

# Lamb Waves and Nearly-Longitudinal Waves in Thick Plates

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## ABSTRACT

We discuss waves created in relatively thick plates by edge excitation at frequency-thickness ( $fd$ ) products that correspond, in principle, to multiple Lamb wave modes. For relatively low values of the  $fd$  product it is clear that Lamb wave modes will be generated, while at large values of the  $fd$  product we observe a bulk (longitudinal wave) in the solid, but influenced by reflections from the plate surfaces. We show that for a range of intermediate  $fd$  products a train of regularly spaced nearly-longitudinal waves is generated. The development of a lead pulse and trailing pulses, all traveling at the longitudinal (bulk) wave speed, is well known and has been explained in the literature. In this paper we describe the transition from Lamb wave generation to the formation of nearly-longitudinal waves with their trailing pulses. We report experimental results and theoretical results, with good correspondence between them. We also examine the transfer of energy from leading to trailing pulses, which means that such nearly-longitudinal waves will not propagate indefinitely; however, we show that they retain ample energy for flaw detection at distances of several meters. Most importantly, we study the interaction of these trailing pulses with cracks, again showing experimental results and theoretical predictions that are consistent with one another. The results suggest that these nearly-longitudinal waves are an attractive option for flaw detection because of their shorter wavelength (as compared to Lamb waves at low  $fd$  products) and because they preserve their pulse train characteristics after scattering.

**Keywords:** Flaw detection, Lamb waves, longitudinal waves, plates, pulse-echo, ultrasonics.

## 1. INTRODUCTION

Extensive research has been directed at the detection of defects in plate-like structures using Lamb waves. This application has been motivated by the fact that pulses formed from Lamb waves propagate with low attenuation for considerable distances in plates. However, the existence of multiple Lamb wave modes presents a significant complication. Lamb waves are typically excited by transducers mounted on the surface of the plate; numerous modes are present when the frequency-thickness product  $fd$  exceeds, say, 3 MHz-mm (in metals such as steel or aluminum). As a result, many researchers have recommended working at low  $fd$  products so that only a small number of modes are excited [1] and other work has concerned the development of transducers that selectively emit particular modes [2, 3]. However, limiting operation to low frequencies is not a completely satisfactory solution because scattering becomes weaker and the resolution is degraded when the defect is small compared to the wavelength.

We have studied the creation of waves in plates by edge excitation [4] at  $fd$  products that correspond to multiple Lamb wave modes; in this paper we briefly summarize those results and present studies of flaw detection. For relatively low  $fd$  it is clear that Lamb wave modes will be generated while at large  $fd$  we will encounter longitudinal waves, possibly influenced by reflections from the surface. We show that for a range of intermediate  $fd$  there develops a pulse train of nearly longitudinal waves. These “trailing pulses” are well known and have been discussed in the literature. We then consider the interaction of these trailing pulses with flaws such as cracks. Results suggest that these nearly longitudinal waves are an attractive option for flaw detection.

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## 2. EXCITATION OF PLATE EDGE

We consider waves generated by a force applied normal to the edge of a plate. Figure 1 shows the geometry under consideration. A plate of thickness  $d$  is excited by a distributed force normal to the edge. A good approximation to this excitation is obtained when the plate is excited by a conventional piezoelectric transducer using grease coupling.



Figure 1. Ideal excitation by force normal to edge (top) and practical case of edge excitation (bottom) by grease-coupled transducer

We first consider the transition from a low-frequency regime, in which only Lamb waves are understood to propagate, to a high-frequency regime in which only longitudinal waves would be generated. For plates with thicknesses typical of structural and aerospace applications, we will show that the frequency need not be very high (the  $fd$  product need not exceed 15 MHz-mm) in order to create nearly-longitudinal waves that propagate for considerable distances. We also obtain insight into the formation of trailing pulses that are known to form when longitudinal waves propagate in plates.

The trailing pulses that form when a nearly-longitudinal wave is excited in a plate are shown experimentally in Figure 2. We excited a steel plate, 19 mm thick and 30.5 cm long, by an edge-mounted piezoelectric wafer (Piezo Systems 5A4E), 15x15x1 mm with a lead backing 3 mm thick, bonded to the plate edge with cyanoacrylate adhesive. The transducer was driven by a five-cycle windowed sinusoid voltage waveform

$$v(t) = \begin{cases} v_0 \sin(2\pi ft) \cdot \left( \sin\left(\frac{ft}{20\pi}\right) \right)^2 & t < \frac{5}{f} \\ 0 & t \geq \frac{5}{f} \end{cases} \quad (\text{Eq. 1})$$

with a center frequency of 2.2 MHz. Figure 2 shows the signal received at a similar transducer on the opposite plate edge, consisting of a pulse at 52.6  $\mu\text{s}$  followed by several trailing pulses. The first arrival at 52.6  $\mu\text{s}$  corresponds to a group velocity of 5.8 km/s, which agrees well with the longitudinal velocity in steel. The leading pulse is followed by several trailing pulses, equally spaced at roughly 5  $\mu\text{s}$ . In this instance the leading pulse is the largest, but at different propagation distances or at different center frequencies one of the trailing pulses may be the largest.

