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Reducing spectrum use in traditional and SFN-based television for uniform and non-uniform deployments

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ABSTRACT

During the last few years, reclaiming TV spectrum for mobile broadband use has been a hotly debated topic in the telecommunications policy agenda. This paper evaluates two ways to improve spectrum efficiency to today's noise-limited single-transmitter broadcast television approach. One way is to increase the transmit power of each broadcaster's only transmitter. The other way is to replace that single transmitter with a multi-transmitter low-power low-tower (LPLT) single frequency network (SFN). In order to quantify their potential benefits, two different scenarios are considered. First, this paper presents the results obtained for a large region in which broadcasters are uniformly distributed, so that the number of TV broadcasts that can be received is roughly the same at any location. In this case, results suggest that increasing power of traditional single-transmitter broadcasters could reduce the amount of spectrum needed for TV by up to 30%, and would be cost-effective for population densities above 30 per square km. A switch to SFNs could reduce the amount of spectrum needed for TV by roughly 60%, but at a higher cost. Results suggest that the LPLT SFN approach could be cost-effective for regions with uniformly distributed broadcasters and population densities above 120 per square km. The study then quantifies spectrum efficiency gains in regions where broadcasters are not uniformly distributed. In particular, it considers the case where U.S. broadcasters in the UHF band continue to serve their coverage areas as of 2015. In this case, the amount of spectrum that can be freed from TV throughout the entire nation using these two approach is considerably smaller, but some additional bands can be freed from TV throughout much of the nation. Moreover, most of these spectrum gains can be obtained when only a minority of broadcasters change their technical approach.

1. Introduction

Spectrum policies have long minimized the cost per area covered of TV transmission while ignoring the opportunity cost of the spectrum. They do that in two ways. First, by choosing the single-transmitter high-power high-tower (HPHT) traditional broadcasting approach, in which elevated sites have to transmit in the range of tens to hundreds of kW in order to cover a large enough —and economically meaningful— area. Second, by using a very conservative frequency reuse approach by making sure that the distances between broadcasters' coverage areas are so large that the effect of interference at the edge of coverage is negligible.

One alternative to this is the use of low-power low-tower (LPLT) Single Frequency Networks (SFN), where multiple synchronous transmitters send the same signal over the same frequency channel, at much lower heights and transmit power (Lewin et al., 2014;

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CEPT-TG6, 2014; Huschke, Sachs, Balachandran, & Karlsson, 2011; Mattsson, 2005). With LPLT SFNs, the building and operational costs are greater, but so is the potential spectrum efficiency. Also, either with LPLT SFNs or with traditional broadcasting the distance between coverage areas could be reduced while keeping coverage areas the same by increasing transmit power, and thereby tolerating more interference at the edge of coverage. Setting the distance between co-channel broadcasters such that interference is negligible might be the right strategy if the goal is to minimize broadcast stations' transmission costs, and if spectrum is considered to be so plentiful that its cost can be ignored. That may have been the world we lived in when regulatory bodies started granting TV licenses, but it is certainly not the world today (Zander & Mahonen, 2013), which is in part why this work revisits this issue and examines the effectiveness of an interference-limited (rather than noise-limited) approach.

Lately, the use of LPLT SFNs has been actively discussed as a means to reclaim a significant amount of TV spectrum. The first formal proposal to change the U.S. broadcast TV transmission architecture to LPLT SFNs was in 2009 (CTIA, 2009), in the context of the 2010 U.S. National Broadband Plan (NBP) (FCC, 2010a). However, LPLT SFN deployments will only become practical in the short term once the new Advance Television System Committee (ATSC) 3.0 TV transmission standard is adopted (ATSC-PT2, 2011; ATSC-TG3, 2013). In Europe, LPLT SFNs has also been considered as a possible way to reclaim a significant amount of spectrum in the discussion of the future of the UHF TV band (Lamy, 2014; CEPT-TG6, 2014; Lewin et al., 2014).[1] Although some forms of SFNs have already been deployed in Europe for a few years, these SFNs have been of the HPHT type (Li et al., 2015; Malmgren, 1997; Meabe, Gil, Li, Velez, & Angueira, 2015; Rebhan & Zander, 1993), as opposed to the more spectrum efficient cellular-like LPLT SFNs considered here and in previous related literature (See Section 2).

This work quantifies the effectiveness of either boosting transmit power or switching to LPLT SFNs to increase spectrum reuse without significantly changing the population served or technical capabilities of current U.S. TV broadcasters. One of the ways this work differs from the existing literature, e.g. (Huschke et al., 2011; Shi, Sung & Zander, 2014; Shi, Obregon, Sung, Zander, & Bostrom, 2014), is that it assumes broadcasters keep the same spectrum licensing model with the same exclusive access rights to their own frequency channel. It is only by comparing spectrum efficiency of LPLT SFNs and traditional broadcasts when both offer the same coverage areas and bandwidths that all of the results of this work are because of the transmission upgrade, and not because of other factors such as bitrate management, source coding, carrier aggregation, etc. Moreover, while this is not the only possible scenario, it is certainly a realistic one, since this is the scenario that requires the fewest changes in business strategy for broadcasters and in spectrum policy for regulators.

The cost-effectiveness of these alternative technical approaches will depend on how the number of broadcasters is distributed across the region under consideration. Thus, this work considers two cases. In the first, this work assumes that all TV service areas are the same size and broadcasters are uniformly distributed, i.e. the number of TV signals that a viewer in any location can receive is roughly the same at any location throughout a region that is much larger than a TV service area. Some parts of the U.S. and other nations may contain broadcasters that are roughly uniformly distributed, but this is certainly not the case everywhere. In a second case, this work considers a highly heterogeneous deployment. To represent such a case, this work uses the Continental U.S., considering all TV stations in the UHF band. Here, the number of UHF TV stations varies tremendously: from 2 to 3 in the smallest of the 210 U.S. TV markets, to 22 in the (#1) New York Market. In the U.S., higher population density means more advertising revenues per square km served, which means that more broadcasters can operate at a profit in such areas. Also note that TV spectrum assignment in the U.S has followed a first-come first-served site-based licensing approach, where a license can be granted in any place as long as spectrum is available.[2] By comparing and contrasting the results of these two cases, this work will discuss insights and implications of the differences in the effectiveness of the technology.

To analyze the uniform deployment case, this paper develops a theoretical model that considers a single frequency channel that is used by multiple identical broadcasters, each of which serves coverage areas of equal size through either a traditional single-transmitter approach or by a LPLT SFN. The model assumes this band is used only for TV, so it measures the extent and efficiency of the spectrum use as the fraction of total area that falls within a TV coverage area. The model shows that both alternatives may increase spectrum efficiency, and it calculates the minimum average population density at which the value of spectrum freed would exceed the cost of changing technology. For doing this, we assume that a traditional television station is in place, and estimate the net present value (NPV) of both the cost of upgrading broadcasting infrastructure, if any, and the change in long-term operating costs when using a cost-minimizing design for each broadcaster. We do not consider the cost to consumers who might need to purchase a converter box, in part because the cost of converters would be negligible when compared to the other costs, as shown in (Bettancourt & Peha, 2015). Moreover, a shift to LPLT SFNs in the U.S. could accompany the adoption of ATSC 3.0, which would require TV households to invest in TV receivers or converter boxes regardless of whether LPLT SFNs are adopted or not.

To analyze the non-uniform deployment case, a different approach and model is used. Here, the number of channels that must be assigned to at least one TV broadcaster somewhere in the nation will be determined in large part by the needs of the few cities with the most TV stations, making it more difficult to clear channels nationwide. Moreover, coverage areas also vary considerably from station to station, so the model uses the actual coverage area sizes and locations of UHF TV broadcasters. When analyzing the effect of increasing transmit power, the model assumes that powers of all broadcasters are increased by the same number of dB. This gives a useful bound on effectiveness, although it is clear that a more complex algorithm could be even more cost-effective. For the case of LPLT SFNs, the model similarly assumes that all broadcasters switch to LPLT SFNs. A modified version of U.S. Federal

[1] In contrast to the U.S., LPLT SFNs are already possible in some parts of Europe that have already adopted the Digital Video Broadcasting Second Generation Terrestrial Standard (DVB-T2).

[2] In this way, current allocations may reflect what fits the traditional noise-limited technical approach rather than what is desired by TV broadcasters. Thus, the assumption that coverage areas remain unchanged will somewhat favor the noise-limited approach.

Communications Commission (FCC) open-source propagation and interference analysis software (FCC, 2016b, 2015b, 2016a) is used, along with a proven efficient SAT Boolean formulation approach (Frechette, Newman, & Leyton-Brown, 2016; Kearns & Dworkin, 2011; Muthukumar, Daruna, Kamble, Harrison, & Saha, 2015) to repack the stations after transitioning to either LPLT SFNs or traditional broadcasters operating at higher transmit powers.

The paper is organized as follows. Section 2 contrast this work with the most relevant literature and previous work about freeing TV spectrum by using LPLT SFNs. Section 3 analyzes the uniform deployment case, including model development and numerical results. In a similar way, Section 4 analyzes the non-uniform deployment case. Section 5 discusses the main insights, conclusions and policy implications of this work.

2. Previous work

Previous studies have proposed a more efficient use of the TV spectrum by changing broadcasters' transmission architecture (CTIA, 2009; Huschke et al., 2011; Shi et al., 2014b, 2014). In 2009, the Cellular Telephone Industries Association (CTIA) and the Consumer Electronics Association (CEA) proposed to the U.S. Federal Communications Commission (FCC) an ATSC 8VSB based switchover to LPLT SFNs (CTIA, 2009). The proposal suggested that with LPLT SFNs the spectral separation distances between TV stations' service areas can be reduced, and this could free up spectrum by repacking them into a smaller portion of the existing TV band. This, without significantly changing the coverage areas or spectrum rights of existing TV broadcasters. The revenue obtained from auctioning off the spectrum freed to Mobile Network Operators (MNOs) would then be used to pay broadcasters' the increased cost of the upgraded network architecture. However, neither cost nor benefits of such a switchover were quantified. This work quantifies these LPLT SFN co-channel separation distances and the potential impact on spectrum efficiency and cost under the same assumptions. Moreover, the ATSC 1.0 8VSB modulation was not be the best for an SFN operation anyway (AMST, 2010; El-Hajjar & Hanzo, 2013; Mattsson, 2005) —particularly for the LPLT case— so this paper assumes the use of an OFDM-based transmission (3GPP, 2008; Digital Video Broadcasting, 2012; El-Hajjar & Hanzo, 2013), consistent with the upcoming ATSC 3.0 (ATSC-PT2, 2011; ATSC-TG3, 2013; ONE Media LLC, 2014).

Other studies have also considered the use of LPLT SFNs, but resembling an industry structure that is different from what we have today in the U.S. and many parts of the world (Huschke et al., 2011; Shi et al., 2014a, 2014b). For example, today in the U.S. each broadcaster operates within its own spectrum block, and serves whatever area can be reached by its transmitter. This is quite different from the way it has evolved in many parts of Europe, where the functions of broadcast transmission and content production/aggregation are not integrated, so TV broadcasters seek carriage over the so-called network multiplexes (Machet, 2010). Similar to this, in Huschke et al. (2011) all broadcast stations in a given region operate over a single provider with an LPLT SFN infrastructure and a block of spectrum much larger than 6 MHz that is in effect shared by all broadcasters. This contrast with the licensing approach in the U.S. and other parts of the world where TV stations have exclusive access to its frequency channel.

