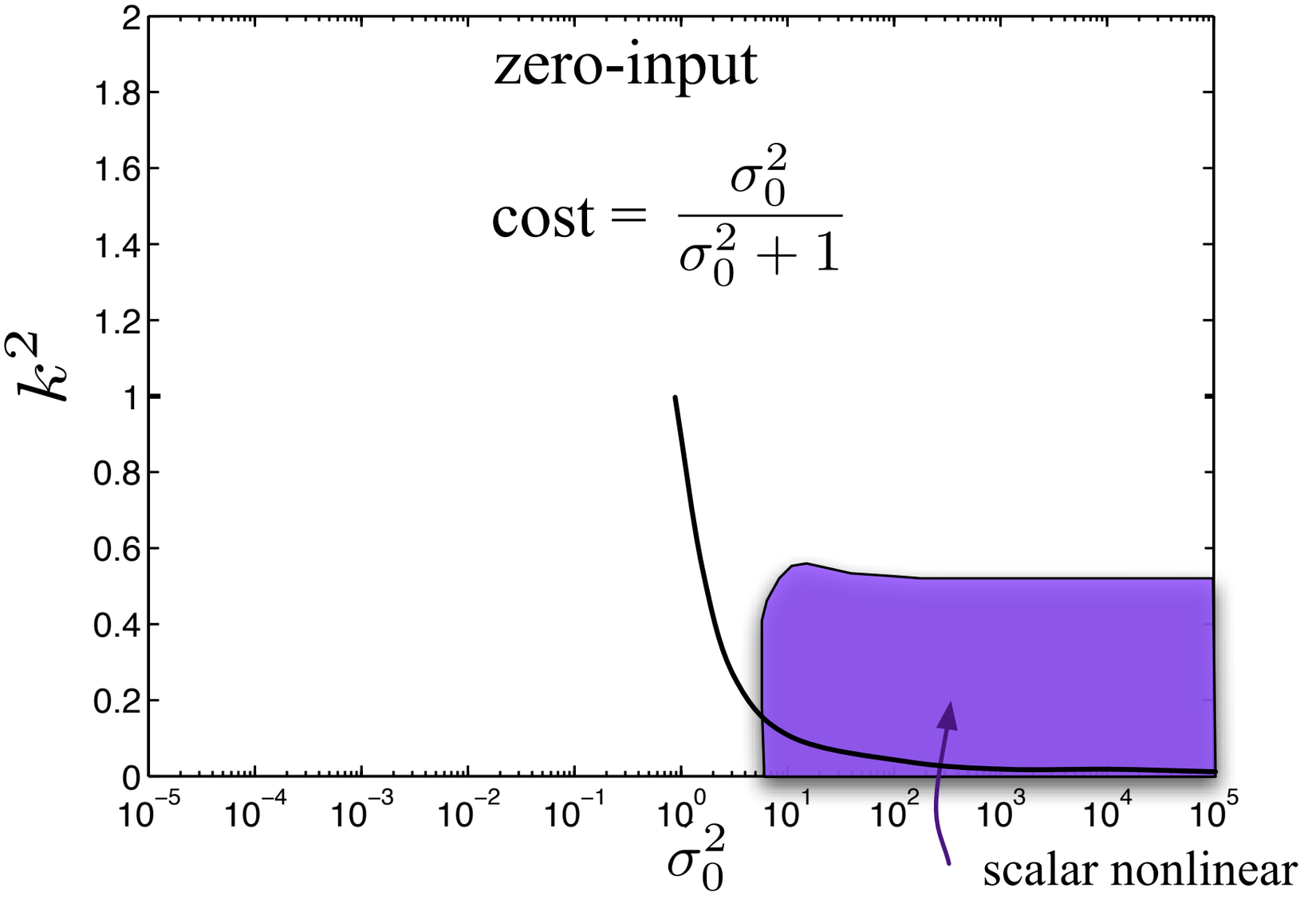

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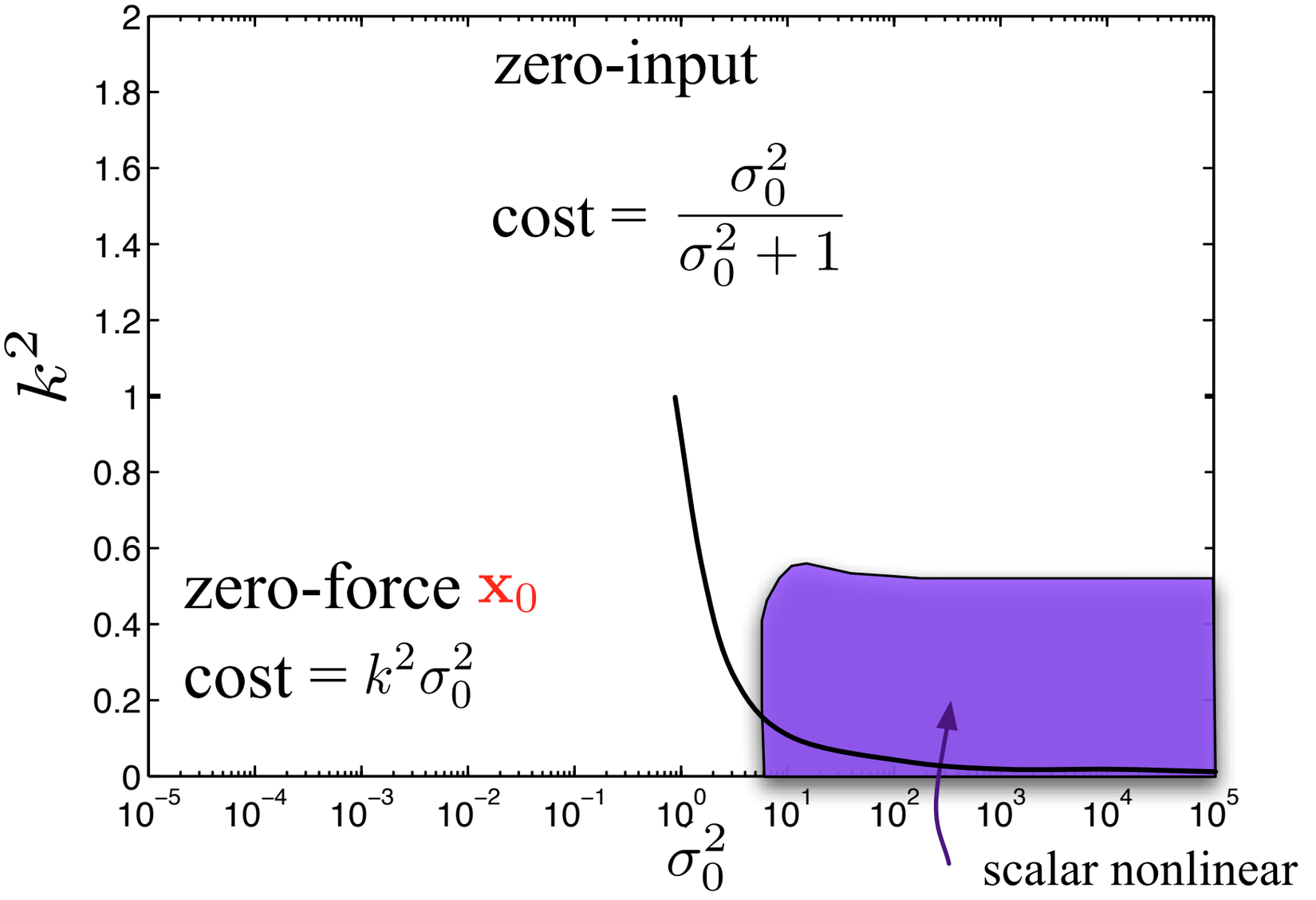