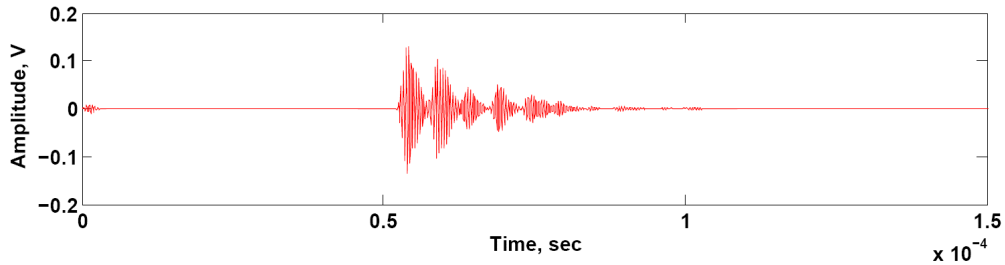


Figure 2. Signal received after a propagation distance of 30.5 cm; the small signal at 0 s is due to electrical stray coupling

Trailing pulses are well known [5] and are associated with boundary reflections; the pulses have been termed “secondary echoes produced by split-off transverse waves” and explained as shown in Figure 3. In general, the transducer creates a longitudinal wave that propagates straight down the plate (a) and a longitudinal wave that can be viewed as striking the plate edge at oblique incidence (b). The latter wave produces a reflected longitudinal wave (c) and a reflected transverse wave (d). Consider now the case of the oblique longitudinal wave (b) approaching the boundary at grazing incidence, such that the reflected longitudinal wave (c) propagates straight down the plate; in that case the reflected transverse wave (d) forms at the critical angle  $\theta$ , and when it strikes the opposite edge it creates another longitudinal wave (e) propagating straight down the plate but delayed with respect to the original pulse (a), along with a reflected transverse wave (f). The process repeats, resulting in additional trailing longitudinal pulses propagating straight down the plate.

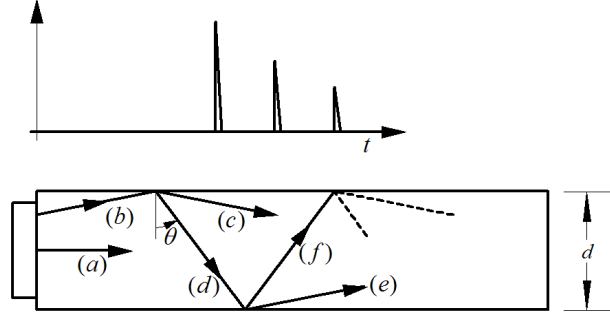


Figure 3. Simple model [5] for formation of trailing pulses as “split-off transverse waves”

In steel, the reflected transverse wave (d) is produced with an angle  $\theta$  near  $33^\circ$ , and the time separation between successive trailing pulses,  $\Delta t$ , can be calculated as

$$\Delta t = \frac{d}{c_t \cos \theta} - \frac{d \tan \theta}{c_l} \quad (\text{Eq. 2})$$

where  $c_t$  and  $c_l$  are the transverse and longitudinal wave velocities, respectively. In steel, at a plate thickness of 19 mm,  $\Delta t$  is predicted to be  $4.7 \mu\text{s}$ , which is in reasonable agreement with the  $\Delta t$  of  $5 \mu\text{s}$  observed experimentally in Figure 2.

However, this simple, classical picture provides no guidance as to the lowest  $fd$  at which the longitudinal pulse propagates for useful distances, nor does it determine how rapidly the leading pulse decays in amplitude. In the following section, we report on our investigations of the generation and propagation of waves in this geometry using finite element simulations. These results provide insight into the generation and propagation of trailing pulses. In addition, these simulations show how a propagating pulse sheds energy by the creation of trailing pulses with an identical shape.

### 3. SIMULATIONS OF WAVE GENERATION

Simulations were performed in the Comsol 3.2 plane-strain structural mechanics mode for a plate 2 cm thick and 35 cm long, and a force in the x direction with the time dependence of Eq. (1) was applied to the left edge of the model. Material parameters of 200 GPa elastic modulus, 0.33 Poisson ratio, and 7.85 specific gravity were used, appropriate for steel. A transient (time dependent) analysis was performed using a step size of  $1/10f$ .

We first consider the effect of pulse center frequency on wave generation. Figure 4 shows the effect of launching a wave with edge excitation at various frequencies by plotting the particle displacement (in the x direction) in the plate at  $50 \mu\text{s}$ . At low frequencies (up to about 450 kHz) the wavefront velocity is significantly slower than the bulk wave longitudinal velocity, and particle displacements are similar to those for low-order symmetric Lamb modes. Figure 5 shows a snapshot of particle velocity at  $175 \mu\text{s}$  in a plate 85 cm long. Two symmetrical Lamb wave modes are visible, a faster mode with highest particle velocity at the top and bottom surfaces, and a slower mode with highest particle velocity at the midthickness. The group velocity for the faster mode, based upon the wavefront position, is estimated as 4.6 km/s,

and the group velocity for the slower mode is estimated as 3.4 km/s. These particular modes can be identified as S1 and S2 modes, respectively, and both of these Lamb wave modes are predicted to be present at an  $fd$  product of 4 MHz-mm.