In the European context, several studies have recently discussed the future of the UHF band and the possibility of a progressive re-farming with the possible use of LPLT SFNs (Lamy, 2014; CEPT-TG6, 2014; Lewin et al., 2014). In Lewin et al. (2014) a cost-benefit analysis (CBA) on the merit of a converged platform in which mobile and terrestrial TV broadcast services share common infrastructure and UHF spectrum is carried out. The analysis assumes a uniform distribution of broadcasters as the GE06 agreement (RRC, 2006) grants equitable access to broadcast spectrum across license areas. It concludes that the incremental benefits and costs of a switchover are still uncertain. In particular, that such a platform would generate substantial network and consumer costs with no guarantee in terms of the spectrum's auctionable value. A new review is recommended within the next 3–5 years, alongside with further technical and economic analysis. Several stakeholders' reports were produced (Qualcomm/ATDI, 2014; Nygren, 2014; Brugger & Schertz, 2014a, 2014b; EBU, 2014a, 2014b), however, there is no widely accepted estimates. so this paper aims to address some of these open issues.

In Huschke et al. (2011), the authors sought to calculate the minimum amount of spectrum that would be needed for TV in the U.S. after a switch to LPLT SFNs. They took nearly the opposite approach; instead of assuming that each TV station must content with a half dozen neighbors of identical coverage area size and technology, they assume that a broadcaster has no neighbors at all. They conclude that today's TV needs could be met by LPLT SFNs with only 85 MHz. Their result may serve as an informative lower bound, but it is based on a number of optimistic assumptions, including the assumption that all of the variable-bit-rate video streams offered today by competing broadcasters in a given city over their own spectrum could be statistically multiplexed within a single wideband channel, and that one can calculate the amount of spectrum needed in the city with the most broadcasters as if there were no broadcasters outside the city that impose co-channel or adjacent-channel constraints. Thus, by including all of 1711 UHF TV broadcasters nationwide in our model, this paper also differs significantly with the work reported in Huschke et al. (2011). Moreover, as noted above, by comparing spectrum efficiency of LPLT SFNs and traditional broadcasting when both offer the same coverage areas and bandwidths, the results of this work set apart the specific effect of the transmission upgrade.

3. Uniformly distributed broadcasters

As mentioned above, in a first case this paper analyzes the effectiveness of either boosting transmit power or switching to LPLT SFNs to increase spectrum reuse with a model that assumes broadcasters are uniformly distributed, i.e. the number of TV signals that a viewer can receive is roughly the same at any location.

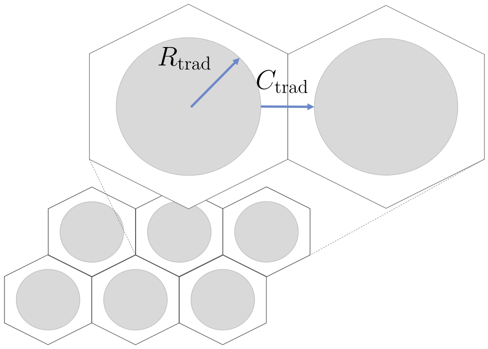

Fig. 1. Hexagonal packing of co-channel traditional broadcasters.

3.1. Model formulation

In this section, this work calculates the maximum spectrum efficiency that can be obtained when identical broadcast networks are deployed in a frequency channel, for both traditional broadcasters and LPLT SFNs. The model measures the extent and efficiency of the spectrum use as the fraction of total area that falls within a TV coverage area. Specifically, it defines spectrum efficiency as the maximum achievable fraction of area covered over a region that is much larger than the coverage area of a single broadcaster (Bettancourt & Peha, 2015). With this definition, spectrum efficiency is then, in general, a function of both the size of the broadcasters' coverage areas and the separation (reuse) distance between those areas. For both technical alternatives, broadcasters' service area size remains constant and is an input to this model. The model shows, for a given required coverage area size, how an increase in infrastructure cost can lead to decrease the separation distance between coverage areas, and thus potentially increase spectrum efficiency by packing broadcasters closer together. For traditional broadcasters, the model obtains the minimum separation distance for a given (increased) transmit power, while for LPLT SFNs, the model obtains the minimum separation distance as a function of the SFN's inter-site distance (ISD).

3.1.1. Spectrum efficiency

Broadcasters are packed in a regular hexagonal tessellation in order to achieve the highest average density of broadcasters on a per area basis, thus to obtain the maximum achievable efficiency of spectrum use. See Fig. 1 and Fig. 2. In terms of signal propagation, the model considers the statistical ITU-R P.1546 model (ITU-R, 2013) instead of a terrain-aware model, so median path loss depends only on the distance from each transmitter.

For a traditional broadcaster, this work makes the approximation that its interference-limited coverage area is sufficiently well represented by a circle centered at the transmitter, with radius R_{trad} equal to the distance between the transmitter and the nearest point on the edge of the coverage area. Under this assumption, the model defines C_{trad} as the minimum distance between coverage areas of two traditional broadcasters.

For the case of a LPLT SFN, the model considers an arrangement of transmitters in a regular hexagonal tiling that forms a reference hexagonal network (RRC, 2006; EBU, 2014). The model makes the approximation that a coverage area can be reasonably represented by a regular hexagon of side R_{SFN}, and it considers a constant separation distance C_{SFN} between two LPLT SFNs coverage areas. As discussed in Bettancourt and Peha (2015), because the edge of coverage is not a perfect line, the distance between two coverage areas is not exactly the same at all points. Thus, the model calculates a value for separation distance that is roughly the average. For the range of numerical values used in this work, it is observed that this is a valid approximation.

From the above, the model estimates the maximum fraction of area that can be covered by traditional broadcasters or LPLT SFNs divided by the area of their respective hexagonal tile in the lattice, which is given by

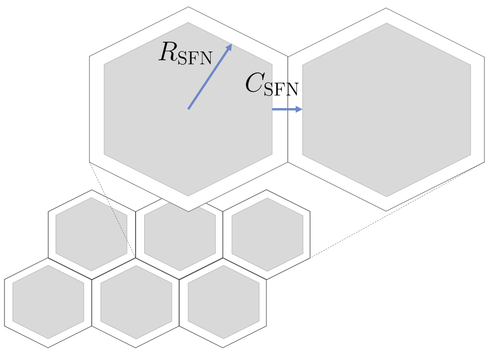

Fig. 2. Hexagonal packing of co-channel LPLT SFN broadcasters.

$$\eta_{\text{trad}} = \frac{R_{\text{trad}}^2}{(R_{\text{trad}} + 0.5C_{\text{trad}})^2} \cdot \frac{\pi}{2\sqrt{3}} \tag{1}$$

$$\eta_{\text{SFN}} = \frac{R_{\text{SFN}}^2}{(R_{\text{SFN}} + 0.5C_{\text{SFN}})^2} \tag{2}$$

3.1.2. Coverage definition

For a meaningful comparison, the model must define coverage area such that if a LPLT SFN broadcaster and traditional broadcaster have the same size coverage area, it is reasonable to view them as being equally effective at bringing TV service to their viewers.

The required coverage area is defined in terms of *coverage probability*, which is a function of the Signal-to-Interference-plus-Noise Ratio (SINR) for a receiver at a particular location. With SFNs in general, the total received signal is the power-sum of multiple OFDM useful and interfering components, so the coverage probability q at any given point can be expressed as

$$\begin{aligned} q &= \Pr\{\gamma \geq \gamma_{\min}\} \\ &= \Pr\left\{\sum_{j=1}^{J} w(\tau_j)S_j \geq \gamma_{\min}N_0 + \gamma_{\min}\sum_{k=1}^{K} U_k + \gamma_{\min}\sum_{j=1}^{J}[1 - w(\tau_j)]S_j\right\} \\ &= \Pr\{S \geq \gamma_{\min}(N_0 + U)\} \end{aligned} \tag{3}$$

where Pr $\{A\}$ is the probability of event A, S_j and U_k represent the received power from the jth wanted transmitter and the kth interferer respectively, and therefore $S = \sum w(\tau_j)S_j$ and $U = \sum U_k + \sum[1 - w(\tau_j)]S_j$ are the total received wanted TV signal and the total unwanted signal (co- or adjacent-channel) respectively, and N_0 is the noise input power of the receiver, all of which are in the linear domain. The function $w(\tau_j)$ is the equalizer weighting function of the OFDM receiver, and represents the constructive portion of the jth signal with relative delay τ_j (EBU, 2013). For traditional broadcasting, J=1 and $w(\tau_1) = 1$. For LPLT SFNs, the model assumes that $w(\tau_j) = 1$, which is reasonable as long as there exists a guard interval (GI) duration T_g such that $w(\tau_j) \approx 1$ across the entire coverage area (EBU, 2013). This occurs when the fraction GI over the total OFDM symbol length is small, and ISD is much smaller compared to the distance that a signal can travel during one GI (Li et al., 2015). For example, for $T_g = 260\mu$s which is equivalent to approximately a 1/16 fraction of a 32 K FFT symbol period in a 6 MHz channel (EBU, 2013), would meet these requirements for a bandwidth $B \leq 8$ MHz and a ISD ≪80 km. This includes the range of parameters considered in this work.

To evaluate $q = \Pr\{\gamma > \gamma_{\min}\}$, the probability distribution function (PDF) of the SINR γ needs to be derived. For this, S_j and U_k are modeled as possibly correlated signals coming from multiple transmitters simultaneously, but with different path losses and possibly different antenna gains. Specifically, S_j and U_k are part of a set of $L = J + K$ correlated log-normal random variables (RV) $\{\Omega_i\}_{i=1}^{L}$ with parameters μ_{Ω_i} and standard deviation σ_{Ω_i} (both in the dB scale). The model assumes a constant cross-correlation model (3GPP, 2006; Szyszkowicz et al., 2010), *i.e.* the cross-correlations in the decibel scale between all wanted-to-wanted, interfering-to-interfering and wanted-to-interfering components are identical to ρ (3GPP, 2006; Ligeti, 2000) and all signals share the same standard deviation $\sigma_{\Omega_i} = \sigma$ (EBU, 2013). It also makes the common assumption that the sum of log-normal RVs is well approximated by another log-normal RV, as exact closed-form expressions for the log-normal sum distribution do not exist to date (Lam & Le-Ngoc, 2007). To obtain the parameters of the log-normal sum distribution, the model uses an extension of the Schwart-Yeh method (Schwartz & Yeh, 1982), specifically, the Safak extension for the case of correlated log-normal components (Safak, 1993). For the mathematical details of obtaining the PDF of γ with correlated signals, refer to Bettancourt and Peha (2015).

One challenge when defining a broadcaster's coverage area is that the spatial distribution of the received signal strength across coverage areas of SFNs and traditional broadcasters are very different. For traditional broadcasting, it could be typically said that any point is within the coverage area if coverage probability q for the broadcaster's signal exceeds some fixed threshold q_{thr}. This means that coverage probability will be close to 100% near the transmitter, and will gradually decrease with distance from the transmitter until the threshold is reached at the edge of coverage. However, if the same definition is adopted with a LPLT SFN, and then the least expensive network that can provide a coverage probability greater than q_{thr} in all points that are covered by the traditional broadcaster is designed, the resulting coverage would be far worse than that for the traditional broadcaster. In this LPLT SFN, a large portion of the coverage area would have a coverage probability close to q_{thr}, including points near the center; whereas for the traditional broadcaster, this would only occur near the edge. Thus, to make the definition of coverage more appropriate for both LPLT SFNs and traditional broadcasters, the model considers two different coverage probability thresholds: a lower threshold q_{thr} near the edge of coverage, and a higher threshold q'_{thr} further inside. Any point with coverage probability greater than the higher threshold q'_{thr} is considered covered. If the set of points with coverage probability greater than q'_{thr} form a contiguous area, as it would be expected in a SFN that is designed to have a large contiguous high-quality coverage area, then the contiguous set of points surrounding this area with $q_{\text{thr}} < q < q'_{\text{thr}}$ are also considered to be within the coverage area.