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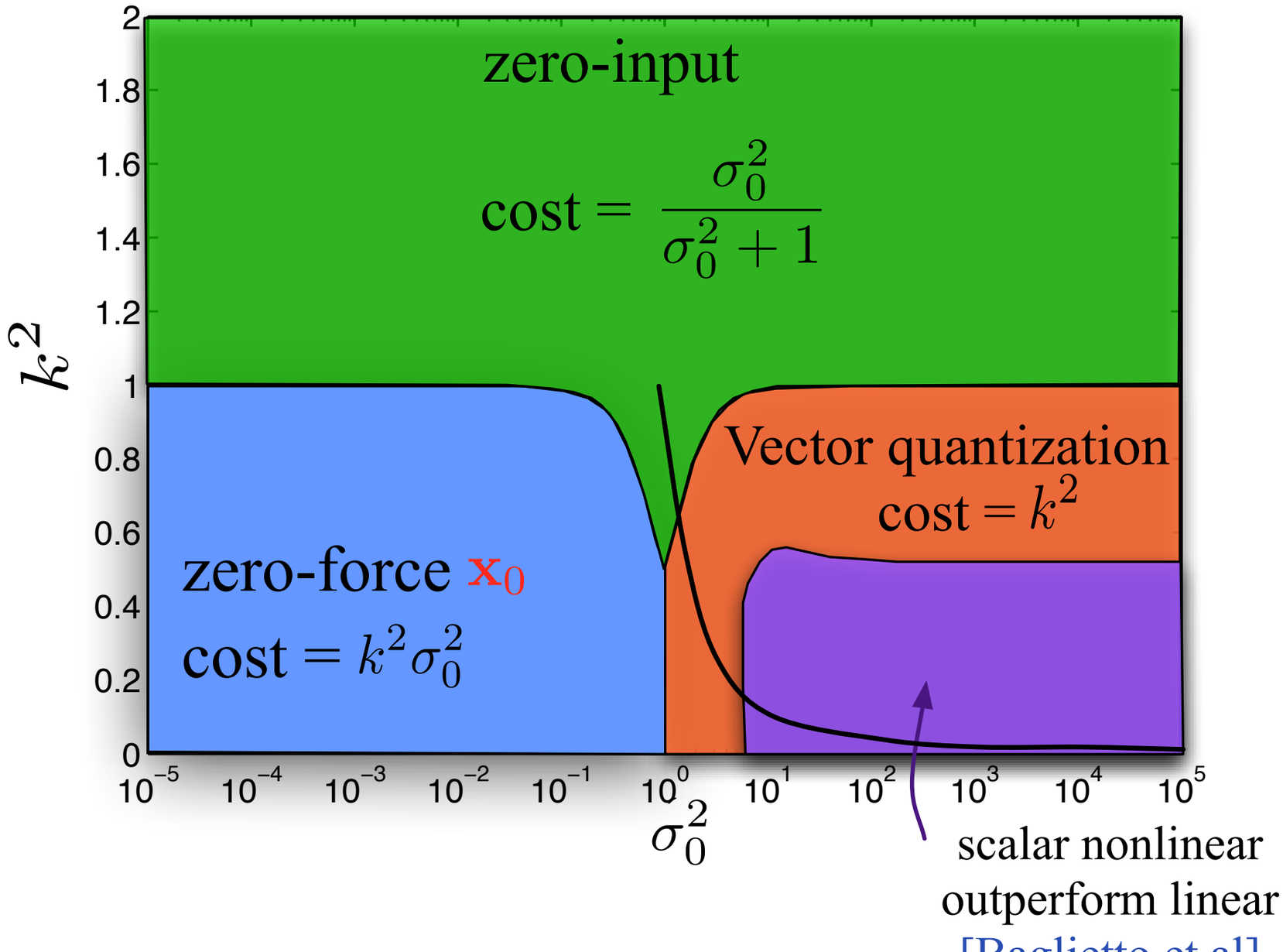
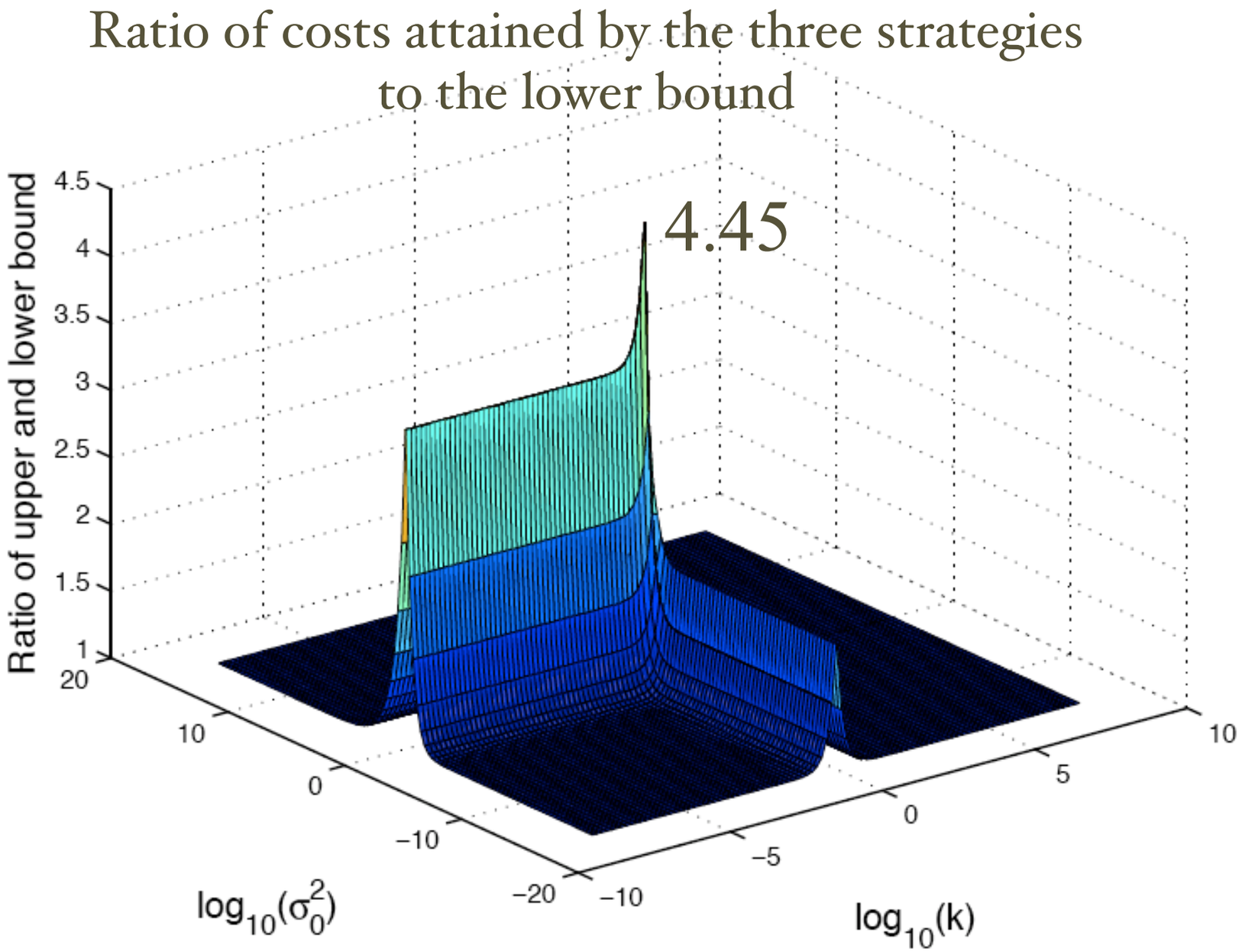
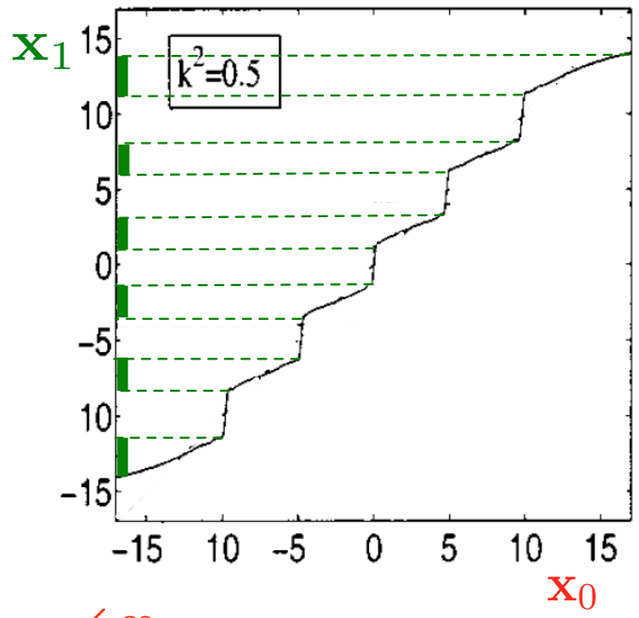


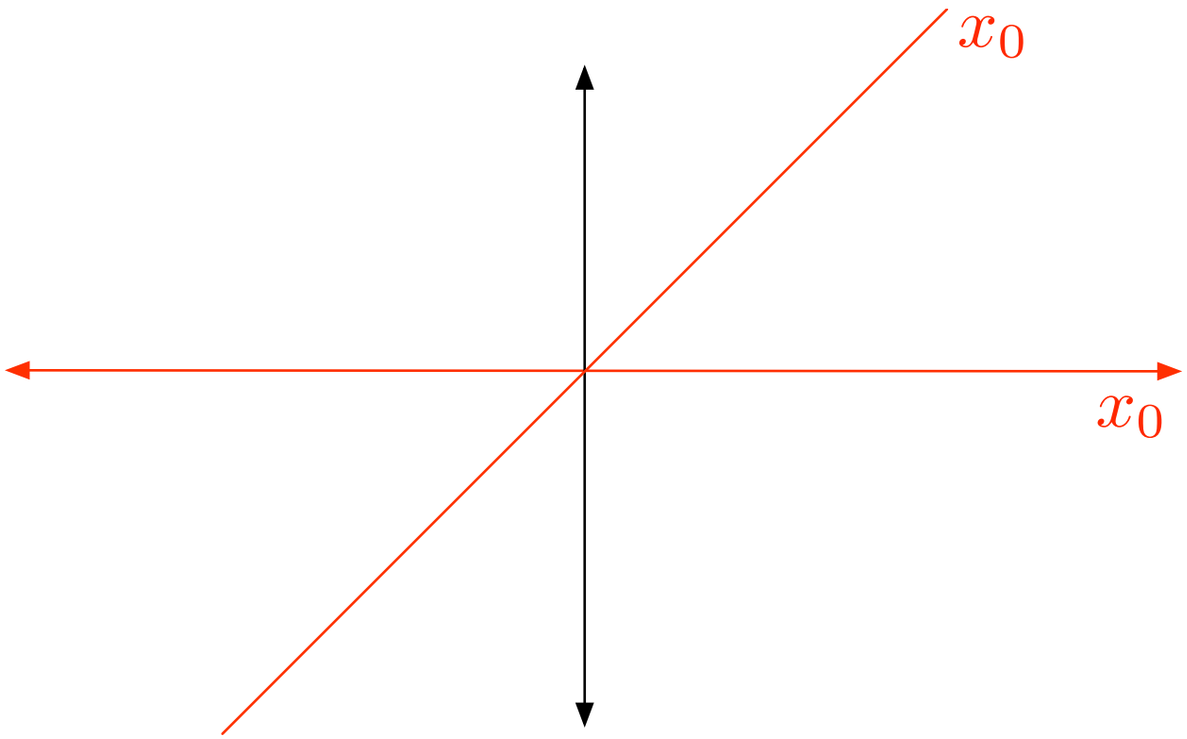
scalar nonlinear outperform linear [Baglietto et al]

Characterizing the optimal cost



Another look at “quantization” to intervals





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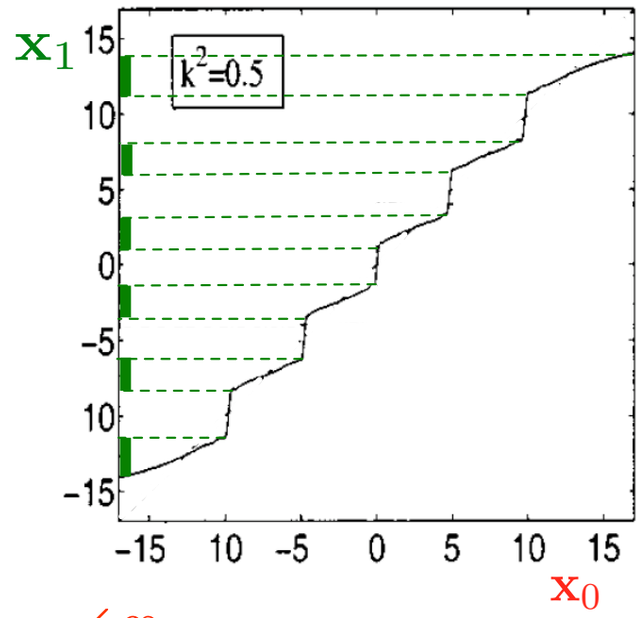