At higher  $fd$  products, theory predicts that a greater number of Lamb wave modes will be present with a considerable range of group velocities, and as a result one might expect a long pulse train with complex character. However, the simulation results in Figure 6 for higher frequency excitations show particle velocities that are nearly uniform across the thickness of the plate, traveling with a group velocity essentially equal to the bulk wave longitudinal velocity. This behavior is fully established at 750 kHz and above (when the  $fd$  product reaches 15 MHz-mm). The trailing portion of the response differs in character from a trailing region of Lamb wave modes.

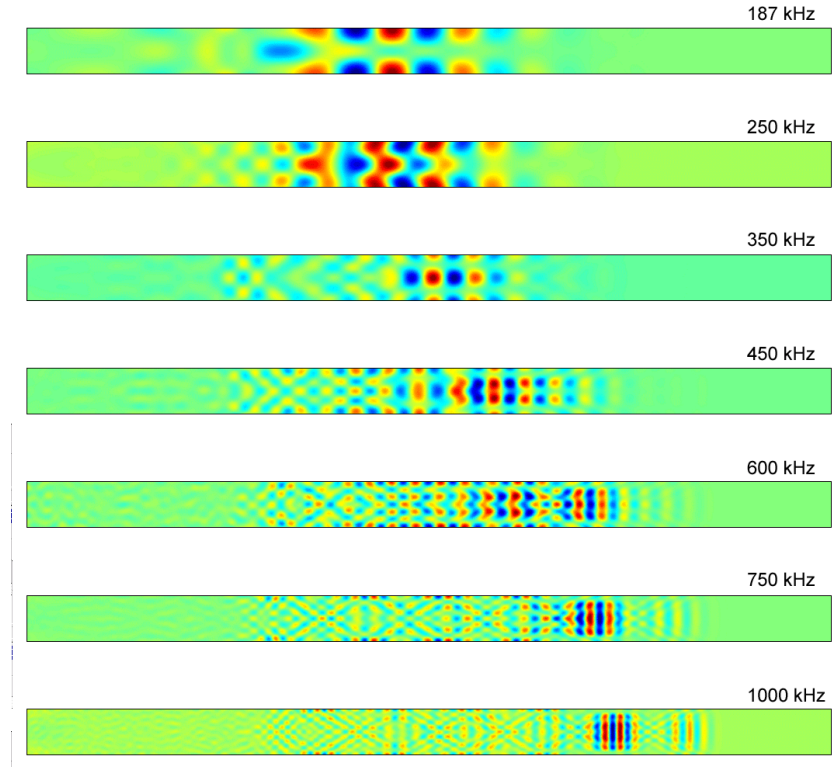


Figure 4. Effect of center frequency on propagating waves for a steel plate 2 cm thick; color shows particle x-displacement at 50  $\mu$ s.

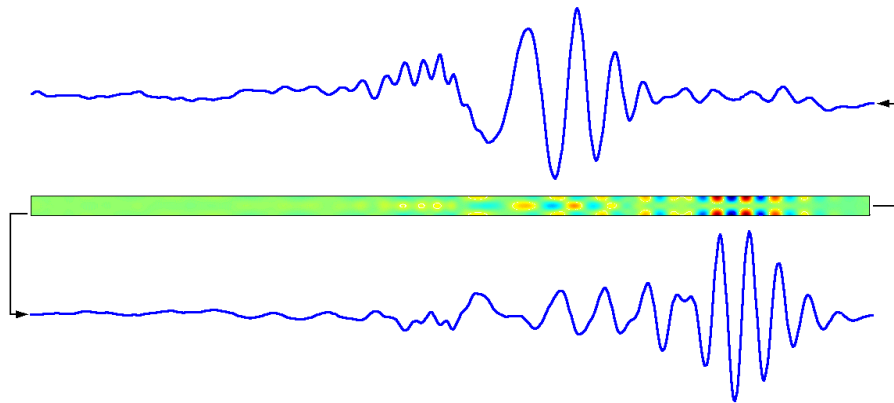


Figure 5. Snapshot at 175  $\mu$ s, center frequency 200 kHz, showing x displacement at midthickness (top) and at surface (bottom)