For traditional broadcasters, the reference antenna currently used for coverage calculations (e.g. by ITU, FCC, others) is a directional antenna with a predefined radiation pattern (FCC, 2004; ITU-R, 1992). In this work, the model maintains this antenna definition for the case of traditional broadcasting, but for the case of SFNs, the reference antenna is omni-directional. An important feature of this scenario is that it considers that every broadcaster has its own frequency channel and multiple transmitters, and the location of each broadcaster's transmitters may have different throughout the coverage area, *i.e.* they might not be co-located. Hence, it may be impractical for many viewers to point an antenna in a direction that is near optimal for all channels. With omni-directional antennas, viewers greatly benefit from signal diversity, while avoiding any need of precisely pointing and/or reorienting antennas based on the location and/or the frequency of transmitters of different TV stations.

3.1.3. Spectrum reuse distances

This section develops the analysis to calculate spectrum reuse distances. This section divides the analysis in two pieces: traditional broadcasters and LPLT SFNs.

By definition, coverage probability needs to be equal to the minimum threshold q_{thr} at the edge of coverage. For traditional broadcasting, this work considers an hexagonal setup where each broadcaster is surrounded by other six co-channel broadcasters. Following Fig. 1, the model calculates the reuse distance C_{trad} along the line between the desired transmitter and any one of its interferers, which corresponds to the minimum distance between two points in opposing coverage areas. For calculation purposes, we only consider the interference from these three closest undesired TV stations, as signal from broadcasters on the opposite side of the coverage area arrive at much lower power levels that are negligible in comparison. From (3), $q = \Pr\{\gamma > \gamma_{\min}\}$ can be expressed as a function of the (median) desired signal and interference μ_S and μ_U, where $\mu_S = P + g_0 - L(R_{\text{trad}}; h)[\text{dB}]$ and $\mu_{U_k} = P + g_0 - L(d_k; h) - \psi[\text{dB}]$, $k \in \{1, 2, 3\}$, where d_k is the length of each undesired path. The function $L(\cdot; h)$ is the median path loss, which depends on the distance between transmitter and receiver for a given transmit antenna height h, and the values g_0 and ψ are the maximum gain and front-to-back (FB) ratio of the directional reference antenna respectively. Hence, the model obtains the relationship between transmit power P and C_{trad} for a given coverage R_{trad}, *i.e.* $C_{\text{trad}} = f(P, R_{\text{trad}})$ subject to $P > P_0$, where P_0 is the transmit power required for a coverage area of size R_{trad} in the noise-limited case. The model parameterizes transmit power as $P = P_0 + \Delta P$ so $\Delta P > 0$ is the *interference margin*. In this way, increasing $\Delta P > 0$ allows to obtain smaller separation distances by allowing additional co-channel interference at the edge of coverage.

For LPLT SFNs, it is assumed that transmitters are deployed in a hexagonal lattice configuration, so they are placed at a constant inter-site distance d_{ISD}, forming an hexagonal reference network of N tiers surrounding a central transmitter site. It is assumed all transmitters are identical, i.e. same antenna height h and same effective transmit power P. To obtain the minimum reuse distance between two LPLT SFNs coverage areas C_{SFN}, two hexagonal SFNs are placed facing each other as shown in Fig. 3. Given the coverage definition in 3.1.2, there are two thresholds $q'_{\text{thr}} > q_{\text{thr}}$, which means a contiguous coverage area is formed by an inner region where coverage prob $q \geq q'_{\text{thr}}$ that is surrounded by an outer region where coverage probability is between q_{thr} and q'_{thr}. As the distance between the infrastructure for two LPLT SFNs D_{SFN} decreases, then the coverage beyond the last tier of transmitters will shrink due to interference from the opposite SFN but it will also reduce C_{SFN}, as C_{SFN} decreases monotonically with respect to D_{SFN}. As the goal is to have LPLT SFNs as close together as possible without this to occur, i.e. without breaching the coverage definition. Hence, the relationship between ISD d_{ISD} and C_{SFN} for a given coverage R_{SFN} can be obtained, i.e. $C_{\text{SFN}} = f(d_{\text{ISD}}, R_{\text{SFN}})$ subject to $d_{\text{ISD}} < d_{\text{ISD}_{\text{NL}}}$, where $d_{\text{ISD}_{\text{NL}}}$ is the maximum ISD possible in the noise-limited case. As shown in Bettancourt and Peha (2015), separation distance $C_{\text{SFN}} = D_{\text{SFN}} - c_1 - c_2$ is a function of both D_{SFN} and d_{ISD}. As distance C_{SFN} monotonically decreases with respect to distance D_{SFN} when holding infrastructure constant, minimizing D_{SFN} is equivalent to minimizing C_{SFN}.

3.2. Numerical assumptions

In this model, the same link budget parameters for coverage calculations are assumed as in current U.S. TV spectrum policy (FCC, 2004). Carrier frequency is set to 615 MHz, as this representative of the UHF band, which is part of the frequency bands already targeted to be reallocated to mobile broadband use in the future. As in FCC (2004), no noise other than thermal is assumed;

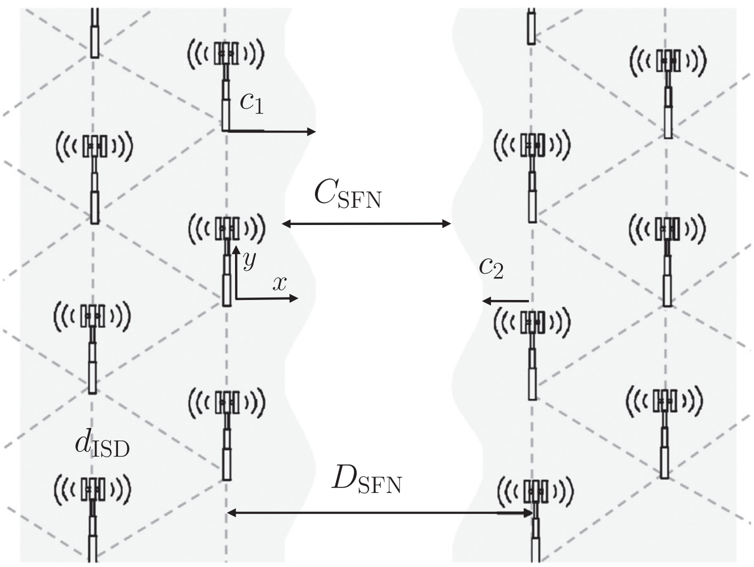

Fig. 3. Hexagonal lattice model for reference LPLT SFN deployment.

Table 1
Link Budget Parameters – Baseline Case.

Parameter	Value	
Carrier Frequency	f	615 MHz
Minimum SINR	$\gamma_{\min}$	15 dB
Thermal Noise Spectral Density	kT	−174 dBm/Hz
Receiver Noise Figure	NF	7 dB
Downlead line loss	L_{line}	4 dB
Shadowing Standard Deviation	σ	5.5 dB
Edge of Coverage probability threshold	q_{thr}	50%
Inner Coverage probability threshold	q'_{thr}	95%
Traditional Broadcasting:		
Antenna height	h	300 m
Directive antenna gain	g_0	12 dBi
Directive antenna FB ratio	ψ	14 dB
LPLT Single Frequency Networks:		
Antenna height	h_i	30 m
Omni-directional antenna gain	g'_0	5 dBi
Transmit power spectral density	P_i	52 dBm/6 MHz

$N_0 = kTB + \text{NF}$, where kT is the background noise spectral density of the receiver, B is the equivalent noise bandwidth of the receiver, and NF corresponds to the receiver's noise figure (in general, these results are independent of B unless stated otherwise). The model assumes a probability threshold $q_{\text{thr}} = 50\%$ for the edge of coverage. Inside SFNs, the model considers $q'_{\text{thr}} = 95\%$ which represents a threshold for high-quality coverage (EBU, 2013; Li et al., 2015). For traditional broadcasting, an antenna height of 300 m is assumed, which is typical for a TV tower in Europe (EBU, 2013), and it represents the median height for a TV tower in the U.S. (FCC, 2013b). For SFNs, antenna height is assumed of 30 m which is the typical height of a cellular transmitter (3GPP, 2006; Lewin et al., 2014), whereas for transmit power, the model considers a range from 43 dBm to 52 dBm over 6 MHz, which is in line with both typical values reported for LPLT SFNs (EBU, 2014; Meabe et al., 2015) and with likely non-ionizing radiation limits (Lewin et al., 2014). A summary can be found in Table 1.

For the directional reference antenna, this work considers the ITU-R BT.419–3 recommendation (ITU-R, 1992), while for the omni-directional reference antenna, it considers a $g'_0 = 5$ dB i gain as for a 1.25λ dipole antenna. For the path loss function $L()$, the ITU-R P.1546 propagation model (ITU-R, 2013) is assumed, considering 90% and 10% time availability for wanted signal and interference respectively (FCC, 2004), regardless of transmission architecture.

For log-normal shadowing, the model assumes $\sigma = 5.5$dB (ITU-R, 2013). In terms of correlation, the model follows 3GPP considering a constant value of $\rho = 0.5$ (3GPP, 2006). This recommendation can be readily applied to the case of LPLT SFNs due to its similarity with cellular networks. For the case of traditional broadcasting, numerical differences are negligible between considering either $\rho = 0.5$, or considering $\rho = 0$ as typically assumed. In this case, given that the coverage definition considers $q_{\text{thr}} = 50\%$, the potential effect of ρ is canceled out; for higher values of q_{thr} it can be found that the $\rho = 0$ assumption is quite pessimistic, leading to larger separation distances.

In terms of costs, this model assumes that a traditional TV station is in place, and estimates the net present value (NPV) of both the cost of upgrading infrastructure, if any, and the change in long-term operating costs when using a cost-minimizing design for each broadcaster. This model considers a evaluation period of 20 years and a 7% real interest rate (OMB, 2015).

With traditional broadcasting, we assume that the change in cost comes from an increase in transmit power, and nearly all of this is the cost of increased energy consumption. This, because partial equipment replacement (such as power amplifiers) is inexpensive compared to energy costs over the evaluation period, and second, the incremental cost of installing the equipment when a broadcaster is already changing frequency for repacking is small. It is worth noting that in the case of the U.S., transmission equipment will most likely be replaced either because of the repacking that follows the Incentive Auction, and/or the migration to ATSC 3.0. To calculate the increase in energy consumption, it is assumed that a transmitter operates 24 h, 365 days per year, consuming a power of P/η_{TX} watts at a cost of c_{kw-h} dollars per kW per hour, where η_{TX} is the power amplifier efficiency of the transmitter. We assume $\eta_{\text{TX}} = 20\%$, as the approximate power consumption for a state-of-the-art DVB-T transmitter of the year 2010 (Huschke et al., 2011) which it is assumed as representative of what can be found in the field today. The model uses $c_{\text{kW}-\text{h}} = \$0.12$, which is close to the average energy cost for commercial/industry use in the U.S. (EIA, 2015). Hence, the energy cost per kW transmitted over the air per year is \$5250, which is equivalent to \$55,000 per kW in present value over the evaluation period, which is the baseline estimate.