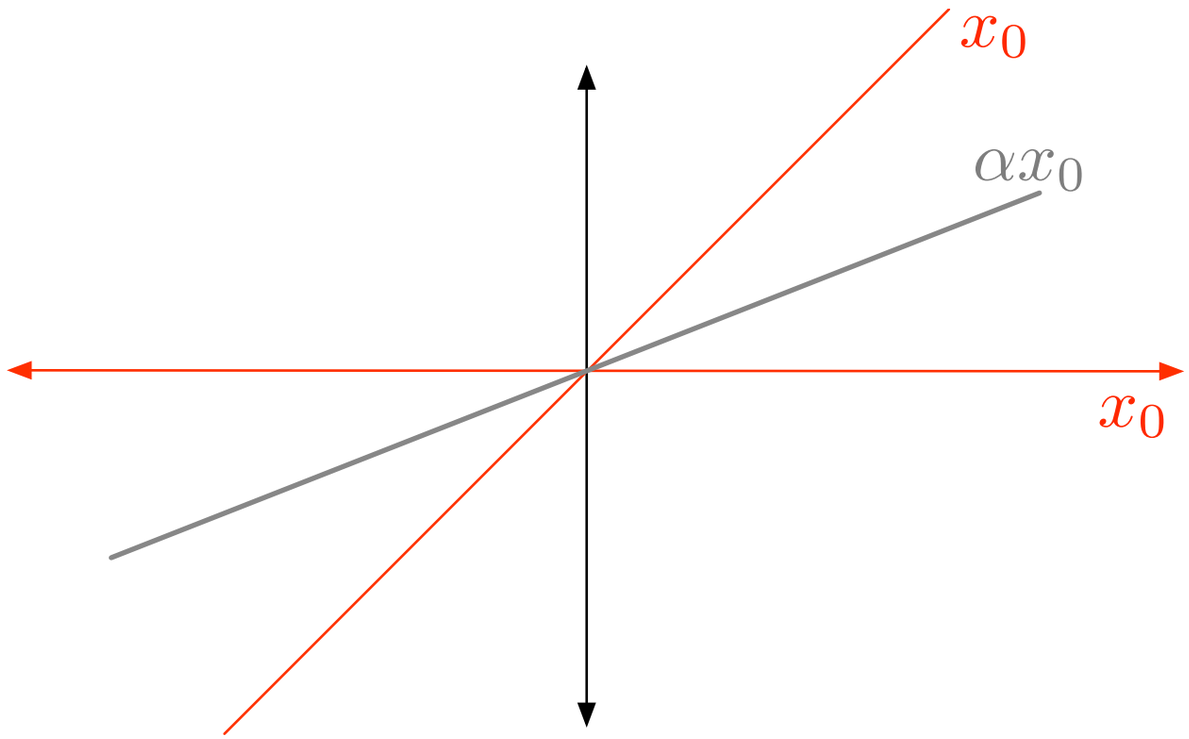

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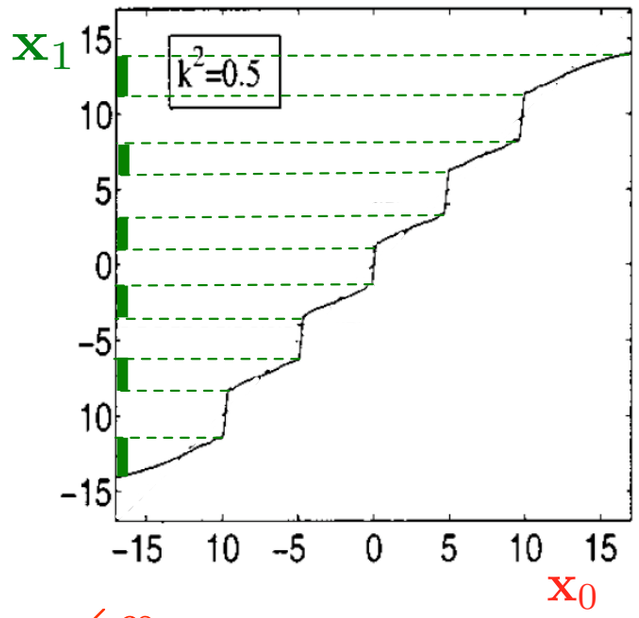

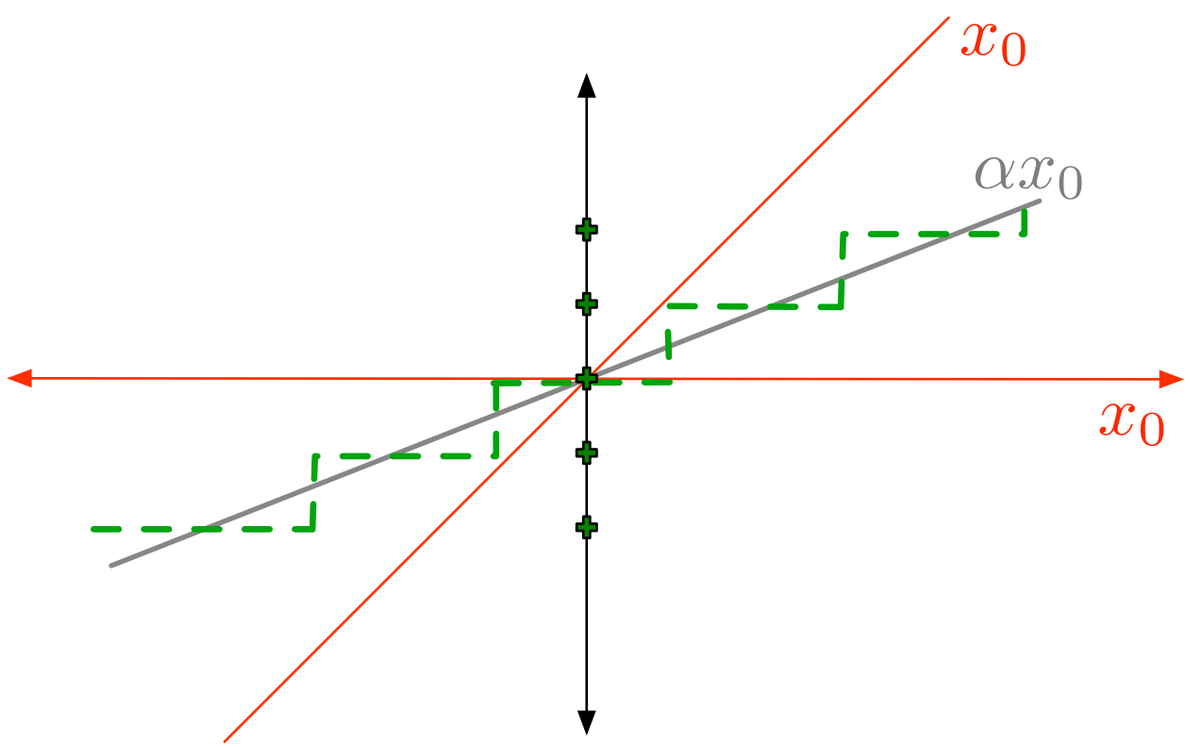

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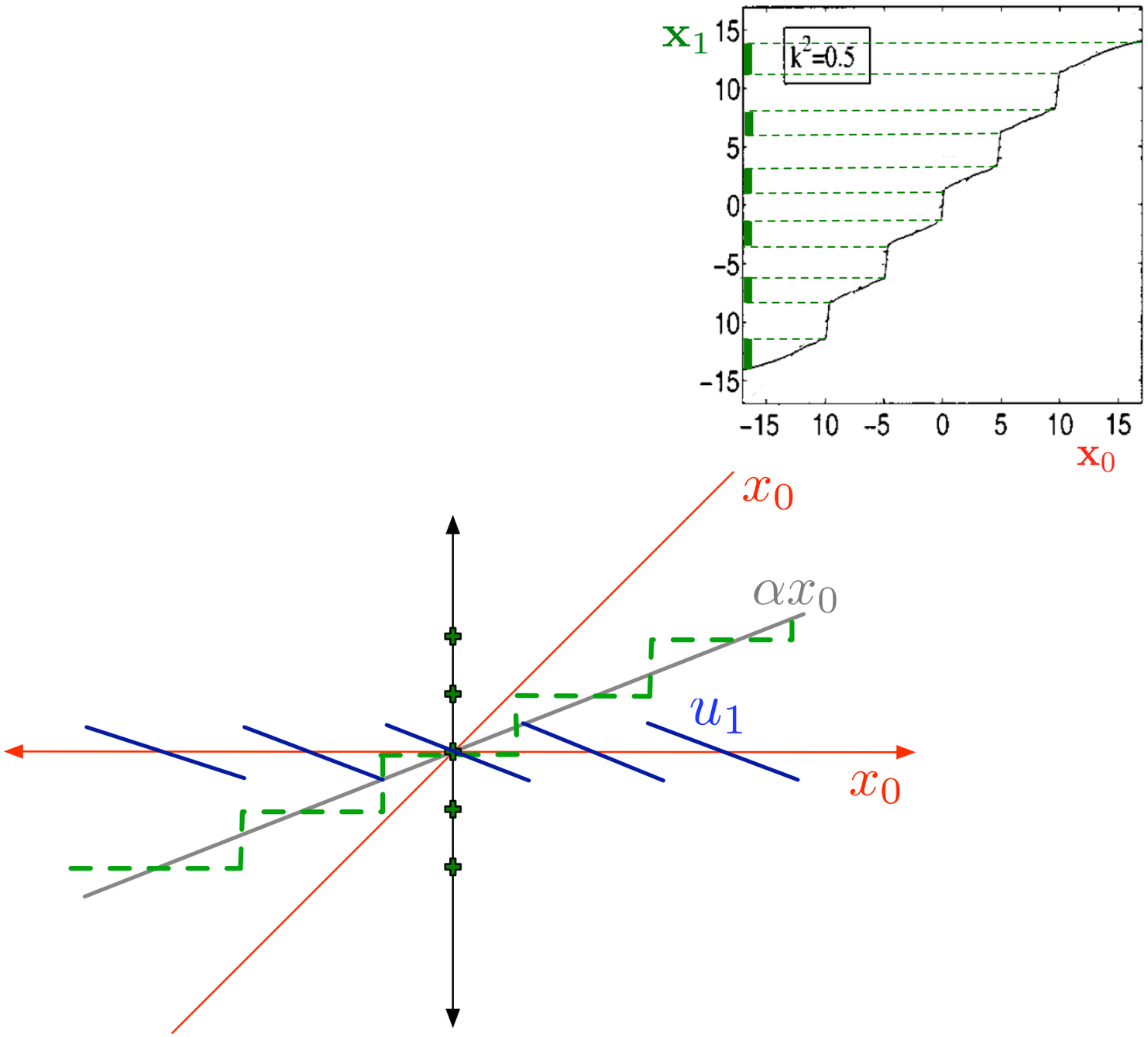