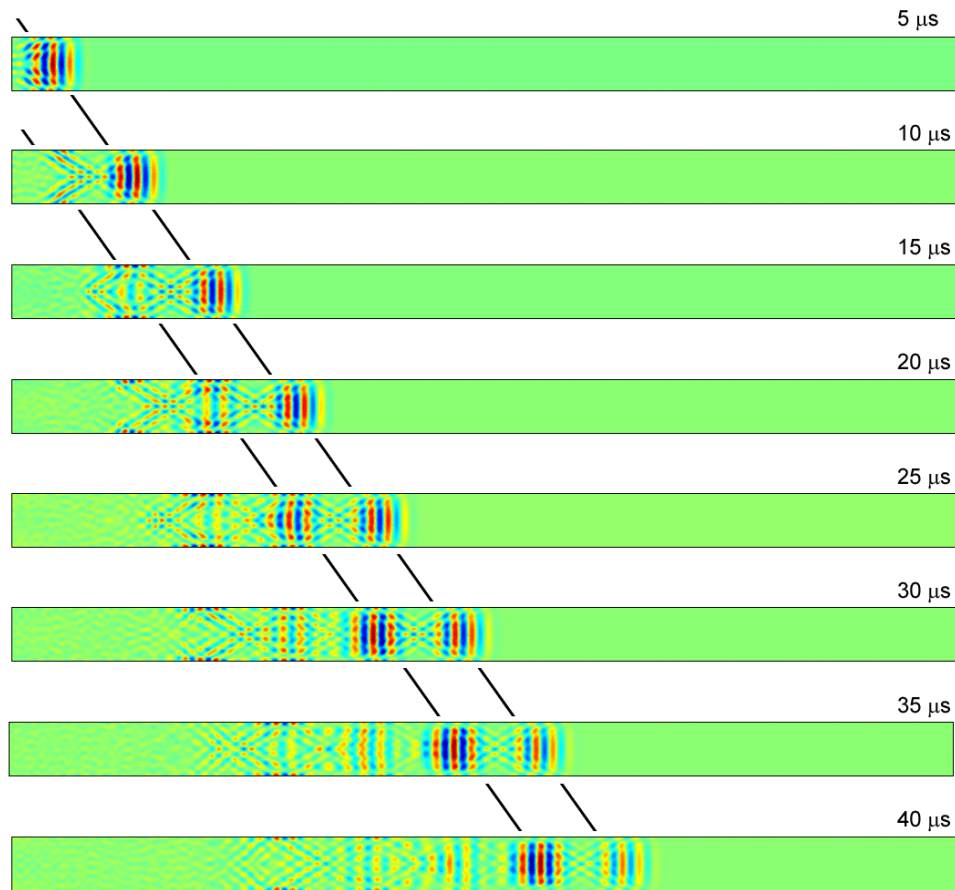


Figure 6. Snapshots showing x displacement in steel plate, 2 cm thick, 1 MHz center frequency

Figure 6 shows snapshots of particle displacement in the x direction at different times for a 1 MHz excitation of a plate 20 mm thick. The simulations show the formation of a near-planar wavefront within the first 5-10  $\mu\text{s}$  of propagation. This wavefront propagates at the longitudinal velocity and gradually loses energy while a second trailing pulse forms and gains energy. Elsewhere we show [4] that the transfer of energy from any pulse to its trailing pulses is dependent on plate thickness, center frequency, and time (path length).

#### 4. SCATTERING FROM FLAWS, SIMULATION STUDY

We next consider scattering of nearly-longitudinal waves from a crack. It is possible that scattering from a crack would result in mode conversion to multiple Lamb wave modes, which would make detection of cracks substantially more difficult. Therefore, we undertook simulation and experimental studies to determine whether such unwanted mode conversion occurs. In order to observe such mode conversion, were it to occur, simulations were performed to include the effect of scattering from a part-thickness slot 1 mm wide and with varying depth. The simulations were performed for a five-cycle windowed sinusoid excitation (Eq. 1) with a center frequency of 1 MHz. The plate was 0.4 m long, and the crack was located 0.2 m from the left edge where the excitation was applied. Figure 7 (top) shows the pulse train approaching the crack at 30  $\mu\text{s}$ , and (bottom) the transmitted and reflected waves after passing the crack at 50  $\mu\text{s}$ .

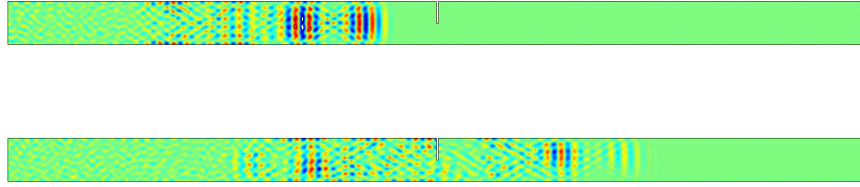


Figure 7. Pulse train, center frequency 1 MHz, with a half-thickness crack 1 mm wide: (top)  $t = 30 \mu\text{s}$ ; (bottom)  $t = 50 \mu\text{s}$ ).

Figure 8, top and bottom, shows the time history of x-direction particle velocity (averaged across the plate thickness) at two locations, 0.10 m and 0.30 m from the left edge, which are symmetrically positioned with respect to the crack. Figure 8 (top) shows the incident wave approaching the first location at approximately  $17 \mu\text{s}$ , and a wave reflected from the crack approaching that same location on its return path at approximately  $51 \mu\text{s}$ . Figure 8 (bottom) shows the transmitted wave, after passing the slot, approaching the second location at approximately  $51 \mu\text{s}$  with nearly the same amplitude and shape as the reflected wave returning to the first location. Examining the wave arriving at the second location, and its counterpart reflected wave returning to the first location, we note that the second pulse is larger than the leading pulse, consistent with the simulations for a total travel distance of 0.30 m without a crack as shown in Figure 8.