For SFNs, a broadcaster requires new infrastructure, and the design choice that most affects cost is the number of transmitters per area covered that each broadcaster operates. There are a variety of ways to deploy a transmitter, including building a new tower, or leasing space on an existing tower. The former would have a higher cost initially, while the latter would have a higher cost in subsequent year. Regardless of which approaches is chosen, we assume that the NPV of the cost of building and operating one LPLT SFN transmitter over the long term, NPV_{site}, is roughly the same for all towers. The model uses $\text{NPV}_{\text{site}} = \$650,000$, which is in line with values reported in FCC (2010b) when considering the evaluation period and the real interest rate considered here; it is assumed

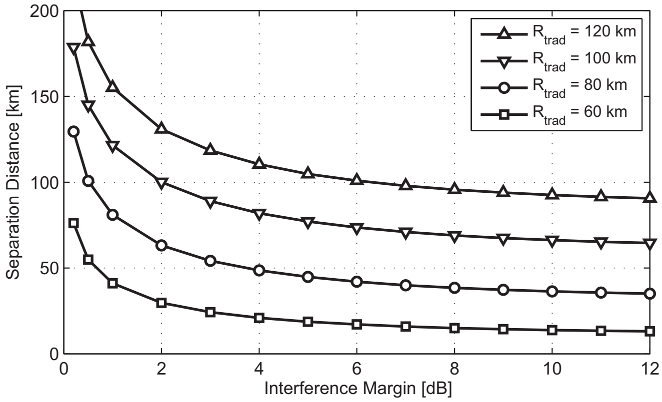

Fig. 4. Traditional broadcasting coverage area separation distance C_{trad} as a function of interference margin ΔP, with service area radius R_{trad} as a parameter.

that the potential cost of building and operating one LPLT SFN site is similar to those of sites in cellular networks (Lewin et al., 2014).

3.3. Results

This section first presents the spectrum efficiency results for each transmission alternative. Afterwards, an analysis of the cost-effectiveness of each alternative is carried out.

3.3.1. Traditional broadcasting

For traditional broadcasting, Fig. 4 shows C_{trad} as a function of the interference margin ΔP, with the required service area radius R_{trad} as a parameter. For any service area size, increasing transmit power always reduces separation distance between coverage areas, but this exhibits diminishing returns. Separation distances are much higher when interference margin is below 1 dB (tolerable interference around 6 dB below noise level or less), which is approximately the current state in U.S. policy (FCC, 2004). When tolerable interference is set at or above noise level, *i.e.* $\Delta P \geq 3$ dB, as in GE06 (RRC, 2006), achievable separation distances are quite smaller. Diminishing returns occur at roughly the same point as measured in dB, although an increase of 1 dB is much more costly if P is high, *i.e.* if coverage area is large, than if P is low. For subsequent results, we assume $\Delta P = 1$ dB as the baseline spectrum efficiency of a traditional broadcaster.

Fig. 4 shows that larger coverage areas require larger separation distances. However, as spectrum efficiency increases with coverage area and decreases with separation distance, it is not clear if spectrum efficiency increases or decreases. Fig. 5 shows how power affects spectrum efficiency. Within this range, the larger the coverage area, the lower the spectrum efficiency, with all other parameters held constant.

3.3.2. Single frequency networks

Fig. 6 shows the relationship between C_{SFN} and ISD. As expected, SFN separation distances are smaller by approximately one order of magnitude compared to traditional broadcasters. Much like increasing transmit power for a traditional broadcaster as shown in Fig. 5, increasing cost by reducing d_{ISD} in an SFN allows a broadcaster to tolerate more interference and thus reduce

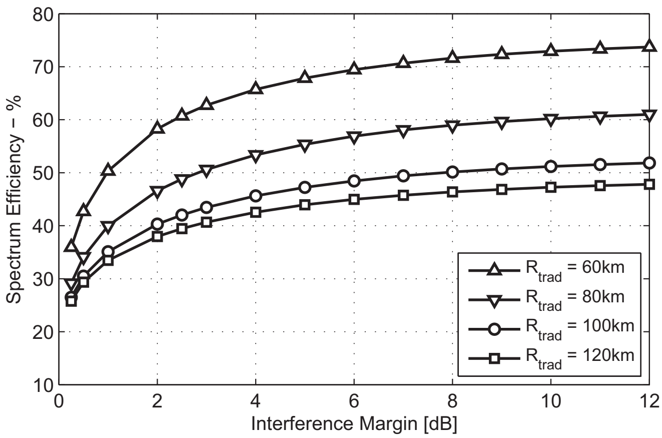

Fig. 5. Spectrum efficiency in traditional broadcasting as a function of interference margin ΔP, with service area radius R_{trad} as a parameter.

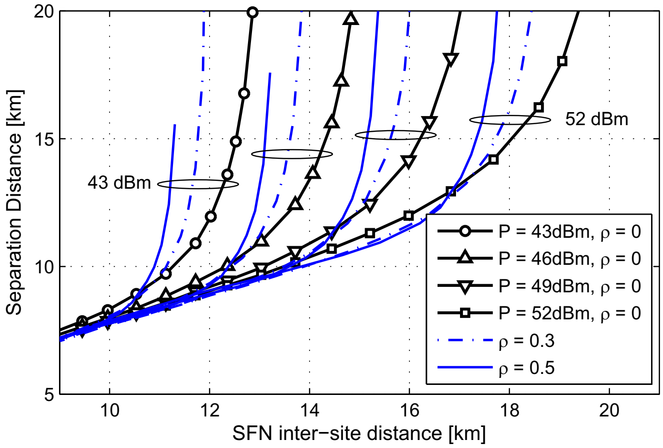

Fig. 6. Separation distance between LPLT SFN coverage areas as a function of ISD, for different values of transmit power P and different values for correlation between signals ρ.

separation distances between coverage areas. Moreover, asymptotic performance is the same regardless of transmit power.

It can be observed that, due to loss in macro diversity, higher correlation leads to a smaller ISD near the noise-limited regime as compared to when signals are uncorrelated, which is a simplifying and somewhat optimistic assumption. However, in the interference-limited regime, correlation slightly improves C_{SFN} but does not significantly change the relationship between ISD and separation distance. These observations are consistent with results in Malmgren (1997).

A TV viewer using the reference antenna within coverage areas of multiple broadcasters will receive all of their signals, because the definition is based on an omni-directional receive antenna. This is reasonable because in the U.S. TV broadcasters choose where to locate and how much to cover. However, European TV spectrum policy is different and it dictates bordering area-based licenses (which also leads to a very different industry structure). In this regard, recent discussions have addressed the feasibility and cost of obtaining separation distances close to 0 km (Lewin et al., 2014; Qualcomm/ATDI, 2014). To analyze this, the model extends the calculation of c_1 and c_2 (Bettancourt & Peha, 2015) (see Fig. 2) by using a directional antenna pointed in the best direction, which is roughly towards the closest transmitter. Results are shown in Fig. 7. Using a directional antenna with a FB ratio of $\psi = 14$ dB makes the asymptotic separation distance fall from about 8–10 km in the omni- case to about 2–3 km. For higher values of FB, a 0 km separation distance is possible and without significantly reducing ISD.

3.3.3. Spectrum efficiency vs cost trade-off

In certain cases, low to moderate gains are desirable if the cost to achieve them is low, while in other cases, large gains in efficiency as those suggested by LPLT SFN performance, can be outweighed by its elevated cost. To analyze this, this work assumes one wants to cover every location across a large area and that the coverage area per broadcaster is fixed. Then, the model considers a change in cost of transmission from an initial traditional broadcasting regime with spectrum efficiency η_1, to a more efficient regime with spectrum efficiency $\eta_2 > \eta_1$ due to either traditional broadcasting with increased transmit power or by switching to SFN transmission. Each channel that the network uses can cover a fraction η of the area. As long as coverage areas in different channels do not overlap, the amount of spectrum needed per broadcast channel delivered to every point in a region is roughly $1/\eta_1$. Hence, the average number of channels N_{ch} that can be saved, by improving spectrum efficiency from η_1 to η_2 per channel of actual content delivered throughout the region is given by

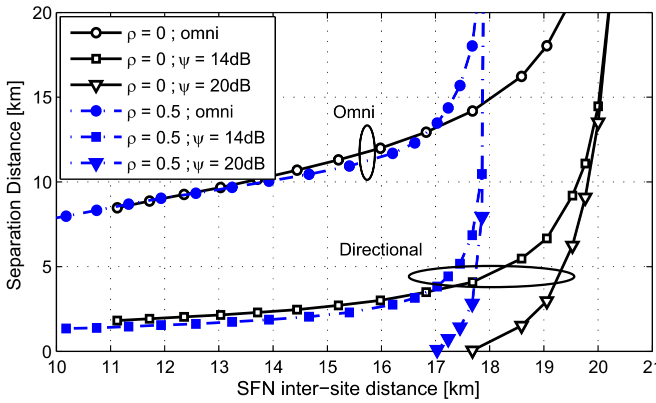

Fig. 7. Comparison between separation distances achievable with omni-directional antennas and with directional antennas with $\psi = \{14\ \text{dB}, 20\ \text{dB}\}$.

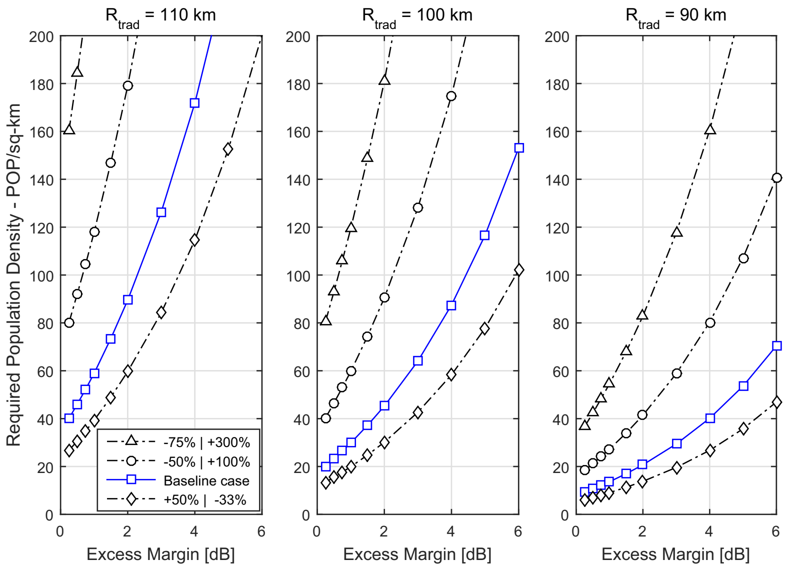

Fig. 8. Required population density for break-even vs. excess interference margin in traditional broadcasting, for different for different values of TV service area radius. Curves represent either the baseline numerical assumptions, or a (percentage) increase/decrease in the value of the spectrum/energy cost, or vice versa.

$$\Delta N_{\text{ch}} = \frac{1}{\eta_1} - \frac{1}{\eta_2}. \tag{4}$$

Eq. (4) obtains the number of channels that can be saved per channel of actual content delivered throughout the region. The increase in cost per area of delivering that content is $\Delta \text{NPV} = (\text{NPV}_2 - \text{NPV}_1)/A_{\text{TV}}$ where NPV_i is the net present value of the per broadcaster cost of transmission in each regime η_i. Thus, the cost per sq-km per broadcast channel that can be freed can be approximated by

$$C_{\text{MHz}-\text{km}^2} = \frac{(\text{NPV}_2 - \text{NPV}_1)}{A_{\text{TV}}} \cdot \frac{1}{B \cdot \Delta N_{\text{ch}}}. \tag{5}$$

The benefit of increasing spectrum efficiency depends on the value of spectrum. If the value of a MHz-sq km is high enough, which often means that if the population density is large enough, then the benefit will be worth the cost. To quantify when this will be the case, this model assumes that value per MHz-POP is known, and that it is constant for all population densities. Hence, population density $\Pi_{\text{POP}} = C_{\text{MHz}-\text{km}^2}/C_{\text{MHz}-\text{POP}}$. Thus, the population density Π_{POP} at which either increasing transmit power in traditional broadcasting or switching to SFNs is worth the cost can be obtained. One way to obtain the value of $C_{\text{MHz}-\text{POP}}$ is by looking back at recent spectrum auctions in the U.S.; in the 700 MHz auction in 2007 spectrum was sold for a national average of \$1.28 per MHz-POP (FCC, 2015a), while the recent AWS-3 auction in 2014 yielded an average of \$2.71 per MHz-POP (FCC, 2015a). This work we considers $C_{\text{MHz}-\text{POP}}$ to be between \$1 and \$3, with \$2 per MHz-POP as the baseline estimate.