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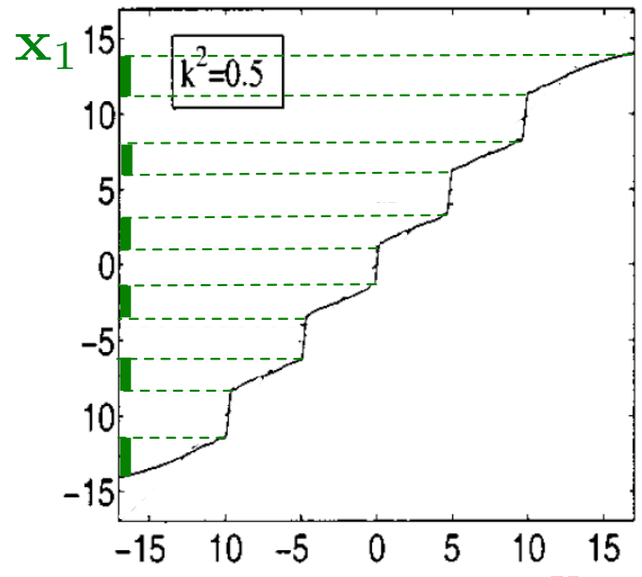
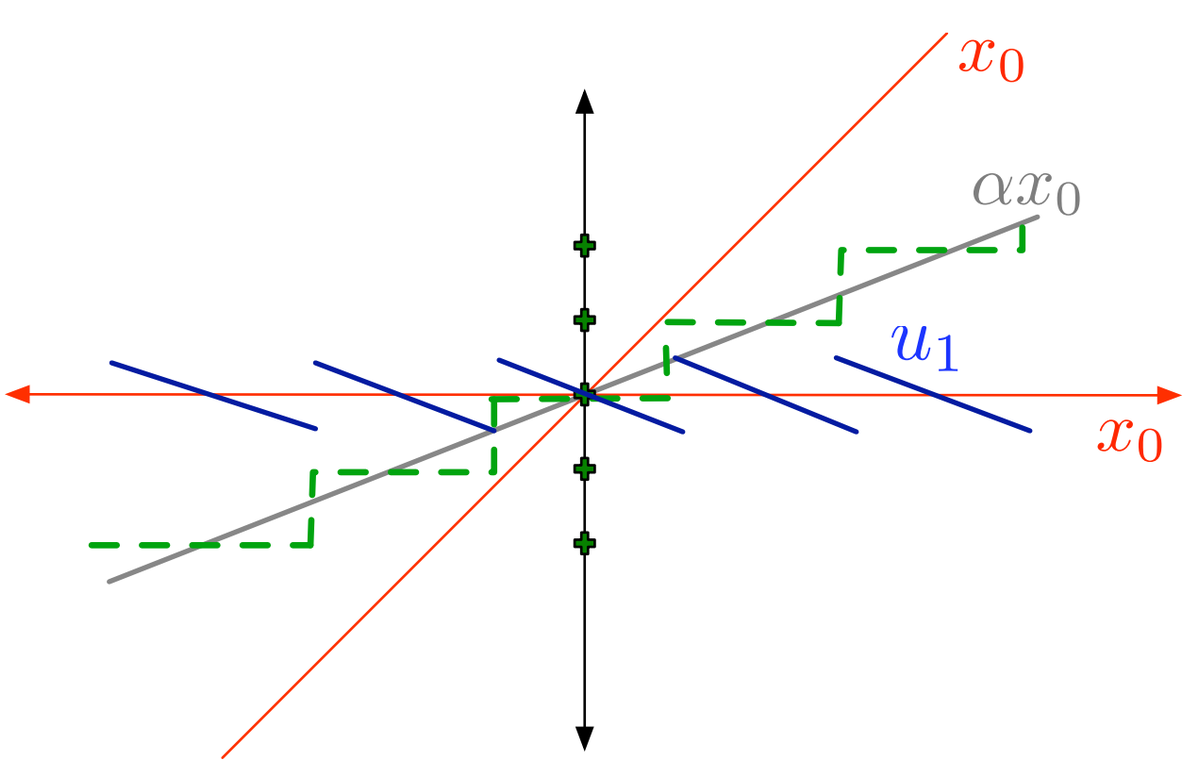
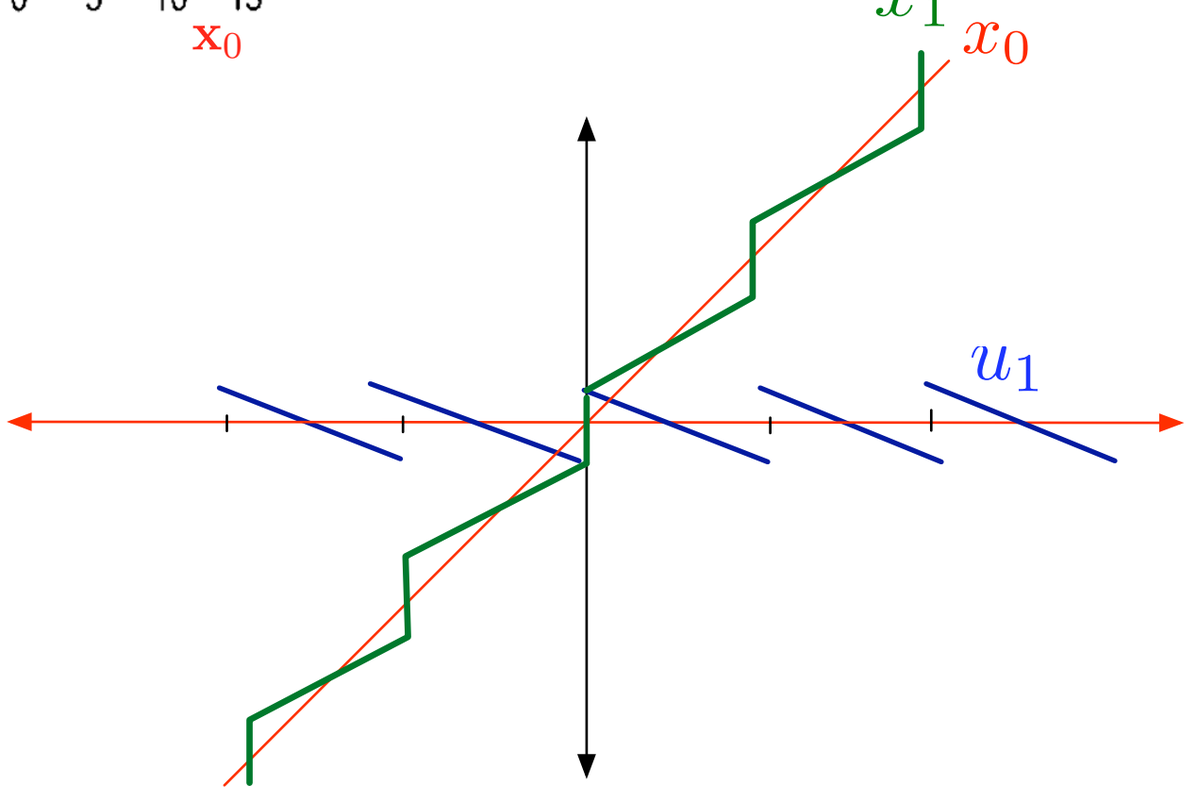
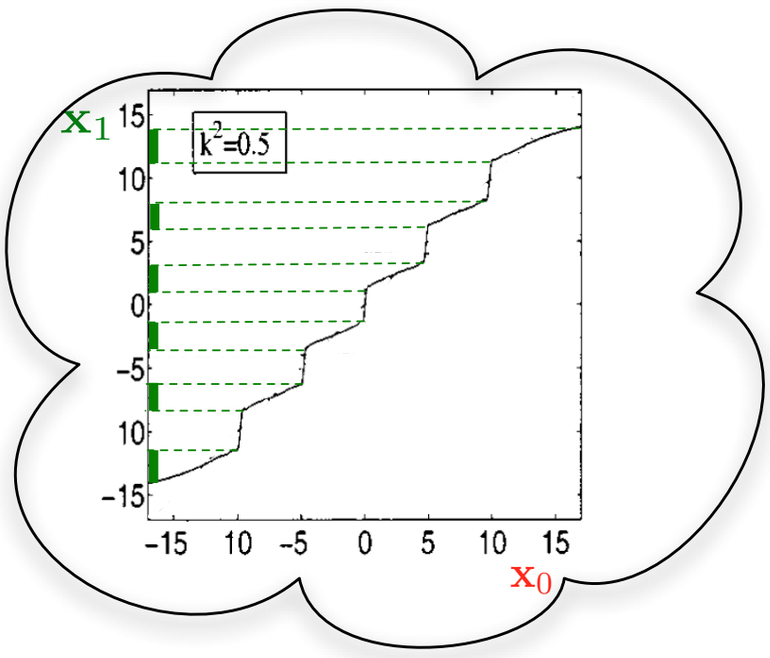






Dirty paper coding based strategy



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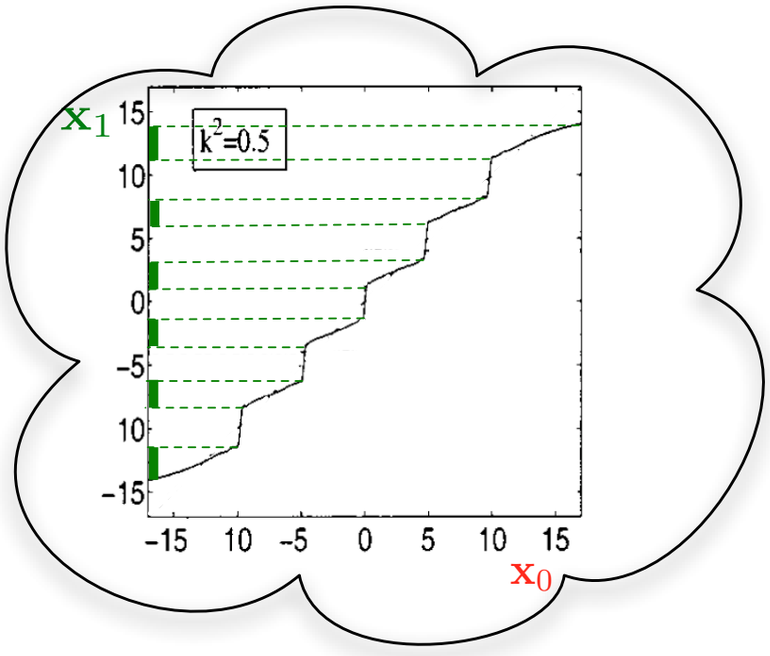