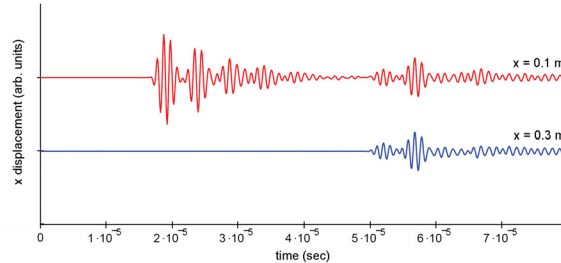


Figure 8. Time history of simulated x-direction particle velocity at two locations

## 5. SCATTERING FROM FLAWS, EXPERIMENTAL STUDIES

Experiments confirm that nearly-longitudinal waves retain their most-important characteristics (velocity, trailing pulse train, and pulse shape) when reflected from a flaw. Figure 9 shows the response recorded at the exciting transducer for a steel plate 19 mm thick and 30.5 cm long; a slot 12.6 cm wide and 5.7 mm deep has been machined into the plate midway along the length. A nearly-longitudinal pulse train returns to the transducer as an echo from the crack, arriving at roughly  $50 \mu\text{s}$ . Another pulse train approaches the transducer starting at roughly  $100 \mu\text{s}$ ; it consists of the original wave reflecting from the opposite edge and twice scattered, together with the wave first reflected from the crack being reflected from the transducer edge and then a second time from the crack. The two wave trains are quite similar and show the same characteristics of nearly-longitudinal waves. From the simulations and experiments we conclude that nearly-longitudinal waves retain their velocity and pulse shape when scattered.

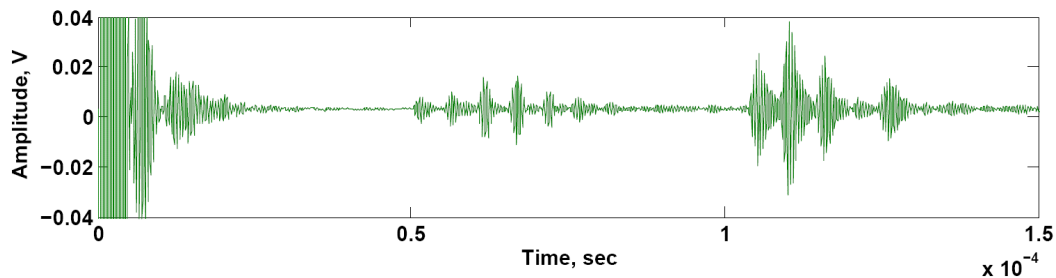


Figure 9. Signal at pulse-echo transducer showing an echo near  $50 \mu\text{s}$  from the slot, and an echo near  $100 \mu\text{s}$  from the far edge

Related experiments were conducted on a steel plate specimen 91.5 cm long and 9.5 mm thick, with a through-thickness crack (slot) 3.2 cm long located 30.5 cm from the right plate edge. A Krautkramer gamma series 5.0 MHz probe was used, for a nominal  $fd$  product of 47.5 MHz-mm, with a USPC-2100 pulser and data acquisition system. Figure 10 is a sketch of the test configuration, and Figure 11 (a through d) shows pulse trains returned as echoes to that transducer.

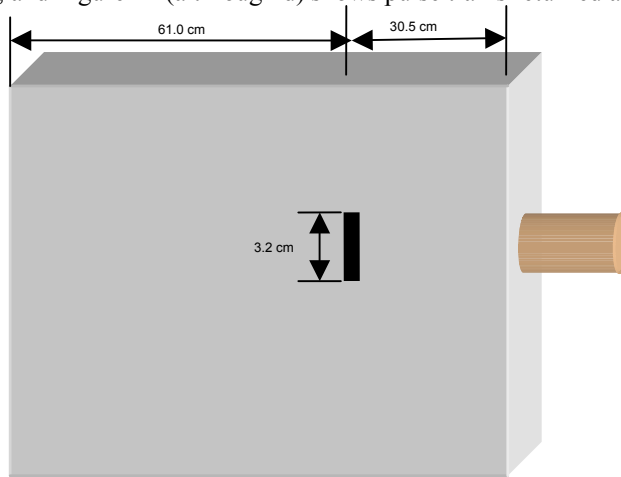


Figure 10. Pulse-echo test, 9.5 mm plate thickness, 5 MHz commercial transducer (12.5 mm aperture), through-thickness crack