For the case of traditional broadcasting, Fig. 8 shows the minimum required population density Π_{POP} at which the cost of increasing transmit power equals the value of spectrum freed, as a function of the excess interference margin. The excess interference margin is defined as the additional transmit power over $\Delta P = 1$ dB. Since a typical full-power broadcaster in the U.S. covers approximately 100 km around the transmitter, R_{trad} is considered being between 90 km and 110 km. For each case, a baseline curve is presented that is obtained for the baseline value of the spectrum $C_{\text{MHz}-\text{POP}}$ of \$2 per MHz-POP and the baseline NPV of the energy cost of \$55,000 per kW. To account for possible uncertainties in both values, Fig. 8 shows additional curves representing either a (percentage) increase/decrease in the value of the spectrum, or a decrease/increase in energy cost as Π_{POP} is directly/inversely proportional to these values. For example, the curve labeled as [+50%| − 33%] represents a 50% increase in the value of the spectrum freed, a 33% reduction in the transmission cost, or any ratio combination equal to 1.5X. As a result, Fig. 8 shows that the break-even population densities are quite low. For example, for a 100 km radius the break-even population density would be 30 POP/sq km if spectrum is worth \$2 per MHz-POP, and 20 POP/sq km if spectrum is worth \$3 per MHz-POP, at the baseline cost and increasing transmit power by only 1 dB. This is small compared to many large areas in the U.S. The population density of the contiguous U.S. is 40 POP/sq km. 29 of 50 states have a population density above 30 POP/sq km, and 36 states have a population density above 20 (USCB, 2010). This suggests that this strategy might be quite cost-effective in a large fraction of the country. Fig. 8 also shows that the larger the coverage area, the more costly it is to free spectrum this way. This translates to a higher required population density to break-even. Thus, the change from noise-limited to interference-limited coverage is even more cost-effective if coverage areas are smaller.

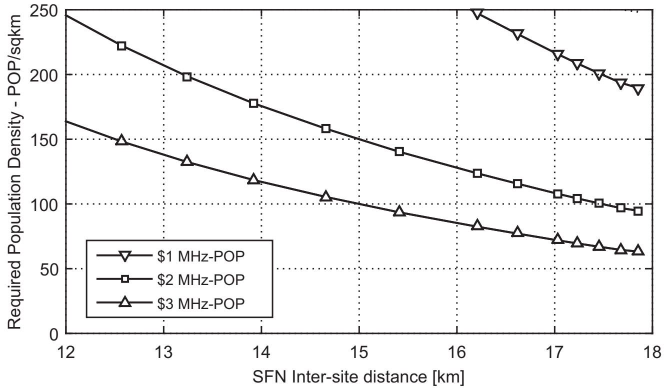

Fig. 9. Required population density for break-even vs. LPLT SFN inter-site distance, for different costs per MHz-POP and for different TV service area radius. Results obtained for $P = 52$ dBm/6 MHz.

Fig. 9 shows the minimum population density Π_{POP} to break-even from a switchover to LPLT SFNs while varying the value of the spectrum freed. Solid lines represent the results for $1, $2 and $3 per MHz-POP for $R_{\text{trad}} = 100$ km. If the smallest Π_{POP} were greater than the population density of New York City, then a switch to LPLT SFNs would always be a bad idea, but this is not the case. For a conservative estimate of $2 in the value of the spectrum, the minimum Π_{POP} is in the order of 100 POP/sq km. At $3 per MHz-POP, LPLT SFNs become cost-effective at a population density of just 67 POP/sq km. For the baseline numerical assumptions, results show that this could be cost-effective in some parts of the U.S., but not in others. The U.S. East Coast corridor between Washington D.C. and Boston is a good example of a region that might benefit from a transition to LPLT SFNs. It is a sufficiently large area and its average population density is at least 145 POP/sq km. This number is obtained from information available on a per state basis, which includes many rural areas. Thus, the actual population density along the coast is even higher.

Fig. 10 shows the minimum population density Π_{POP} to break-even while varying the cost per LPLT SFN site. If the cost per site could be reduced by one third or two thirds, LPLT SFNs become cost effective at around 60 POP/sq km and 20 POP/sq km respectively, assuming the baseline value of the spectrum freed of $2 per MHz-POP. Thus, if the cost per SFN site can be reduced substantially, LPLT SFNs could be cost effective in a much larger fraction of the country. Moreover, Fig. 10 shows that for a sufficiently low cost per site, reducing the value of the spectrum in half to $1 per MHz-POP is shows a smaller increase in Π_{POP} than doubling the cost per site.

Fig. 11 shows the approximate fraction of spectrum that can be freed as a function of the incremental cost per sq km covered. A modest increase in transmission power can save over 30% of the spectrum, and transitioning to an LPLT SFN can save over 60%, although an LPLT SFN is only worthwhile where spectrum is sufficiently valuable to justify the cost.

Fig. 12 shows the approximate fraction of spectrum that can be freed with these two technical approaches when maximizing the value of spectrum freed minus the cost incurred. To show the results' sensitivity to numerical assumptions, three scenarios are considered: Scenario A assumes both cost and value of the spectrum are at the baseline levels. Scenario B differs from baseline in that the value of spectrum is now $1 per MHz-POP. Scenario C assumes the baseline $2 per MHz-POP, but the cost per LPLT SFN site is now a third of the baseline value. Fig. 12 also shows the amount of spectrum that can be freed as a function of population density with both approaches when the value of spectrum freed exactly equals the cost, so that any point in between these two curves yields a benefit minus cost that is superior to what we have with the noise-limited policy of today. For Scenario A (baseline), the difference between benefit and cost achievable with LPLT SFNs exceeds that achievable with interference-limited single-transmitter

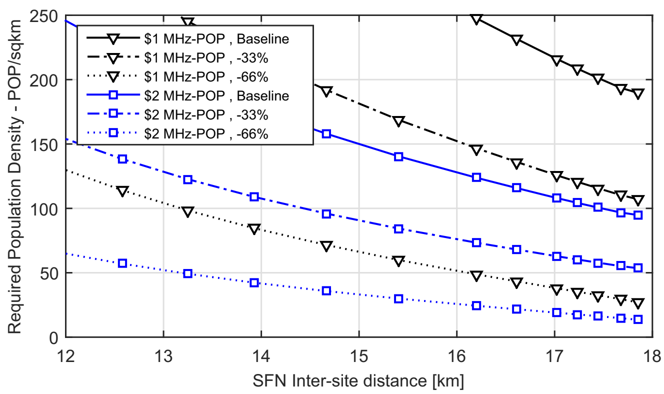

Fig. 10. Required population density for break-even vs. LPLT SFN inter-site distance, for different costs per MHz-POP and for different TV service area radius. Results obtained for $P = 52$ dBm/6 MHz.

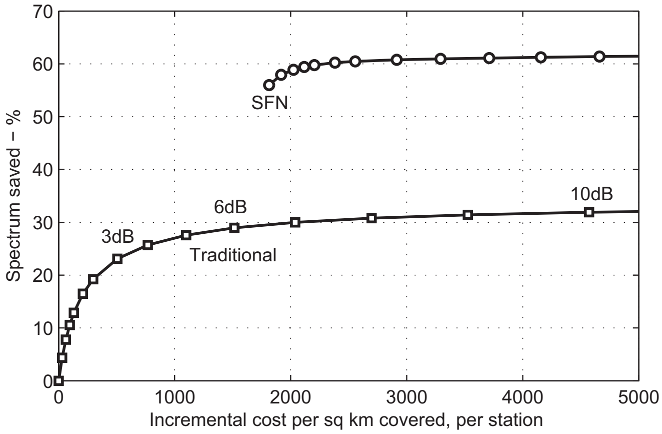

Fig. 11. Maximum percentage of spectrum savings vs. incremental cost per sq-km covered for each strategy for one station with R_{trad}=100 km.

systems when the population density exceeds the vertical line just above 120 POP/sq km. As a result, if spectrum is worth $2 per MHz-POP, then regions with population density above roughly 120 POP/sq km should adopt LPLT SFNs, regions with population density between 30 and 120 POP/sq km should use a traditional broadcast architecture but with increased transmit power, and regions with under 30 POP/sq km should maintain the current approach. In Scenario B, where spectrum is half as valuable, the population density at which the difference between benefit and cost achievable with LPLT SFNs exceeds that achievable with interference-limited single-transmitter systems is twice as high, i.e. above 240 POP/sq km. Thus, not surprisingly, the cost-effectiveness of a transition depends greatly on the value of spectrum that might be freed. In Scenario C, LPLT SFNs are always better than traditional broadcasting with increased transmit power for any population density where LPLT SFNs are cost-effective. Moreover, the transition from noise-limited traditional broadcasting to SFNs is cost-effective in this scenario at a population density of just 20 POP/sq km.