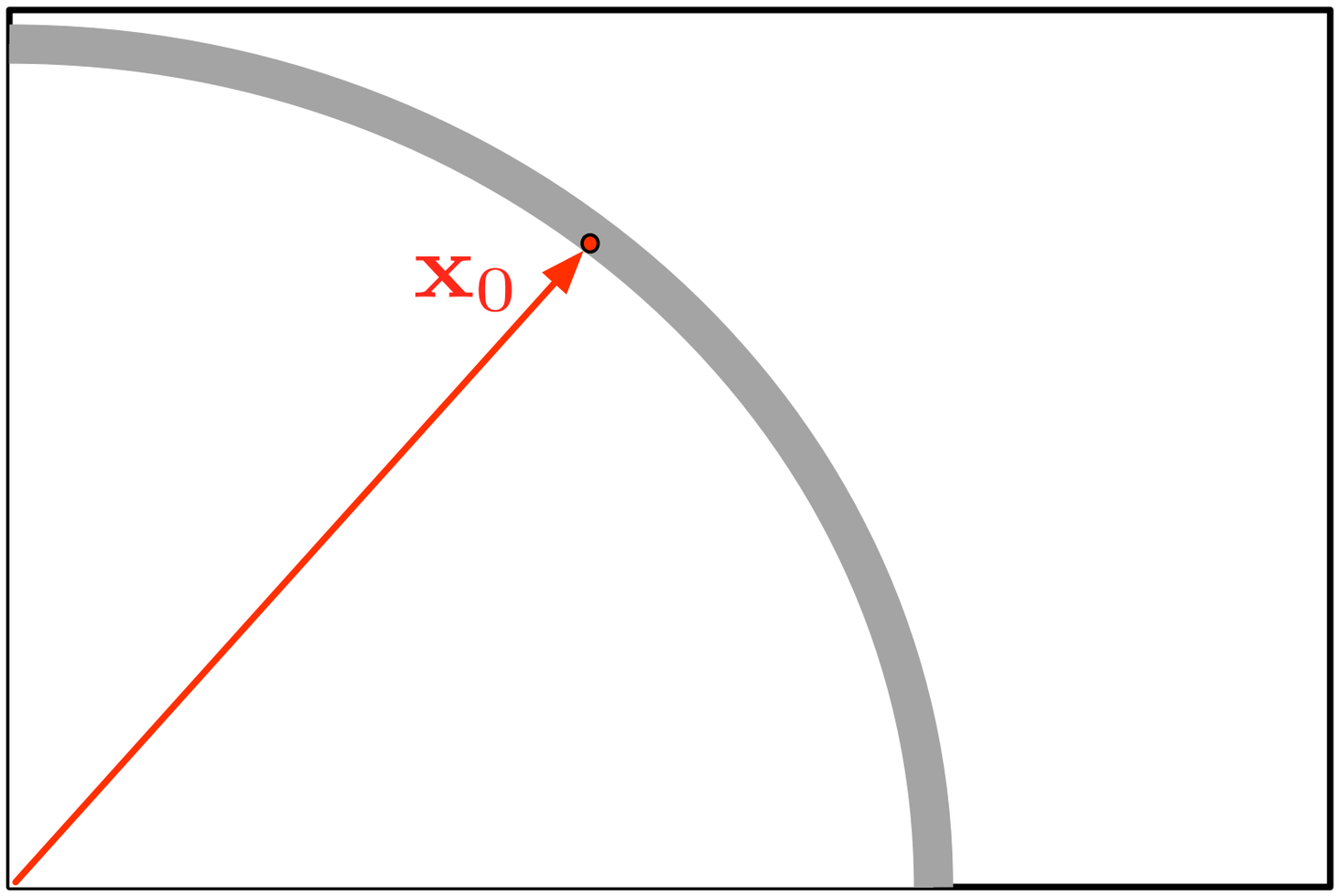

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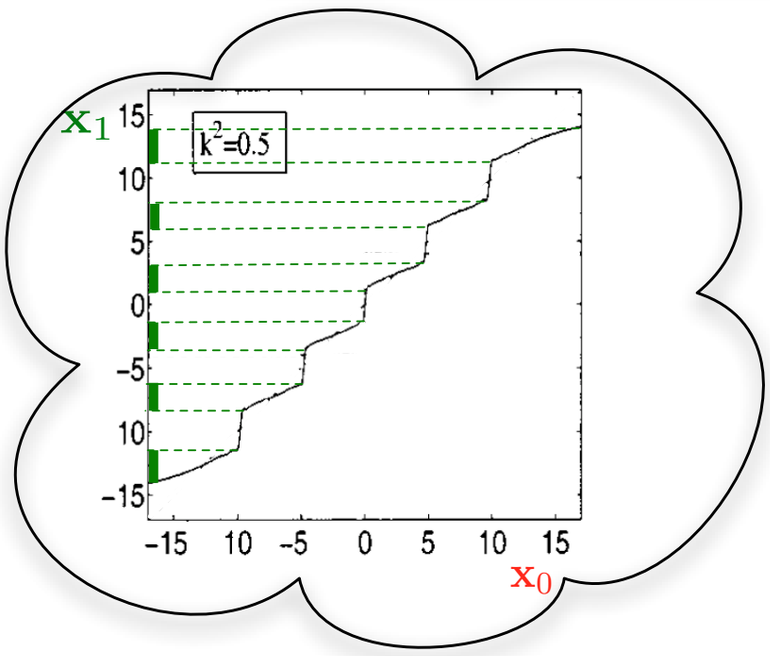
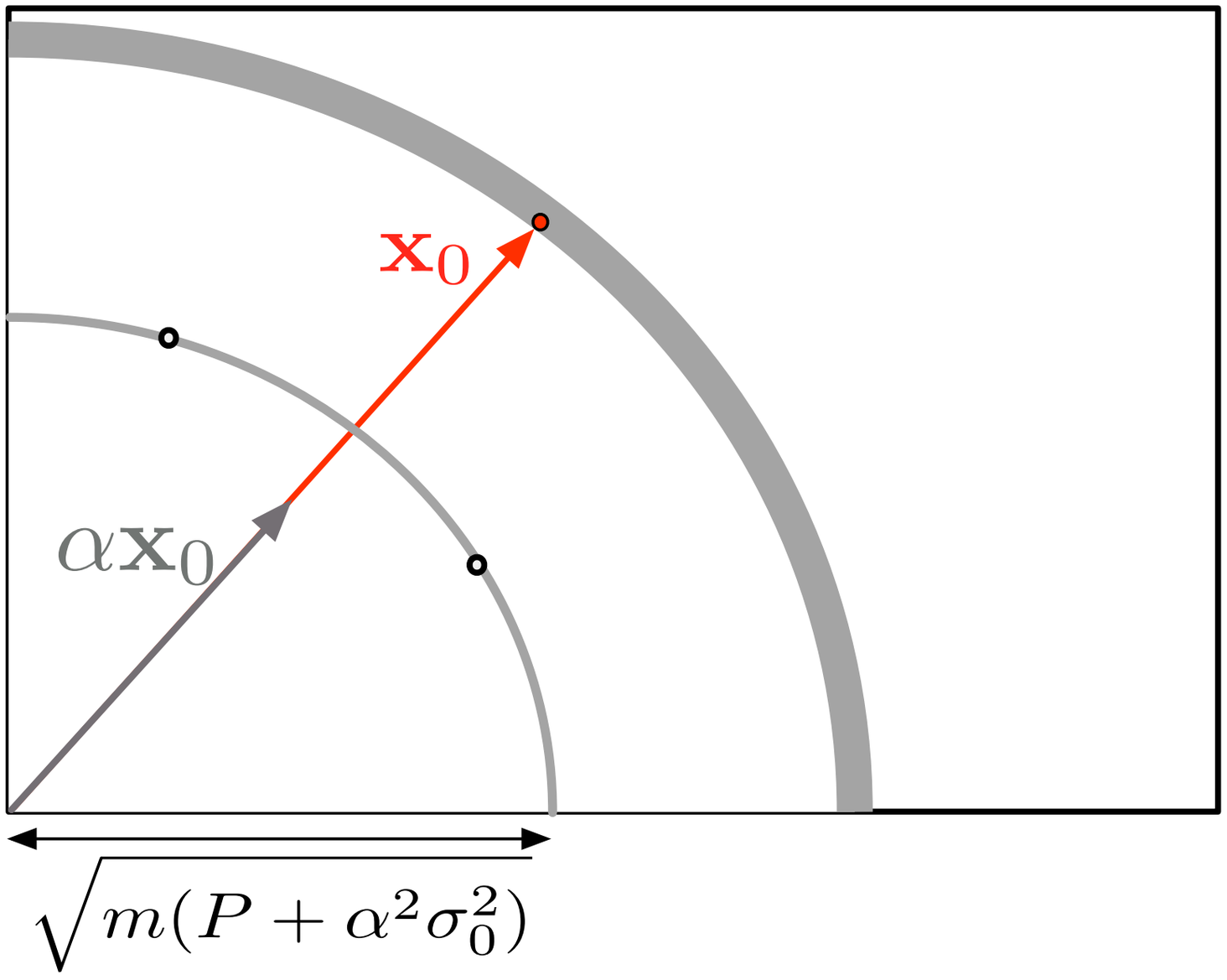
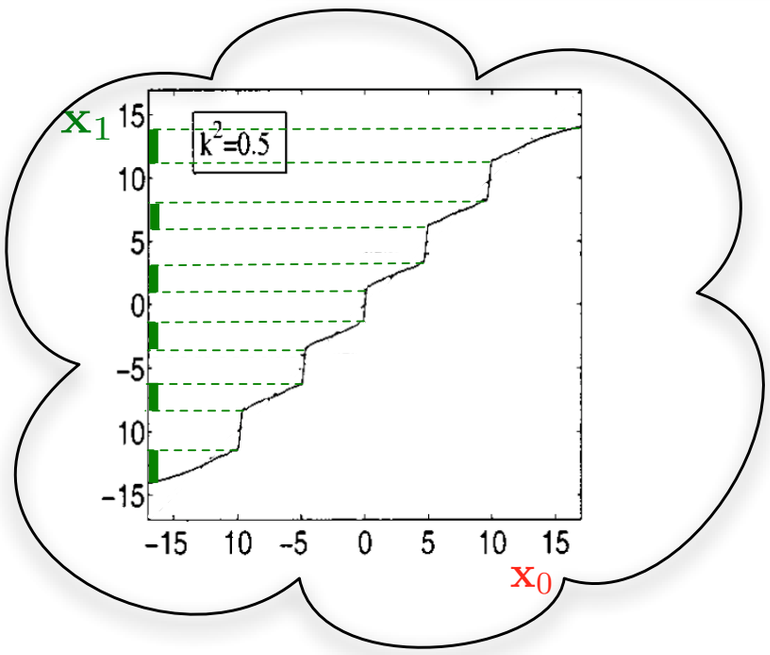
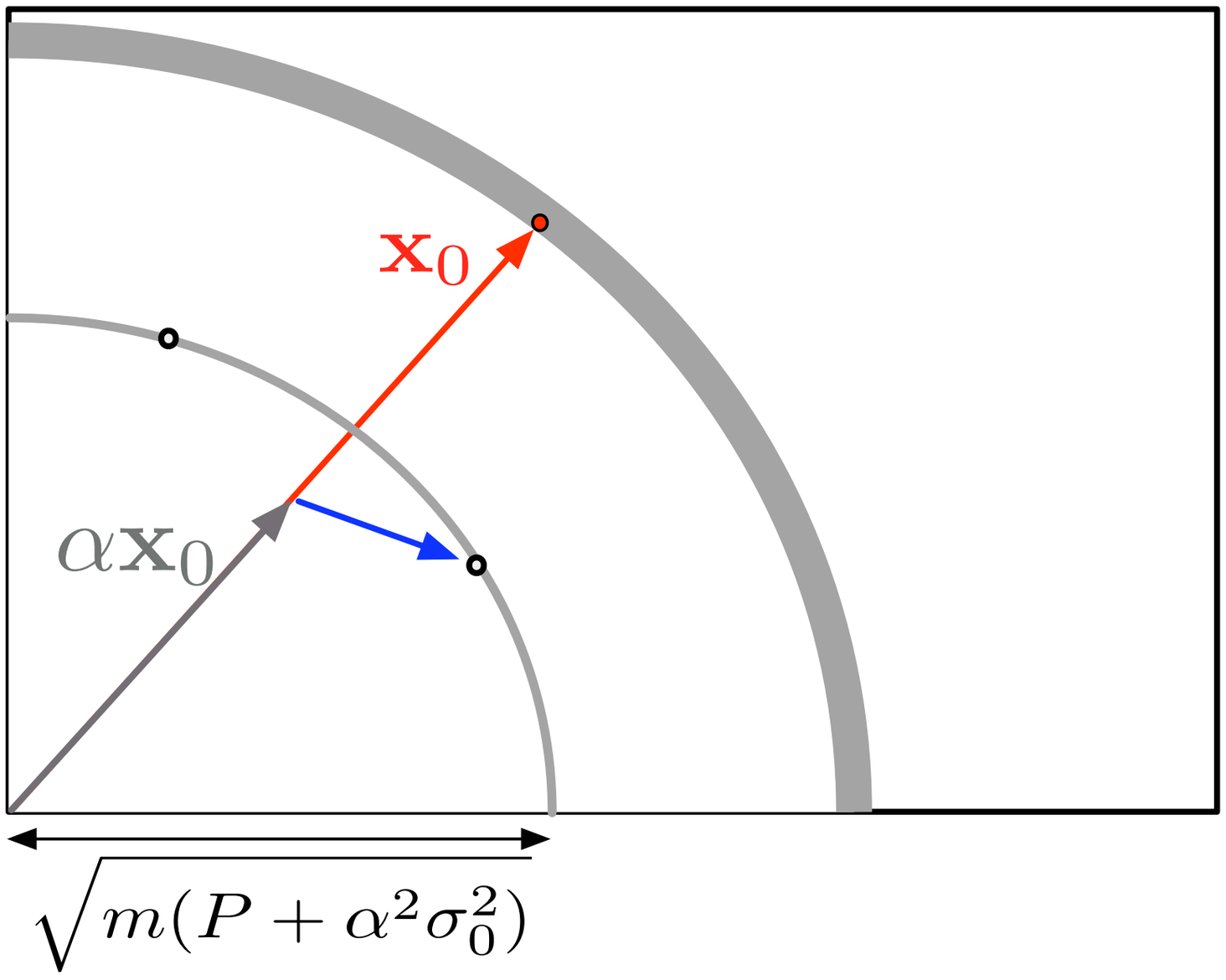