Figure 11a shows the first echo arriving near  $103 \mu\text{s}$ , which is consistent with the total path length of 61 cm and a bulk wave velocity of 5.9 km/s. The leading pulse is already diminished in comparison the first trailing pulse, and the pulse spacing of  $2.6 \mu\text{s}$  corresponds well with the prediction obtained from Eq. 2. Figure 11b shows the second echo, scattered twice by the crack and once by the plate edge, arriving near  $206 \mu\text{s}$ . The leading pulse is scarcely detectable, and the third pulse (the second trailing pulse) is largest in magnitude, slightly greater than the fourth pulse. Figure 11c shows the third echo, and with the sensitivity setting used in recording that plot the first arrival occurs near  $315 \mu\text{s}$  and the strongest pulses in this train are respectively the third and the fourth in the train. This suggests that the original leading pulse and first trailing pulse were below the threshold of observation, that the first arrival was originally the second trailing (third overall) pulse, and that the strongest pulses send in this record were originally the fifth and sixth overall. (Figure 11c is complicated somewhat by the fact that an echo from the far edge would arrive near this same time, diffracted on its return path by the crack.) Figure 11d shows the fourth and fifth echoes, arriving roughly near 420 and  $525 \mu\text{s}$ , but we do not attempt to trace the identity of the individual pulses in those trains.

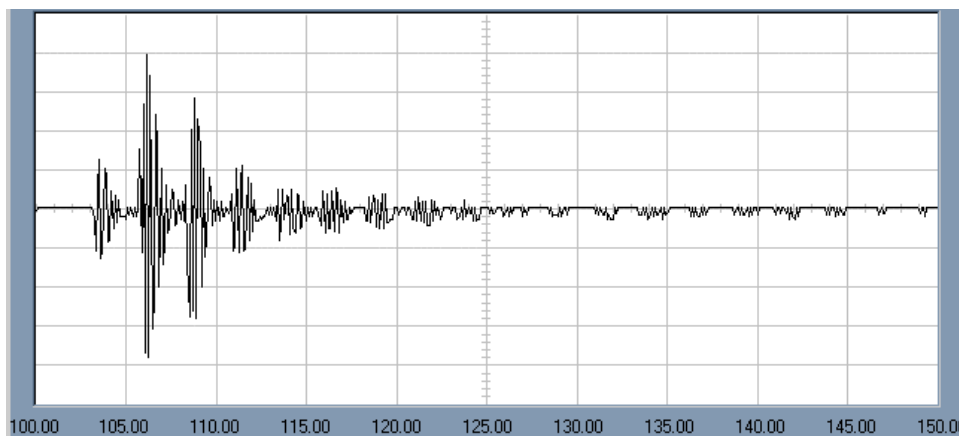


Figure 11a. First echo from crack arrives near  $103 \mu\text{s}$ , trailing pulses observed at approximately  $2.6 \mu\text{s}$

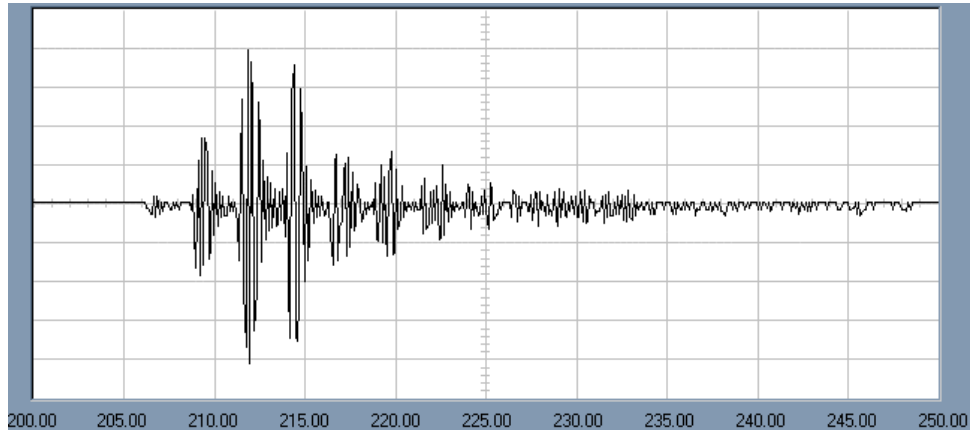


Figure 11b. Second echo from crack arrives near 206  $\mu$ s

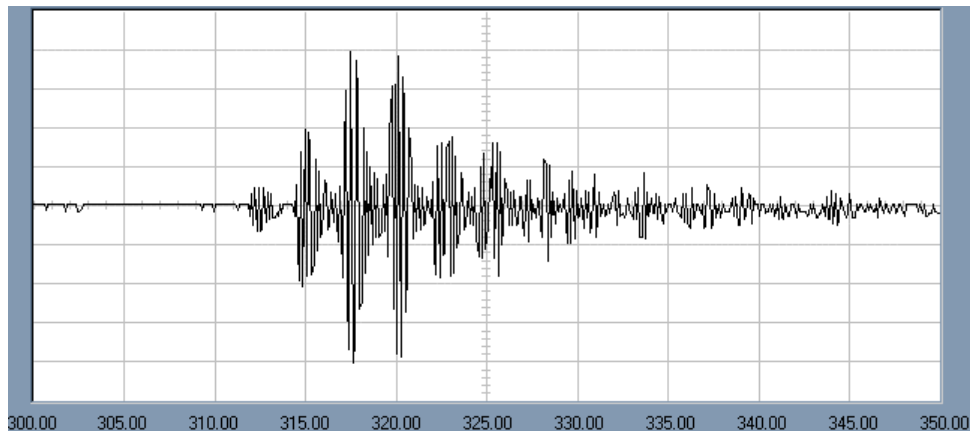


Figure 11c. Third echo from crack arrives near 315  $\mu$ s (roughly coincident with echo from far edge of plate)