4. Non-uniform distribution of broadcasters

In the uniform deployment it was shown that not only switching to LPLT SFNs can decrease the minimum separation distance between two broadcasters' coverage areas, but also when all broadcasters increase their transmit power by the same number of dB. This section applies that finding into a highly non-uniform deployment scenario. This scenario is represented by the U.S. TV stations

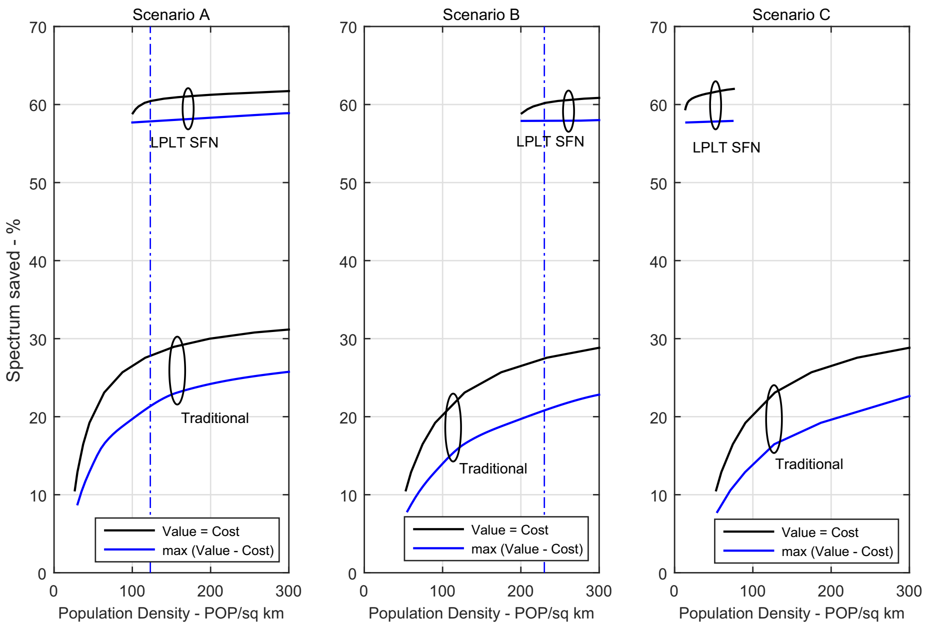

Fig. 12. Maximum percentage of spectrum savings vs population density for (a) value of the spectrum freed equals cost, and (b) maximum net benefit. Scenario A assumes both cost and value of the spectrum are at the baseline levels. In Scenario B the value of spectrum drops to $1 per MHz-POP. Scenario C considers $2 per MHz-POP and that the cost per LPLT SFN site can be dropped in two thirds from the baseline value.

in the UHF band as of 2016. In this section, and for each technical alternative, the amount of spectrum needed to support all TV stations nationwide is calculated —by solving the channel repacking problem (Frechette et al., 2016)—, assuming that this is the best metric to show the effectiveness of both alternatives. We will relax this assumption later.

In Section 3, the theoretical model assumes broadcasters' identical service areas are preserved, but their spatial location can change, which may only be possible with a central planning regulatory regime. What occurs in the U.S. and some other countries is largely due to a decentralized regime where broadcasters can choose their geographic locations at will and there are no major warranties in terms of equitable access to the TV spectrum. In contrast with the model in Section 3 where any reduction in separation distances —by definition— would increase spectrum efficiency, in the non-uniform deployment case the impact of reducing separation distances operates in a different way. First, not only broadcasters' service areas remain constant, but their geographic position is fixed as well. Second, and more meaningfully, increasing cost to reduce minimum separation distances may or may not help in reducing the allocated spectrum overall. In practice, the co-channel interference constraint between two TV stations may vanish if they, for example, switch to LPLT SFNs. This would allow the flexibility for them to use the same frequency channel. However, it is not the change in the number of pairwise interference constraints between TV stations what matters, but the global optimal solution to the channel repacking problem.

In the repacking problem, the amount of spectrum needed is determined by minimizing the number of allocated channels while meeting stations' co-channel and adjacent-channel interference constraints (Frechette et al., 2016; Kearns & Dworkin, 2011). More generally, the objective is to know if repacking of all TV stations is feasible within a given number of available channels, and if not, what is the maximum number of stations that can be packed within these available channels.

Recently, in preparation for the Incentive Auction (FCC, 2014), the U.S. Federal Communications Commission (FCC) released open-source software, data and methodologies that facilitates solving the repacking problem (FCC, 2016b, 2015b, 2016a). In particular, this work uses the TVStudy software to both (a) obtain existing coverage areas that need to be preserved under repacking and (b) analyze interference that would occur after boosting transmit power in traditional broadcasting. With this, repacking problem can be solved by following a proven efficient SAT Boolean formulation approach (Frechette et al., 2016; Kearns & Dworkin, 2011; Muthukumar et al., 2015).

4.1. The frequency assignment problem

This repacking problem is notoriously difficult to solve as a mixed-integer program (MIP).[3] However, for the FCC interference graph, good performance has been obtained via the Boolean satisfiability (SAT) formulation we use herein (Frechette et al., 2016; Kearns & Dworkin, 2011). To obtain the stations' interference constraints, this work uses the FCC's TVStudy software, which was developed for the repacking that would follow the Incentive Auction (FCC, 2016b). The TVStudy software was designed for evaluating coverage areas and performing interference analysis between TV stations..[4] The output produced by TVStudy ("pair study") is then processed by the FCC Constraint Generator (FCC, 2015b) to obtain the list of pairwise interference constraints that populate the repacking problem. The results in this paper are built upon and by modifying these two pieces of software.

As the TVStudy software is very intensive in computational resources, so this work adopts the so-called proxy channel approach (FCC, 2013a); stations' coverage areas and population covered are calculated using their actual assigned (14–51) channels, but interference analysis is performed as if all stations operated over the same proxy channel where stations' coverage is "replicated".[5] Following (FCC, 2013a), Ch. 20 is used as the proxy channel, because it well represents the propagation characteristics of the UHF band for the purpose of our analysis, which focuses on repacking feasibility.[6]

The repacking problem is feasible if, for a given set F of available channels, there exist a channel assignment that satisfies all the constraints for all stations in $i \in N$. To create such a frequency plan, the mutual interference status of all pairs of stations is needed. As of 2016, there are N=1711 TV stations assigned to the UHF band, including both full-power and Class A (CA) stations.

As shown in Kearns and Dworkin (2011), for each station i and channel $k \in \{1, \ldots, F\} \cup \{0\}$, the binary variable $x_{i,k}$ takes value 1 if station i is assigned to channel $k \in F$, and 0 otherwise; if a stations has been assigned to channel k=0, this means that the station was left out of the repacking. More formally, the conditions to define the SAT problem are expressed as (Kearns & Dworkin, 2011):

- Each station is assigned to at most one channel
- Each station is assigned to at least one channel
- Each station must satisfy all co-channel interference constraints
- Each station must satisfy all adjacent-channel interference constraints
- No more than b stations can be left out of the repacking:

A quick and informative test that can help understand the repacking solution is by listing all maximal cliques of the co-channel

[3] This is due to adjacent-channel constraints. Without them, the problem reduces to a much more tractable graph coloring problem (Frechette et al., 2016).

[4] This open-source software integrates the FCC Media Bureau's CDBS database, terrain elevation databases, and the assumptions contained in the FCC OET-69 Bulletin (FCC, 2004)

[5] Refer to FCC (2016b) for a more detailed discussion on how each station is replicated by slightly adjusting its transmit power to maintain roughly the same coverage area.

[6] In June 2, 2014, the FCC released an updated dataset that replaces the use of proxy channels with the specific pairwise channel assignments for determining potential interference constraints, along with an analysis showing that this should have a minimal impact on feasibility (Kearns & Dworkin, 2011)

interference graph. In this context, a clique is a subset of TV stations such that any pair of station i, j is allowed to share the same channel, and a maximal clique is a clique that cannot be extended by including one more TV station, *i.e.* it is not part of a larger clique. The largest maximal clique is a lower bound of the minimum number of channels of the repacking solution. If the co-channel interference graph contains a clique of size k, then at least k channels are needed.[7] On the other hand, the size of these independent maximal cliques can be roughly interpreted as the minimum number of channels on a "local" (geographic) basis.

The FCC interference graph is very large but also very sparse. Fig. 13 shows a "heat map" that represents the total number of UHF TV stations that could potentially be received in any single point across the contiguous U.S. TV stations are highly concentrated in the most populated areas, which is a distinctive feature of the predominant first-come first served licensing approach in the U.S. The implication of this is that many of the channels contained in the UHF band are there to provide service for quite a "long tail" of TV stations, both in terms of area and in terms of population. It is expected that reducing separation distances between coverage areas will reduce the number of pairwise constraints, and this should reduce the size of the spectrum allocation. However, these highly dense spots (e.g. New York City, San Francisco) may end up defining the size of the spectrum band.

4.2. Interference constraints

In general, a station can share a channel with another station only if when doing so, the effect of harmful interference is "sufficiently small" across the stations' noise-limited coverage area. The traditional way of assigning broadcasters to channels while making sure interference is sufficiently small is through noise-limited *packing*, which requires that the interference be small at the edge of coverage for any TV broadcaster. An alternative approach allows interference from previous channel assignments to persist, and makes sure that additional interference beyond what was there before is sufficiently small. The latter approach is labeled as FCC *repacking*, as this is the criteria that is in use for the Incentive Auction (FCC, 2014). In this case, the FCC's definition for "sufficiently small", which is also adopted in this work, is based on population served: two TV stations i and j can only be assigned to channels s and c if when doing so, the existing population served in their current channel assignment, called *interference free* population P_i, P_j, does not decrease by more than 0.5% on either station. This is

$$(x_{i,s}, x_{j,c}) = \begin{cases} \dfrac{P_{i,s}}{P_i} \geq 99.5\% \text{ and } \dfrac{P_{j,c}}{P_j} \geq 99.5\% & (1, 1) \\ \text{otherwise} & (0, 0) \end{cases} \tag{6}$$

where $P_{i,s}$ and $P_{j,c}$ are the interference-free population served when stations i and j use channels s and c respectively. In the noise-limited *packing* case, P_i and P_j are instead the population served in the noise-limited regime.

Before determining the exact coverage and population served, the TVStudy software establishes for each TV station the so-called "noise-limited contour" C_i as an upper bound of the area in analysis: this contour represents the points where the signal-to-noise ratio (SNR) at a reference receiver equals the minimum threshold $\gamma_{\min} = 15$ dB, as predicted by the FCC's F(50,90) propagation curves (FCC, 2004). The area within this contour is then divided into square "cells" (2 km by 2 km), and for each cell, the software uses the irregular-terrain Longley-Rice model (FCC, 2004) to determine if coverage is impaired due either to terrain blockage of the signal (*i.e.* SNR is below threshold) or to interference from other stations (*i.e.* SINR is below threshold). The sum of population within cells where coverage is both within the noise-limited contour and not impaired is the station's *interference-free* population.[8]

4.2.1. Single-transmitter traditional broadcasting

To obtain the pairwise constraints for the case of increased transmit power, power needs to be increase in both stations by the same number of decibels. To obtain a correct result with the TVStudy software, the software needs to consider the exact same set of 2 km by 2 km cells, and thus the same noise-limited contour that was used to compute P_i and P_j (*i.e.* for $\Delta P = 1$ dB) when obtaining $P_{i,k}$ and $P_{j,k}$.

To determine if a 2 km by 2 km cell is impaired by co-channel interference, FCC rules define a minimum Desired-over-Undesired (D/U) ratio (FCC, 2004). For our purposes, in order to incorporate the effect of increased transmit power, $\Delta P - 1$ is added as a correction term to the FCC definition as

$$D/U_{\min} = \gamma_{\min} + 10\log_{10}[1 - 10^{-(x+\Delta P-1)/10}]^{-1}[\text{dB}], \tag{7}$$

where x is the amount by which the desired signal exceeds the minimum required for TV reception $\gamma_{\min} = 15$dB, as defined in FCC (2004). Thus, $x = C/N - \gamma_{\min}$ [dB]. When transmit power is boosted, S/I ratios across each study cell within the noise-limited contour stay the same. However, the second term in the right hand side portion of (7) strictly decreases with ΔP, which reduces the D/U minimum threshold instead of increasing the effect of interference as signal strength goes up. Essentially, as shown in Bettancourt and Peha (2015), the stronger the signal is (near edge of coverage), the more interference a cell can tolerate. On the other hand, adjacent-channel constraints do not change as we boost power, because the criteria are based entirely on S/I rather than SINR.