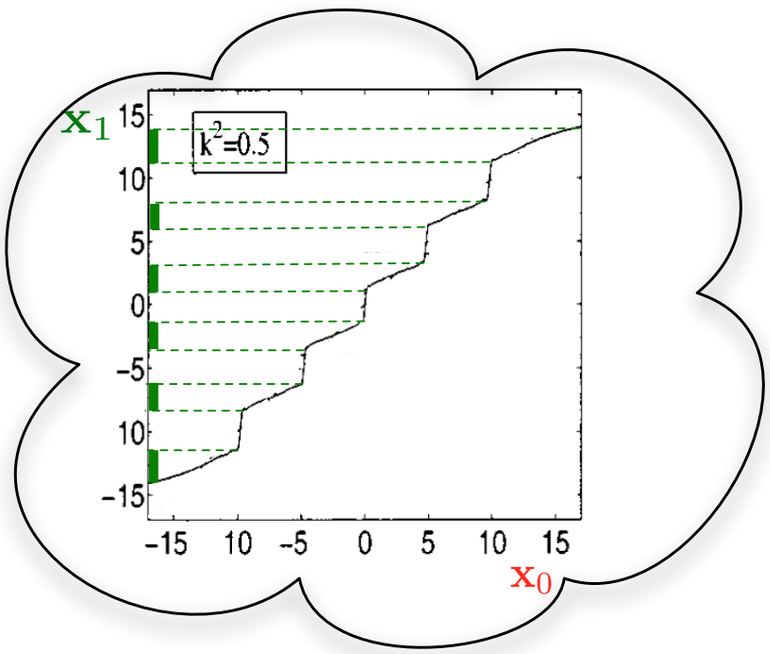


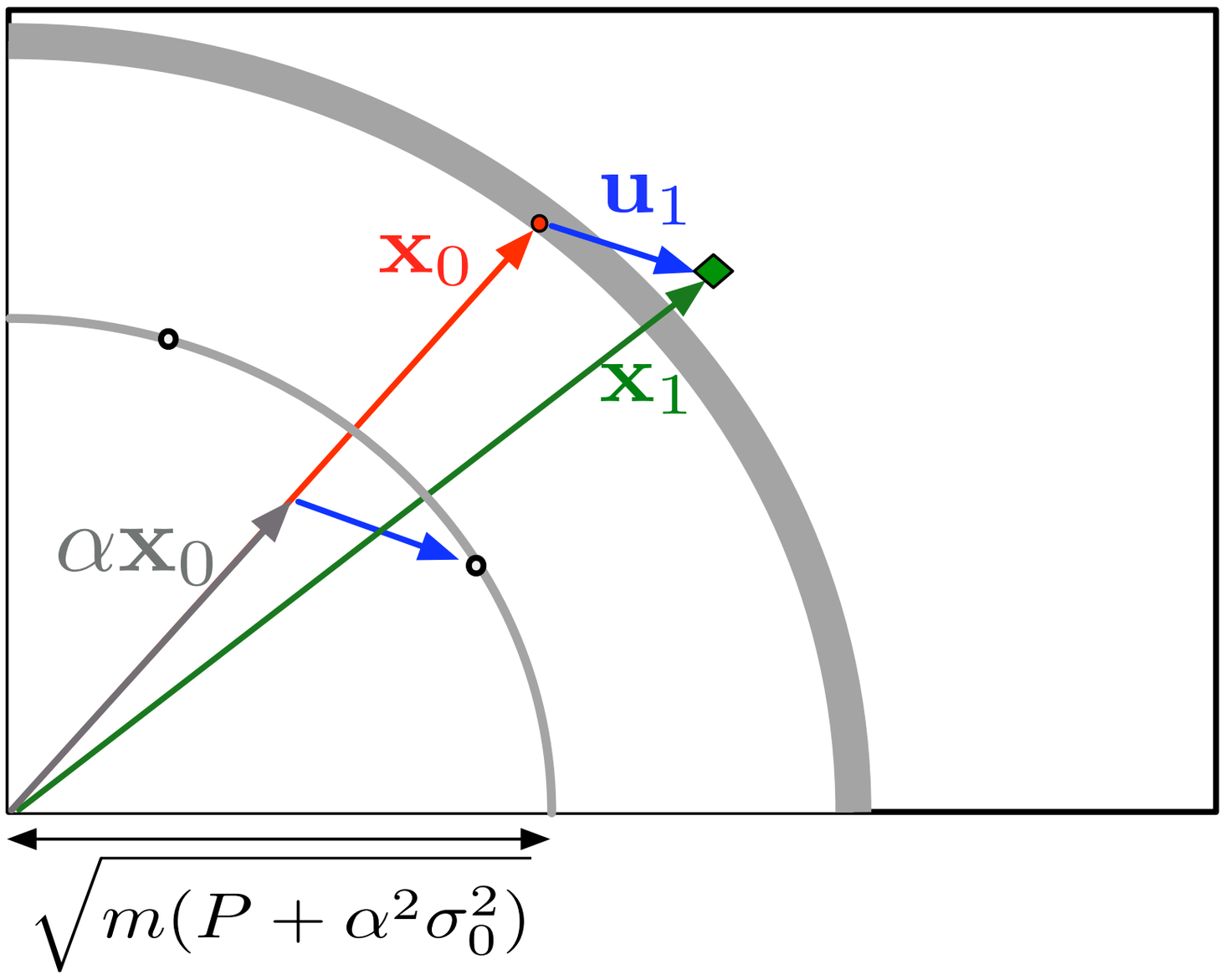
Dirty paper coding based strategy





Dirty paper coding based strategy





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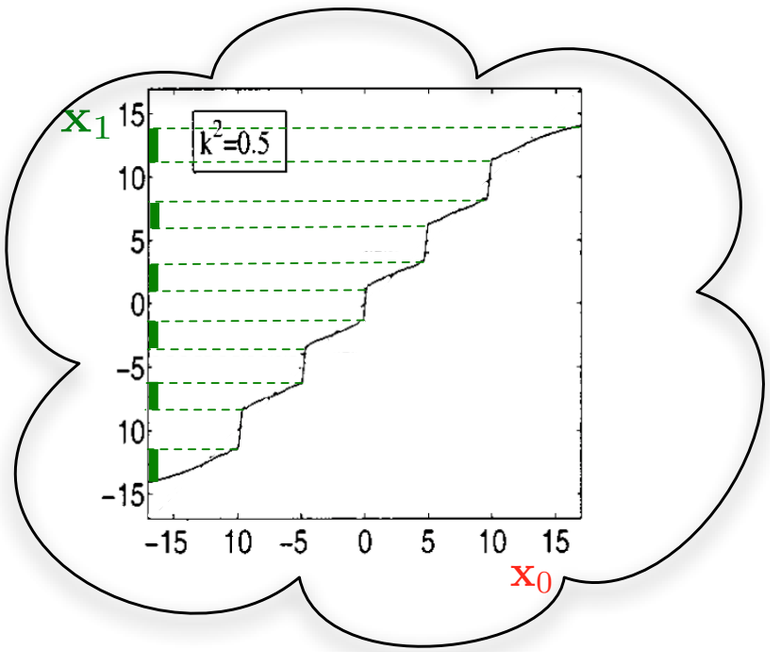
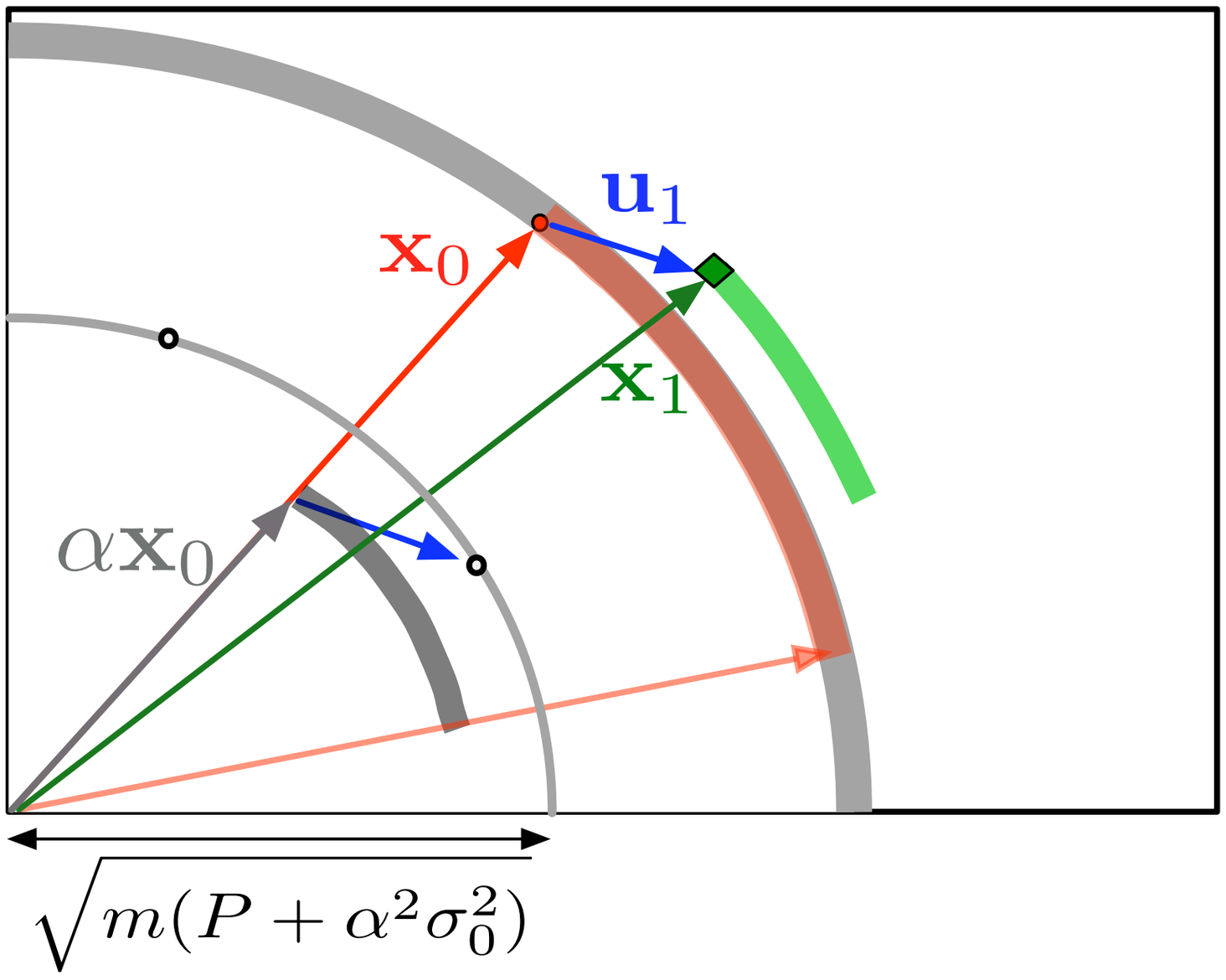
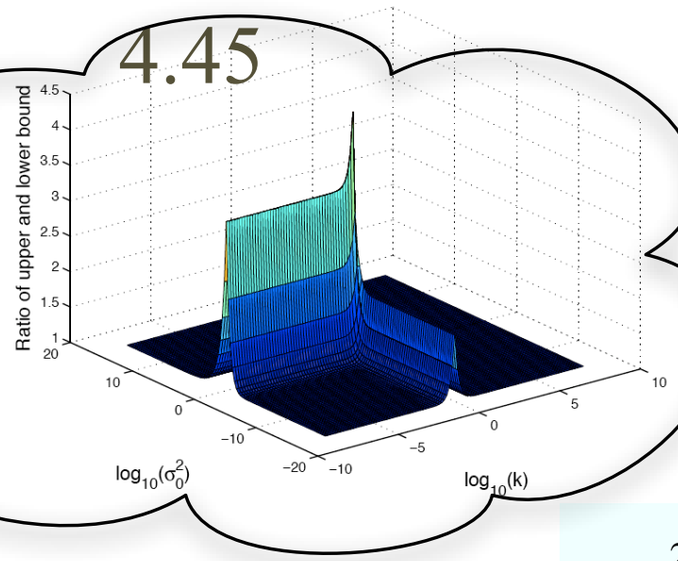