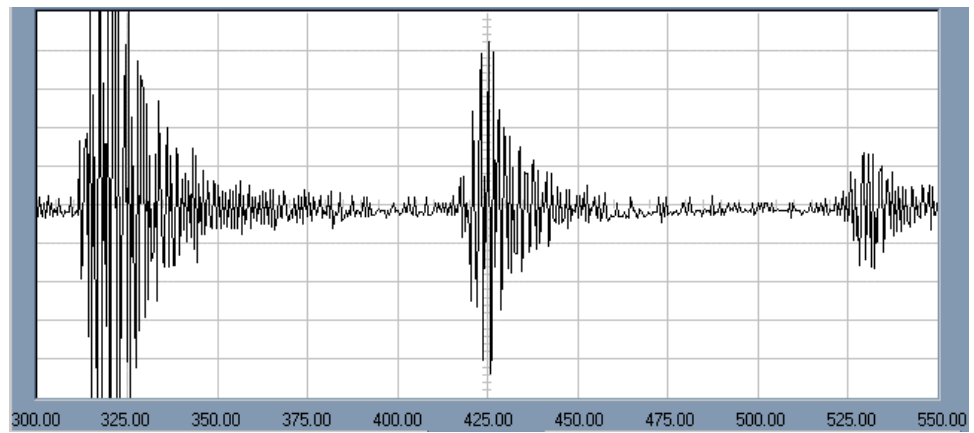


Figure 11d. Fourth and fifth echoes from crack arrive near 420 and 525  $\mu$ s, respectively (note change in time scale)



The experiment depicted in Figure 10 was repeated with the probe placed at the left edge of the plate, 60.5 cm from the crack location. Figure 12a shows the first echo arriving near 206  $\mu\text{s}$ , with the leading pulse scarcely detectable and the third pulse (the second trailing pulse) largest in magnitude, slightly greater than the fourth pulse; this essentially reproduces the record shown in Figure 11b, which was a second echo but with the same total path length. Figure 12b shows an echo arriving near 315  $\mu\text{s}$ , representing a reflection from the far edge, which would have been diffracted when passing the crack location, and an echo arriving 415  $\mu\text{s}$ , representing the second echo from the crack. We note that the echo from the relatively small crack is easily detected even after multiple echoes and even after the wave has traveled a path length of almost 3 m. We observe that the crack could have been detected at even greater distances; our experiment was limited not by amplification or sensitivity, but by the time limit of signal acquisition, slightly more than 600  $\mu\text{s}$ .

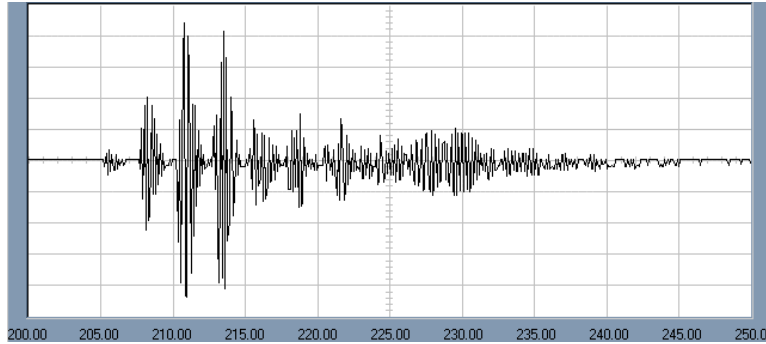


Figure 12a. First echo from crack arrives near 206  $\mu\text{s}$  (transducer has been moved to left edge of plate in Figure 10)

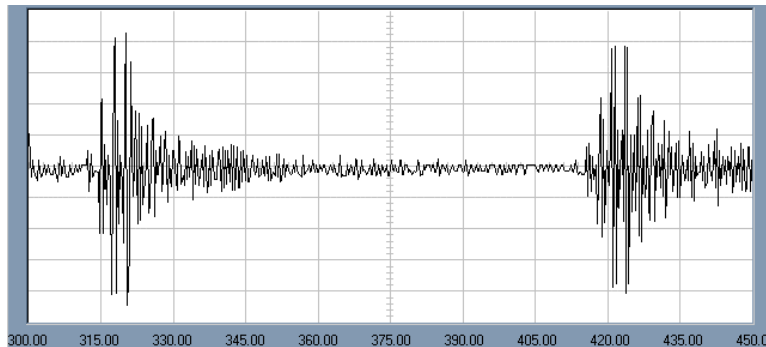


Figure 12b. Second echo from crack arrives near 415  $\mu\text{s}$

Finally, crack detection was tested on a large plate 19 mm thick with a through-thickness crack, 6 cm long, 96.4 cm from the plate edge. Figure 13 shows the first echo arriving near 327  $\mu\text{s}$ , which corresponds precisely with the travel time for a longitudinal pulse and therefore is evidence that the pulse is the original leading pulse. The spacing between trailing pulses is approximately 5  $\mu\text{s}$ , as observed earlier in Figure 2 with a plate of the same 19-mm thickness.