[7] In practice, this bound is quite tight when the interference graph is sparse.

[8] Note that the F(50,90) curves and the Longley-Rice method are two very different propagation models: depending on terrain, cells inside the contour could be impaired, while cells with service can be found outside the contour. A good source to check this mismatch can be found at http://www.rabbitears.info/maplist.php, which contains a list of Longley-Rice U.S. coverage maps. It is out of the scope of this paper to analyze the FCC methodology itself, but to use it as a representation of how channels could be assigned by the agency.

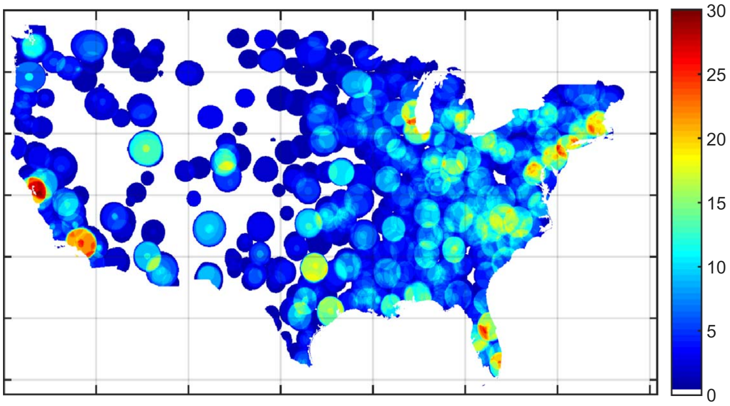

Fig. 13. Geographic distribution and coverage density of available UHF TV stations across the U.S.

From a software implementation standpoint, a very minor modification is needed to include (7) and to indicate TVStudy that $\Delta P = 1$ dB when obtaining P_i and P_j, *i.e.* when both stations i and j are in their original assigned channel, and $\Delta P > 1$ dB when obtaining the population $P_{i,k}$ and $P_{j,k}$ served in the replicated channel.[9]

4.2.2. Low-Power Low-Tower single frequency networks

For LPLT SFNs, co-channel interference constraints are obtained using a different approach, that is based on polygons rather than on a grid as for traditional broadcasting. In (Bettancourt & Peha, 2015), it is shown that a single distance d is accurate and representative enough of the required separation distances between the coverage areas of LPLT SFNs of different sizes but built with the same given ISD. In this regard, two LPLT SFNs “interfere” with each other if any point within SFN i's noise-limited contour and any point within SFN j's noise-limited contour are within d km of each other. For this purpose, D_i represents the minimum contour that contains all points within the noise-limited contour and those within a distance d (other names for D_i are buffer zone, polygon expansion/dilation, etc). If the noise-limited contour C_i of the desired TV station i overlaps with the interfering station j's D_j, then these two stations cannot share the same channel. In the limiting case, if the separation distance is 0 km, then C_i=D_j. On the other hand, as opposed to traditional broadcasting, adjacent-channel constraints can be safely ignored altogether if LPLT SFNs (Huschke et al., 2011; Lewin et al., 2014) are assumed, and as considered in this analysis. Although results are not shown here, when using LPLT SFN transmitters the so-called *punch hole* effect is minimum or non-existent due to both the low transmit power and the use of transmit antennas with a very narrow vertical radiation pattern, much like what occurs in cellular networks (Huschke et al., 2011; Lewin et al., 2014).

4.3. Results

For solving the repacking problem, there are several SAT solvers. For example, authors in Kearns and Dworkin (2011) and Muthukumar et al. (2015) considered Picosat.[10] This work uses Clasp,[11] which is part of the Incentive Auction FCC repacking tool SATFC (Frechette et al., 2016) and has been optimized for this particular problem.[12] In addition, to better understand the effect on interference constraints, a modified version of the well-known Bron-Kerbosch algorithm is applied for finding the co-channel interference graph's maximal cliques (Eppstein et al., 2010).

Results show that a change in technology could allow a significant amount of spectrum to be freed from TV use nationwide, although not as much spectrum as was freed in the case of uniformly distributed broadcasters of Section 3. In the noise-limited packing regime, increasing power by 3 dB, *i.e.* $\Delta P = 4$ dB, clears 2 channels or 12 MHz of spectrum nationwide, and switching to LPLT SFNs frees 9 channels, equivalent to 54 MHz nationwide. A summary of these results, along with the lower bounds obtained based on the size of the largest clique (clique number) in each case can be found in Table 2. From Table 2, we can observe, as claimed previously by Kearns and Dworkin (2011), that the size of the largest clique is a tight lower bound, as adjacent-channel constraints do not play a very significant role.

In addition to the TV channels that can be freed entirely through a change in technology, it is also important to consider the extent to which channels could be freed in much of the country, at least if spectrum policies allow a band to be used for TV in some regions and for other purposes in other regions. In Fig. 14 we show the cumulative distribution function (CDF) of the maximal

[9] We hard code this in TVStudy by modifying the input value to the function that implements the ramp function, `dtv_codu_adjust`. In both the 1.3.1 and 2.0.1 version of TVStudy, this function is declared inside a file called `study.c`. To indicate TVStudy when to declare $\Delta P = 1$ dB or not, a simple `if..then..else` statement that looks for these cases via an internal variable.

[10] http://fmv.jku.at/picosat/

[11] http://www.cs.uni-potsdam.de/clasp/

[12] We can acknowledge a quite superior performance of Clasp against Picosat in the ability to quickly find repacking solutions, and hence ensuring that our results are accurate.

Table 2
Repacking Results for both Traditional Broadcasting and LPLT Single Frequency Networks, for either noise-limited packing and for the FCC repacking problem.

Noise-limited packing	ΔP = 1 dB	ΔP = 4 dB	SFN, 0 km
Clique number	38	37	31
Required channels (SAT)	40	38	31
Spectrum savings (SAT)	–	12 MHz	54 MHz
FCC repacking	ΔP = 1 dB	ΔP = 4 dB	SFN, 0 km
Clique number	31	31	31
Required channels (SAT)	33	32	31
Spectrum savings (SAT)	–	6 MHz	12 MHz

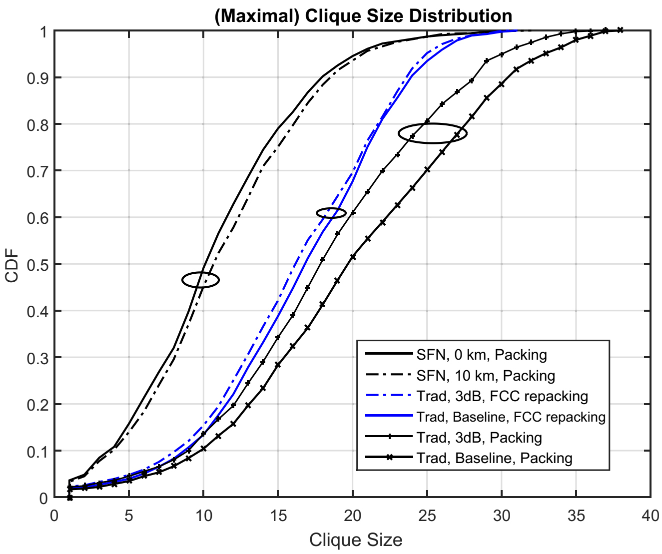

Fig. 14. Cumulative Distribution Function (CDF) of maximal cliques associated to the graph representing nationwide co-channel interference constrains. Results are obtained for traditional broadcasting and LPLT SFNs, for either the noise-limited packing or the FCC repacking problem.

cliques' size induced by the respective co-channel interference graphs. For the LPLT SFN case, results are shown for separation distances of 0 km and 10 km, while for traditional broadcasting, it is considered that all TV stations' increase their transmit power by 3 dB. The point at which each curve reaches 1.0 is the size of the largest clique (or the *clique number*), shown in Table 2.

As previously observed, in those parts of the country where clique size is the largest, the difference in clique size between the LPLT SFN approach and the traditional noise-limited approach is 7 channels. Considering parts of the country where clique size is at the 95th or 90th or 80th percentile, the difference between these two approaches is even larger, at 11, 13 and 13 channels, respectively. This indicates that it is probably possible to free even more spectrum in these regions, if there is a way to put spectrum that has been freed in part of the nation to use, as the FCC recently made possible in the U.S. (FCC, 2014). However, in the predominately rural regions with smaller clique sizes, the savings is smaller. For example, at the 20th and 10th percentile, the difference is 7 and 6, respectively. The observations above are true for SFNs with both 0 km and 10 km distances between coverage areas, since that difference has little impact. A similar pattern can be observed when comparing noise-limited packing at current transmit powers with noise-limited packing with a 3 dB increase in powers, but the differences are much smaller. For example, where the largest clique sizes differ by just 1 channel at the 100th percentile, *i.e* nationwide, the differences at the 90th, 80th and 70th percentiles are 2, 3 and 3 channels, respectively. For the case where repacking rules allow coverage areas to shrink, Fig. 14 shows that the difference in clique size is also somewhat greater at percentiles below 100, although the difference is still modest.

On the other hand, if comparing SFNs to traditional architectures under the FCC repacking approach wherein coverage areas are allowed to shrink, looking only at the amount of spectrum freed nationwide as shown in Table 2 misses a larger impact. Although drastically reducing reuse distances with SFNs provides no apparent gain on a nationwide basis to the maximum clique size, the difference between CDFs in Fig. 14 in places other than the tail regions is significant. This suggests that the spectrum needed in much of the nation would be greatly reduced, but not in the market with the most TV stations, where stations may be experiencing significant interference at edge of coverage already.

The effect of increasing transmit power by 3 dB in a single transmitter system has much less impact under the assumptions of FCC repacking, where coverage areas are allowed to retain existing interference that was previously above threshold, than in noise-

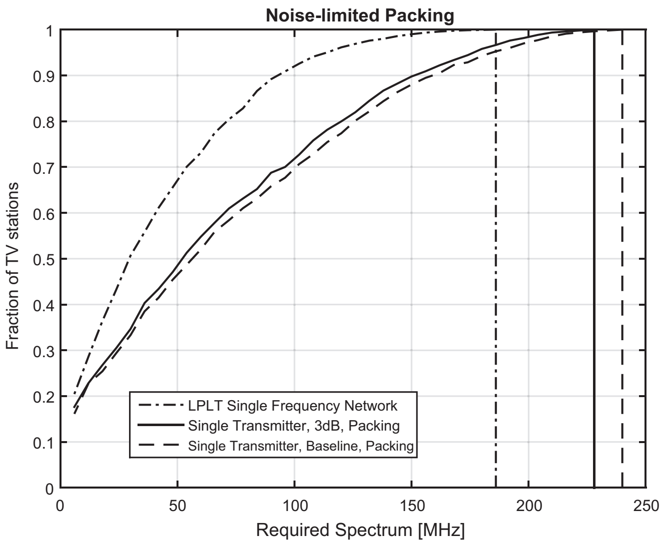

Fig. 15. Fraction of today's U.S. UHF TV stations that can be packed vs spectrum available for UHF television.

limited packing where they cannot. As with the result above, this may in part be because current coverage areas are already interference-limited in some places. As shown in the previous section, increasing transmit power when a system is already interference-limited will increase spectrum efficiency, but there are diminishing returns with each increase.