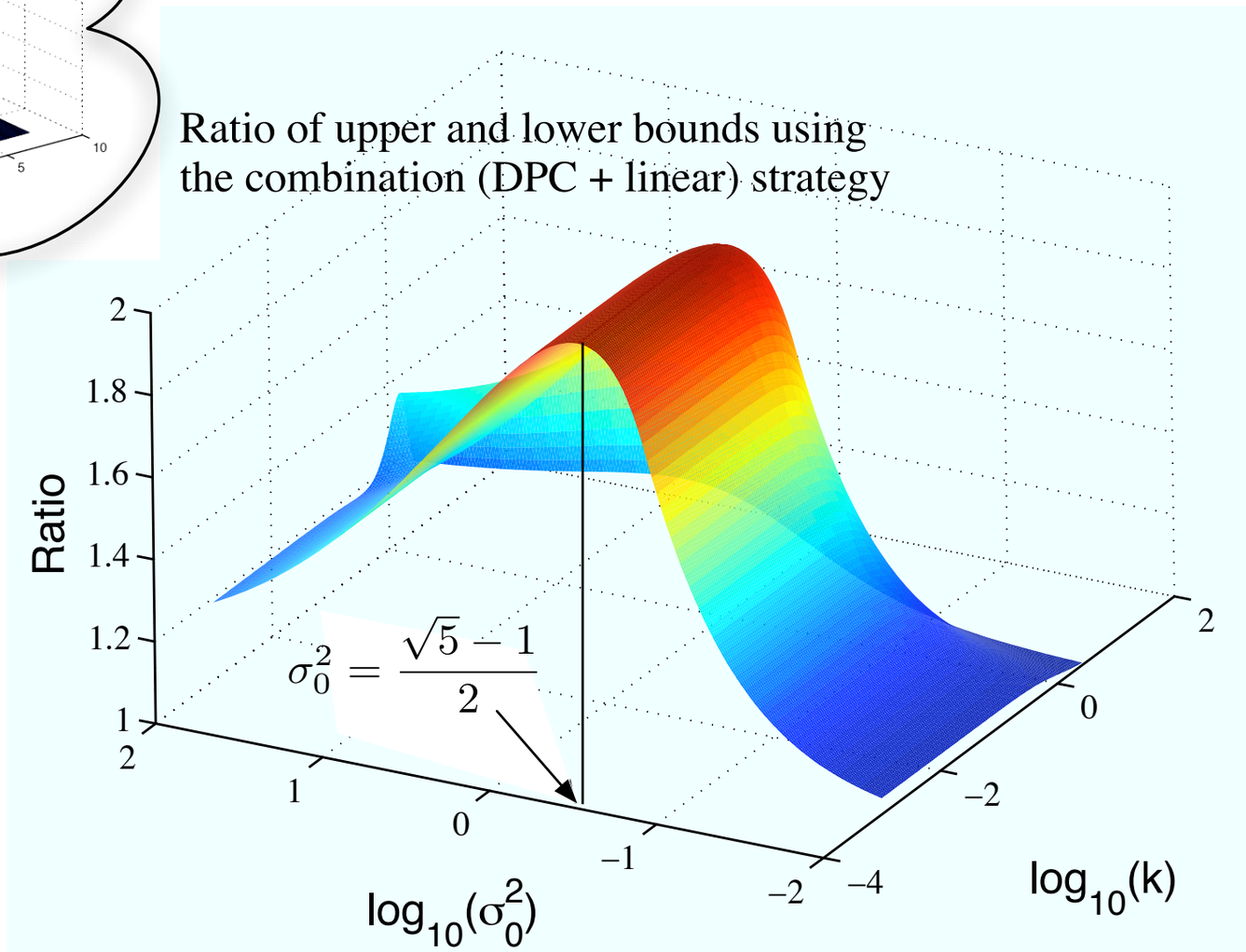


Optimal cost to within a factor of 2 !

4.45

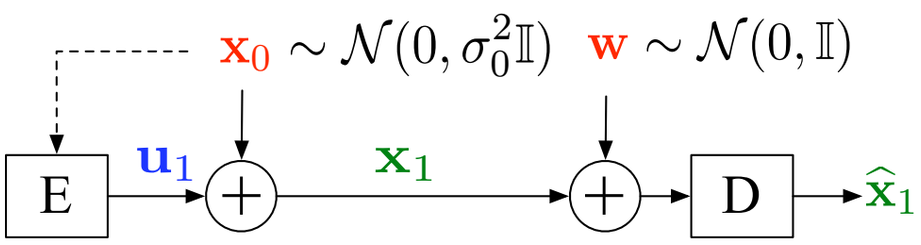


2.00

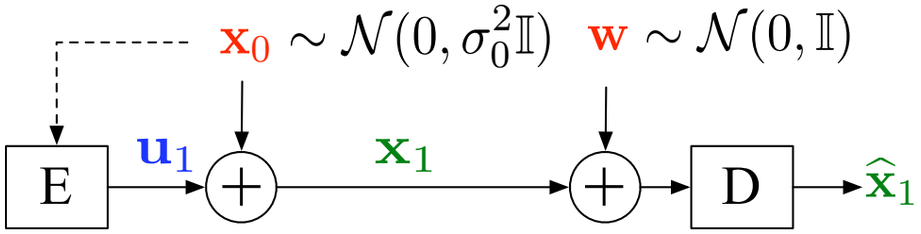


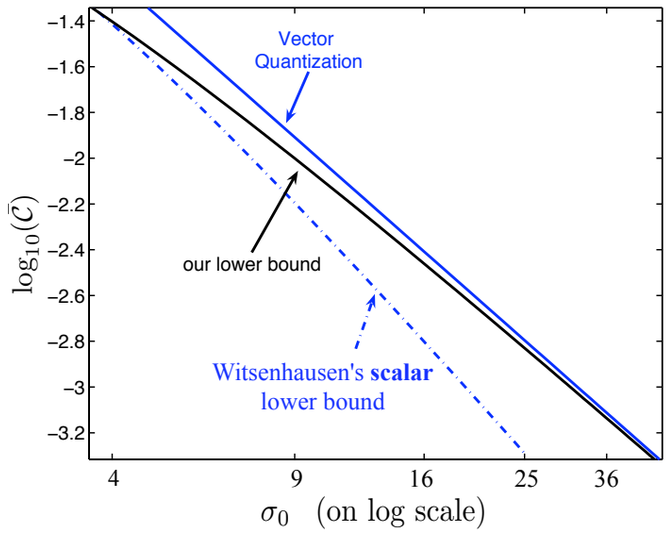
Summary

Summary

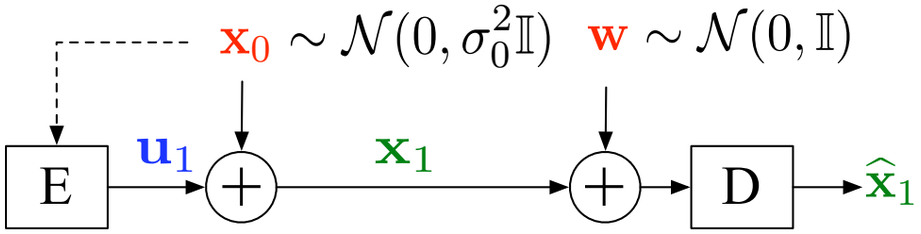

$\mathbf{x}_0 \sim \mathcal{N}(0, \sigma_0^2 \mathbb{I})$
$\mathbf{w} \sim \mathcal{N}(0, \mathbb{I})$
E
$\mathbf{u}_1$
$\mathbf{x}_1$
D
$\widehat{\mathbf{x}}_1$

Summary


$\mathbf{x}_0 \sim \mathcal{N}(0, \sigma_0^2 \mathbb{I})$
$\mathbf{w} \sim \mathcal{N}(0, \mathbb{I})$
E
$\mathbf{u}_1$
$\mathbf{x}_1$
D
$\widehat{\mathbf{x}}_1$


Vector Quantization
our lower bound
Witsenhausen's **scalar** lower bound
$\log_{10}(\bar{\mathcal{C}})$
−1.4
−1.6
−1.8
−2
−2.2
−2.4
−2.6
−2.8
−3
−3.2
4
9
16
25
36
σ_0 (on log scale)