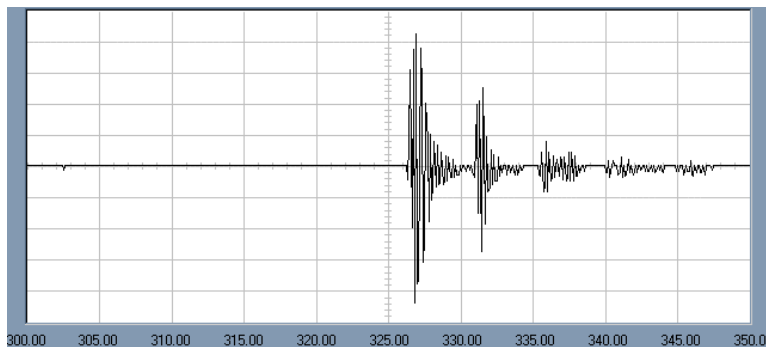


Figure 13. First echo from crack arrives near 327  $\mu\text{s}$  (crack 96.4 cm from plate edge, 19 mm plate thickness)

## 5. SUMMARY AND CONCLUSIONS

We have reviewed the generation of waves in plates by force excitation normal to a plate edge. Simulations and experiments both show that multiple Lamb modes are created at low  $fd$  products, while for  $fd$  greater than 15 MHz-mm a nearly longitudinal pulse is generated which is followed by several trailing pulses. During propagation, energy is transferred from the leading pulse to trailing pulses by boundary reflections from the surfaces of the plate. Simulations and experiments both show that the pulse train retains its integrity over distances of at least 1 m, and subsequent experiments show the pulse train to retain its integrity over distances greater than 3 m. We suggest that these nearly-longitudinal waves are attractive as an alternative to Lamb waves for the detection of defects in plate-like structures. An important advantage of the nearly-longitudinal modes is the shorter wavelength which will result in enhanced scattering from small defects.

We simulated the scattering of a pulse train by a part-thickness slot in a plate 19 mm thick, in which both the transmitted and reflected pulse trains show no sign of mode conversion to Lamb waves, and then obtained experimental confirmation of that observation. Finally, we studied pulse-echo signals reflected from a small through-thickness flaw in plates 9.5 and 19 mm thick, tested with a commercial ultrasonic flaw detection system using a 5.0 MHz probe. Echoes were unambiguous even after multiple reflections and even with a total path length approaching 3 m.

It is instructive to compare the nearly-longitudinal transients as they are described here to Lamb waves (harmonic waves) as they are most commonly described mathematically, with particular attention to practical implications for flaw detection. Setting aside the influence of material attenuation, which is relatively modest in materials such as steel or aluminum, and temporarily setting aside the geometric spreading of the wavefront, a Lamb wave can be viewed as a harmonic wave, extending indefinitely in time and space. The nearly-longitudinal wave described here is a transient disturbance that sheds its energy into an ever-changing train of trailing pulses. However, the practical question is not whether it will propagate to infinity, but whether it will propagate far enough to permit useful flaw detection. The measurements summarized in Figures 12 and 13 show that an ultrasonic pulse can propagate considerable distances, approaching 3 m, with sufficient energy and preservation of pulse shape to provide unambiguous pulse-echo flaw detection. The short wavelength of the relatively high-frequency pulse forms an unambiguous echo from flaw sizes of practical interest, and the absence of multiple modes makes it easy to interpret the echoes, in contrast to the challenge when multiple Lamb wave modes are present. Whereas Lamb waves in plates are correctly termed guided waves, the nearly-longitudinal wave, excited at relatively high frequency by an edge-mounted transducer, is also a guided transient disturbance with great practical utility.

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## REFERENCES

1. J.-B. Ihn and F.-K. Chang, "Detection and monitoring of hidden fatigue crack growth using a built-in piezoelectric sensor/actuator network: I. Diagnostics," *Smart Mater. Struct.* 13, 609-620, 2004.
2. Zhu, W., and Rose, J. L., "Lamb wave generation and reception with time-delay periodic linear arrays: A BEM simulation and experimental study," *IEEE Trans. Ultrasonics, Ferroelectrics and Freq. Ctl.*, 46, 654-664, 1999.
3. Giurgiutiu, G., "Lamb Wave Generation with Piezoelectric Wafer Active Sensors for Structural Health Monitoring," *Proc. SPIE- Smart Structures and Materials 2003: Smart Structures and Integrated Systems*, 5056, 111-122, 2003.
4. Greve, D. W., Zheng, P., and Oppenheim, I. J., "Excitation of longitudinal and Lamb waves in plates by edge-mounted transducers," *Proc. IEEE Ultrasonics Conference*, New York, October 2007.
5. Krautkramer, J., and Krautkramer, H., *Ultrasonic Testing of Materials*, 4th fully rev. ed., Springer-Verlag, 1990.