Up to now, the requirement has been that all stations need to be packed. Although clique results give meaningful intuition, they do not tell directly how many channels are needed for TV and how many can be freed if the number of stations were reduced slightly in the specific locations where this would have the greatest impact on spectrum needs, *i.e.* from the largest cliques. For this, the iterative method proposed in Kearns and Dworkin (2011) can shed light. Fig. 15 shows the maximum fraction of stations that can be packed for a given amount of spectrum.

If it were possible through something like an incentive auction to remove a few broadcasters in big cities, the advantages of changing technology would be even greater. For example, while Table 2 showed that switching from traditional noise-limited to SFNs would free 9 channels nationwide, Fig. 15 shows that the number of channels freed by switching to SFNs would increase to 12 if 5% of stations were removed from locations where TV spectrum is in greatest demand. Moreover, in the case where no broadcasters are removed, it could be considered that the channels containing the last 5% of stations to be mostly but not entirely freed, as they would only be used in the few cities with the greatest demand for TV spectrum. Thus, a switch to SFNs frees 9 channels entirely, and increases the number of channels that are mostly freed by 3. Fig. 16 compares SFNs vs traditional now under FCC repacking. Table 2 showed that switching from traditional noise-limited to SFNs with the FCC repacking approach would free 2 channels nationwide, Fig. 16 shows that the number of channels freed by switching to SFNs would increase to 8 if 5% of stations

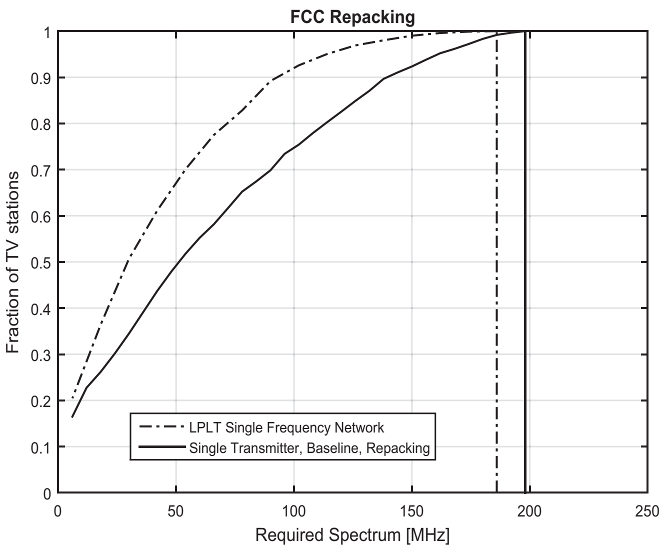

Fig. 16. Fraction of today's U.S. UHF TV stations that can be packed vs spectrum available for UHF television.

Table 3
States needed to switch to LPLT SFNs in order to reduce the size of the largest clique. Other states remain using traditional broadcasting architecture. States that form a contiguous area have been grouped in brackets. Note that states are added cumulatively to the SFN set from clique numbers 38 to 31.

Required Clique Number	States needed to switch to SFNs	U.S. Area	U.S. Population	UHF Stations
37	{NY, NJ, MD, DC}	2.4%	11.7%	7.0%
36	{MA}	2.7%	13.9%	8.5%
35	{CT, PA, VA}	5.7%	22.1%	16.2%
34	{VT} {NC} {CA}	13.0%	38.4%	28.7%
33	{IL, WI}	17.0%	44.6%	34.2%
32	{RI} {ME} {IN} {GA}	21.3%	50.9%	40.3%
31	{KY}	22.6%	52.4%	42.4%

were removed. This is further evidence that looking only at the number of channels freed nationwide misses a significant part of the benefit.

Finally, because the results above suggest that much of the benefit of changing technology for all TV stations occurs in small fraction of the U.S., this work considers a scenario where the transmission architecture is changed only in a small fraction of the U.S. This would greatly decrease cost, and potentially increase cost effectiveness. To test this, the impact on maximal clique size of converting broadcasters in some states with relatively high population density to SFNs is observed, while leaving the broadcasters in other states as traditional noise-limited systems.

For all station pairs where both stations are within one of these states, pairwise constraints are assumed to be the ones obtained in the LPLT SFN case. In all other cases, pairwise constraint obtained in the traditional broadcasting case are considered. In step 1, broadcasters in New York, New Jersey, Maryland and the District of Columbia are converted to LPLT SFNs. This includes 7% of all U.S. broadcasters. These states include 11.7% of the U.S. population, but only 2.4% of its area, which shows that they are densely populated. In step 2, the same broadcasters, plus the broadcasters in Massachusetts, or 8.5% of U.S. broadcasters. In step 3, the states of Connecticut, Pennsylvania, and Virginia are added. By the 7th step, all broadcasters in 18 states (including DC) are converted. This procedure continues until it reaches the clique number previously obtained in Table 2 for the case where all stations in the U.S. switch to LPLT SFNs. The results are summarized in Table 3.

By converting only the broadcasters in these states, which include 42% of all broadcasters, it is possible to achieve the same 7-channel reduction in (maximal) clique size as was achieved if 100% of broadcasters switch to LPLT SFNs, but at less than half the cost. For the much smaller cost of converting 8.5% of broadcasters in 5 states, it is still possible to reduce maximal clique size by 2 channels, or 29% of the number of channels potentially freed if all stations convert to SFNs. Obviously, converting broadcasters to LPLT SFNs on a state by state basis is a somewhat arbitrary, and an optimal algorithm could presumably yield greater spectrum gains for a given number of broadcasters to be converted.

5. Conclusions and policy implications

This paper analyzes two technical strategies that increase spectrum reuse for TV broadcasting while keeping coverage area size and bandwidth per TV broadcaster unchanged. One is to boost the transmit power in traditional single-transmitter broadcasting and the other is to switch to LPLT SFNs. The impact of both strategies on spectrum efficiency is compared from the results of two cases: where TV stations are uniformly distributed throughout a large region and stations all have the same sized coverage areas, and where stations have the same size and coverage areas as existed in the U.S. in 2016, which is very far from uniformly distributed. In both cases, this work finds that a change in technology can free a significant amount of spectrum, and that the LPLT SFN approach frees more spectrum, although at a greater cost. However, the amount of spectrum freed in these two cases differ both quantitatively and qualitatively. Underlying reasons for this may be, at a higher degree, the large disparities in population density across the U.S. that promote TV stations geographic distribution, in addition to the first-come-first-served TV spectrum assignment policy in the U.S. to a lesser extent.

In the case where broadcasters are distributed roughly uniformly, this work finds that increasing transmit power in a traditional single-transmitter architecture can free up to roughly 30% of the spectrum used for TV, assuming that the stations are packed together as closely as possible both before and after the transition. Switching to an LPLT SFN can free up over 60% of the spectrum used for TV. However, these increases in spectrum efficiency come at a cost that cannot be ignored. It is only worth adopting a new strategy for TV broadcasting if the value of the spectrum freed exceeds that cost. If spectrum is worth $2 per MHz-POP, then a region with 30 POP/sq km or more would benefit from a switch to an interference-limited approach in traditional broadcasting rather than the nearly noise-limited approach. Because LPLT SFNs free up considerably more spectrum per area but are also more expensive, LPLT SFNs make sense only where population density is even higher. If spectrum is valued at $2 per MHz-POP, then LPLT SFNs would be cost effective for population densities of roughly 120 POP/sq km or more over an area large enough to include multiple broadcast markets. If the cost per LPLT SFN tower can be reduced by around 60–70% from the baseline value (derived from FCC, 2010b), then LPLT SFNs would be even more cost-effective. In this case, it would be worthwhile to switch from traditional noise-limited broadcasters to LPLT SFNs at a population density of just 20 POP/sq km or more, and traditional single-tower broadcasting with elevated transmit power is never preferable to LPLT SFNs. This result may be particularly important because the cost per transmitter site may be lower in some countries other than the U.S. Streamlined tower siting combined with more extensive tower sharing may someday bring those costs down in the U.S. as well.

In the scenario where broadcasters are distributed as in the U.S., increasing the transmit power of every broadcaster by 3 dB can free 12 MHz nationwide, and a transition to LPLT SFNs can free 54 MHz nationwide. This is significant, but considerably less than in the previous case. This difference occurs because in a nation where the TV stations' density varies as much as it does in the U.S., the amount of spectrum needed nationwide for TV depends primarily on the TV needs of a few large cities, and there is a limit to how much improving frequency reuse can help in these cities. Traditionally, channels with TV broadcasters cannot be licensed for any other purpose, so a technology transition should be judged only by the number of channels it can free nationwide. However, the FCC has recently decided that, for the first time, it will allow some channels to be licensed for TV in some parts of the country and for cellular in other parts. Thus, the benefits of changing TV technology can be greater in this case, because LPLT SFNs make it possible to clear some channels in most of the country, as well as making it possible to clear channels entirely. Nevertheless, the amount of spectrum entirely or mostly freed when all TV broadcasters change their technology appears to be less cost-effective in the case where broadcasters are not uniformly distributed.

On the other hand, it is found that in the non-uniform case, as opposed to the uniform case, there is no need for all broadcasters to make the technology transition to free spectrum nationwide. In areas, nations, or groups of nations with a highly uniform distribution of broadcasters, this is absolutely necessary. In the case of the U.S., similar gains in spectrum efficiency with LPLT SFNs are possible at less than half the cost by adopting LPLT SFNs in just 15 states. In general, it can be expected that in the most cost-effective solution, some broadcasters would adopt LPLT SFNs, some would operate a single transmitter at higher power, but the majority would remain noise-limited single-transmitter systems. The changes would occur in or near large cities, where the demands for spectrum are greatest. Further research is required to find the most cost-effective approach for the U.S., or any other part of the world with similar setup, by identifying more effective ways of determining which broadcasters should either switch to LPLT SFNs or increase their transmit power in order to maximize the ratio of benefit to cost. In addition, these numerical results assume that broadcasters all increase transmit power by the same 3 dB, but better results are achievable if stations can increase their transmit powers by different amounts.

During this work, it is surprising that even having 0 km separation distance between coverage areas via SFNs is not enough to counteract the effect of the non-uniform distribution. This suggests that there is an opportunity for policymakers in the U.S. to revisit the TV licensing rules in a way that it could help in narrowing the gap between the spectrum efficiency we see in the uniform case and in the U.S. case. Some broadcasters might want to modify and/or move their coverage area rather than keep it exactly the same, and regulations might consider that possibility when making a technology change and repacking. That was not an option that was considered in the Incentive Auction, but —setting the complexity of the auction itself aside— maybe it should have.

Finally, even when overall benefits exceed overall costs, broadcasters cannot be expected to incur all the costs of a technology transition while others derive all the benefits. In such as case, further research will be needed in terms to help policymakers to develop appropriate allocation mechanisms for both benefits and costs. So far, this work has considered only the value of the spectrum and network deployment cost in this analysis, but there might be additional benefits for broadcasters as well. For example, a LPLT SFN architecture may present opportunities to geographic targeted advertising, hyper-local content, or the ability to transmit content in a hybrid Multicast/Unicast fashion Shi et al., (2014a, 2014b). Additionally, more work is needed to assess how spectrum management agencies would need to coordinate the work required to move broadcasters from the existing frequency assignment to new channels, in order to minimize service disruption to TV viewers.

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