Summary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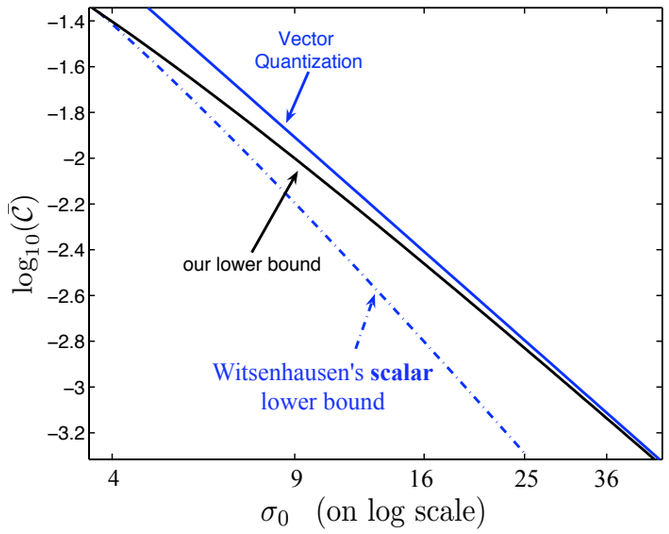
$\widehat{\mathbf{x}}_1$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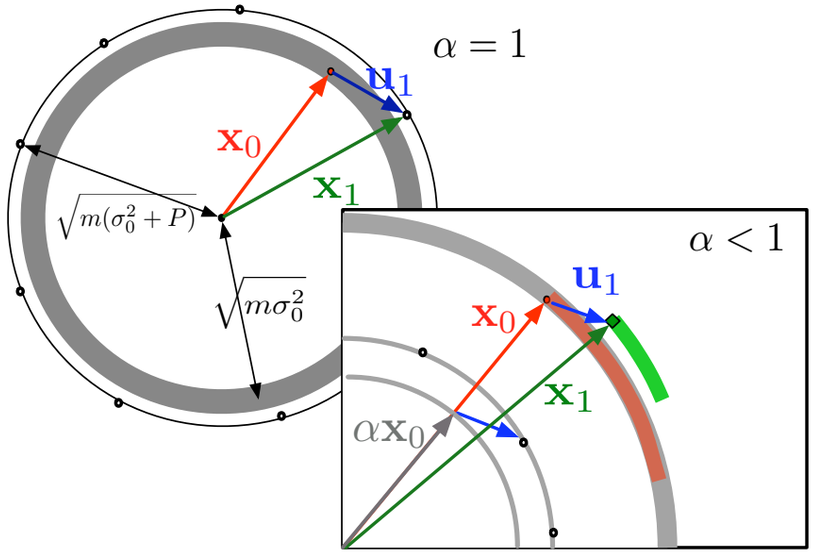
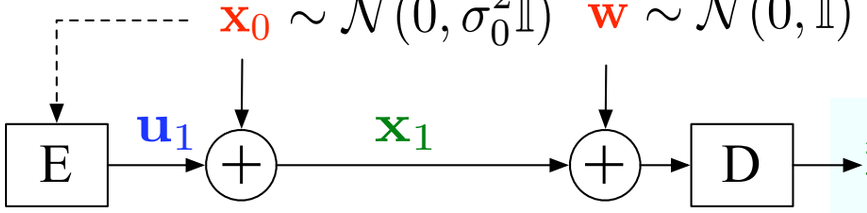
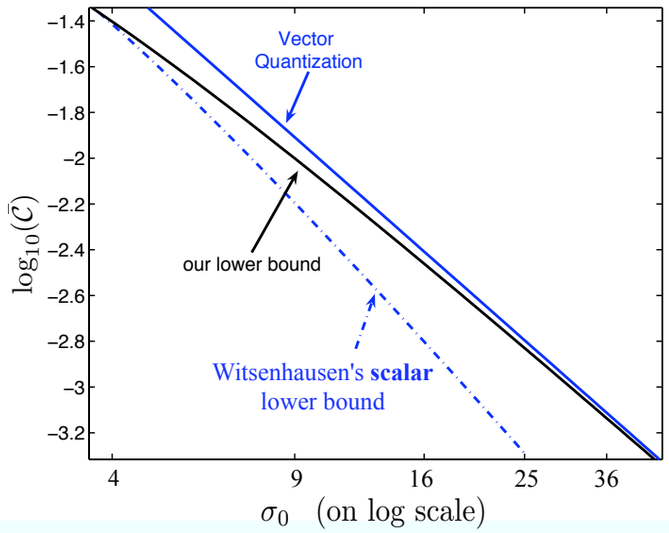
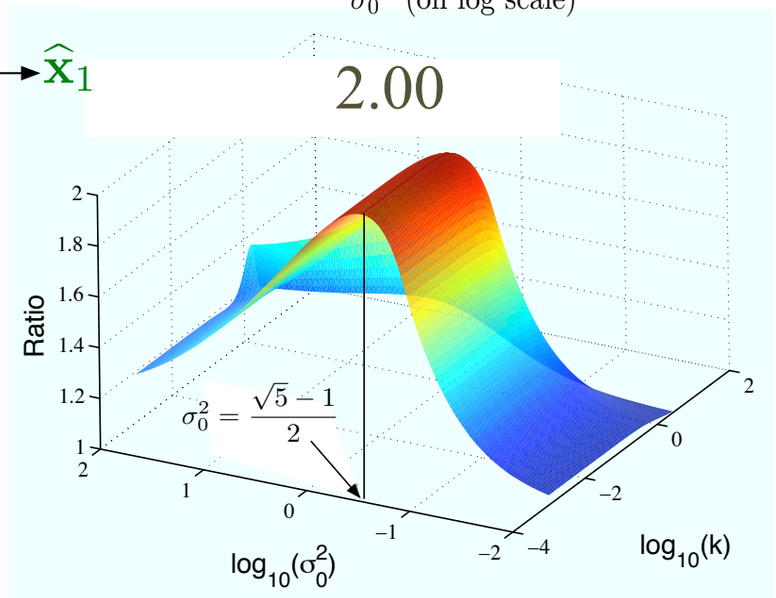
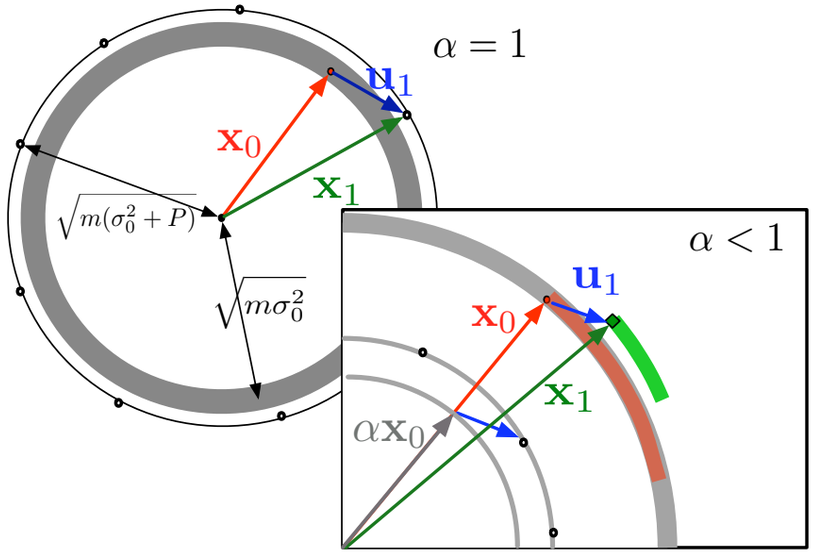

$\alpha = 1$
$\mathbf{u}_1$
$\mathbf{x}_0$
$\mathbf{x}_1$
$\sqrt{m(\sigma_0^2 + P)}$
$\sqrt{m\sigma_0^2}$
$\alpha < 1$
$\mathbf{u}_1$
$\mathbf{x}_0$
$\mathbf{x}_1$
$\alpha\mathbf{x}_0$

Summary

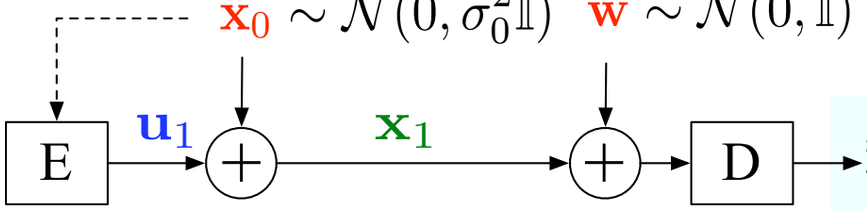

$\mathbf{x}_0 \sim \mathcal{N}(0, \sigma_0^2 \mathbb{I})$
$\mathbf{w} \sim \mathcal{N}(0, \mathbb{I})$
E
$\mathbf{u}_1$
$\mathbf{x}_1$
D
$\widehat{\mathbf{x}}_1$

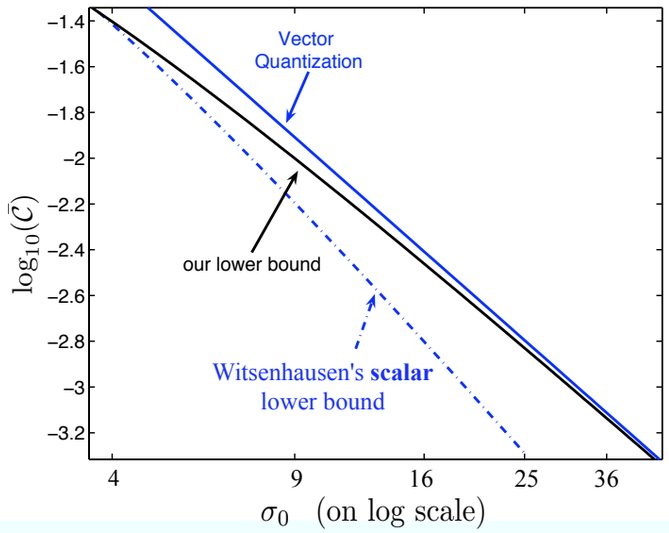

Vector Quantization
our lower bound
Witsenhausen's **scalar** lower bound
$\log_{10}(\bar{\mathcal{C}})$
σ_0 (on log scale)

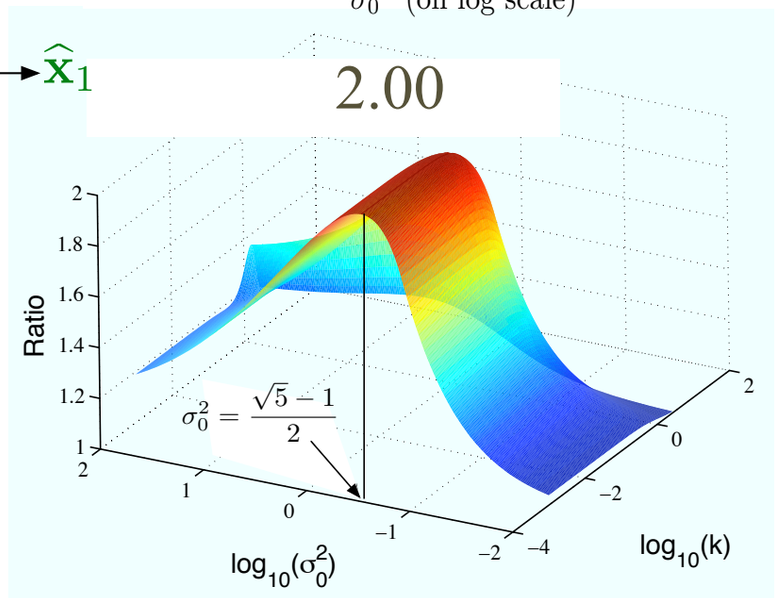

2.00
Ratio
$\sigma_0^2 = \frac{\sqrt{5}-1}{2}$
$\log_{10}(\sigma_0^2)$
$\log_{10}(k)$

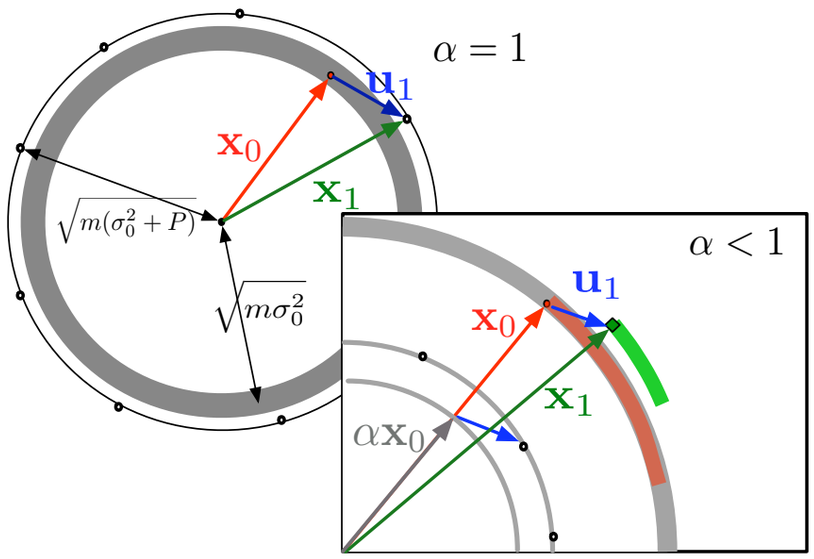

$\alpha = 1$
$\mathbf{u}_1$
$\mathbf{x}_0$
$\mathbf{x}_1$
$\sqrt{m(\sigma_0^2 + P)}$
$\sqrt{m\sigma_0^2}$
$\alpha < 1$
$\mathbf{u}_1$
$\mathbf{x}_0$
$\mathbf{x}_1$
$\alpha\mathbf{x}_0$

Summary









Computational aspects in the proceedings paper

Journal version "Vector Witsenhausen counterexample as Assisted Interference Suppression". Pulkit Grover and Anant Sahai.

Available at : http://www.eecs.berkeley.edu/~pulkit/