Noncoherent Compressive Sensing with Application to Distributed Radar

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***Abstract*—We consider a multi-static radar scenario with spatially dislocated receivers that can individually extract delay information only. Furthermore, we assume that the receivers are not phase-synchronized, so the measurements across receivers can only be combined noncoherently. We cast this scenario as a compressive sensing reconstruction problem, where the vector of unknowns consists of complex baseband coefficients of tentative targets at discrete positions within a region of interest. The difference to previous work is that each receiver has to have a separate set of variables to account for the noncoherent measurement model. This leads to multiple reconstruction problems that are individually ill-defined, but can be regularized by a shared sparsity pattern, as studied in jointly- or block-sparse reconstruction problems. We evaluate this approach in a simple scenario with three receivers and three closely spaced targets. Using the popular basis pursuit and orthogonal matching pursuit algorithms, we find that targets can be fully resolved and that the position estimation error is close to the Cramér-Rao lower bound based on an estimator that knows the number of targets.**

I. Introduction

In radar, to detect and localize targets, a transmitter emits a waveform and a receiver observes the superposition of reflections of this waveform emanating from the targets. Neglecting Doppler effects due to target motion, the received baseband signal can be characterized as a linear combination of delayed transmit signals corresponding to the targets, each weighted by a complex coefficient. The complex baseband coefficients quantify the attenuation and phase shift caused by the interaction with the target, its radar cross section (RCS), which depend on the specific type of the target and its orientation as presented to the receiver. In this sense the radar processing problem consists of detecting how many targets compose the received signal and estimating their corresponding weights and delays [1], [2].

To simplify this challenging task, it is commonly reduced to only a detection problem by discretizing the search space into range-, angular-, and/or Doppler-bins [1]. Then a matched filter is used to evaluate the likelihood of a target being present for each bin – but agnostic of all other bins. If the transmit waveform and receiver array are designed appropriately only likelihoods of directly neighboring bins will be correlated, and since only few bins will be occupied at any point in time, these neighboring detections can be fused in a post-processing stage.

Compressive sensing (CS) is a topic that has lately gained much attention in the applied mathematics and signal processing communities [3], [4]; it involves reconstructing an underlying vector of unknowns based on an under-defined linear measurement model under the additional assumption that the vector of unknowns is sparse, i.e, most unknowns are zero. The application of CS to radar processing [5]–[7] uses the fact that in radar the received signal can be modeled as a linear combination of waveforms corresponding to the target bins. The vector of unknowns then consists of the complex baseband coefficients of hypothetical targets in each bin, which is sparsely populated with targets. The main appeal in using CS in radar processing is that it allows for joint bin processing without putting unrealistic assumptions on targets appearing only in centers of bins of a given size. This leads to good target resolution without additional post-processing [5] and works even if the waveform design cannot be controlled [7].

In [7] a bi-static setup was considered, where the receiver could not extract any angular information (no receiver array). This limited the system to only extract delay estimates that place each target on an ellipse around the transmitter-receiver axis. We investigate a similar setup, but where we want to combine the measurements made by multiple spatially dislocated receivers. The conventional approach would be to process each receiver separately (using matched filter or CS processing), extracting multiple sets of delay estimates, and fusing these estimates in a subsequent stage (intersection of ellipses). Instead, we would like to jointly process all receiver measurements using CS recovery algorithms.

In the previously outlined CS model multiple receivers can be combined, if their received signals can be formulated using a joint or *coherent* linear measurement model, based on the same underlying sparse vector of complex baseband coefficients. This is generally true if all receivers are spatially colocated, as in a receiver array, because then they will observe the same target RCS. When receivers are spatially dislocated, the underlying vector of unknowns will not be identical, either due to variations in the target RCS as viewed from different angles, or simply because it is not feasible to tightly synchronize the local oscillators in each receiver leading to unknown phase rotations (see also discussion in [8]). This is denoted as a *noncoherent* measurement model.

Discretizing the region of interest into a common set of target bins, each sparse vector contains different coefficients, but will present the same sparsity pattern. This can be cast as multiple separate linear reconstruction problems corresponding to each receiver, with independent variables per receiver – but linked through the shared sparsity pattern. Similar reconstruction problems have been studied under the names jointly- or block-sparse; see [9] and reference therein.

We illustrate the application of this to the task of multi-static

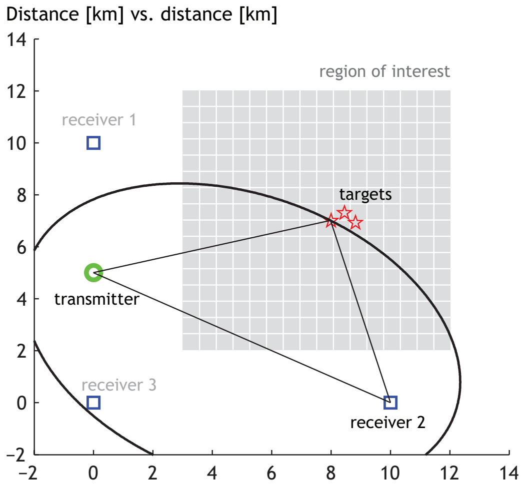


Fig. 1. We consider a multi-static setup with one transmitter and three widely separated receivers that individually do not have angular sensing capabilities (no array); delay estimates place targets on ellipses around the transmitter-receiver axis.

radar processing. Specifically we consider a scenario where three receivers observe multiple targets and use the popular basis pursuit (BP) and orthogonal matching pursuit (OMP) algorithms to detect and localize targets. Since targets naturally do not appear in bin centers, no specific non-zero support needs to be recovered. Instead we focus on target localization error, which means that we measure how close the detected bin is relative to the original target position. This naturally leads to much closer spaced bins relative to what would be considered in a matched filter. We find that even though the matched filter output has many local maxima, the three targets can be recovered with few false alarms. Furthermore the localization error is close to the Cramér-Rao lower bound based on only additive Gaussian noise and a known number of targets.

II. Setup & Conventional Approach

A. System Model

In a multi-static radar setup, a transmitter emits a waveform that is reflected off targets and arrives at the receivers with certain delay and attenuation. Assuming complex baseband formulation, the received signal at the n-th receiver is

$$r_n(t) = \sum_p \xi_{p,n} s\left(t - \tau_{p,n}\right) + w_n(t), \tag{1}$$

where $w_n(t)$ is a Gaussian noise process of spectral power N_0. Each complex attenuation factor, $\xi_{p,n}$, is related to the radar cross section (RCS) of the p-th target as seen by the n-th receiver. The delay $\tau_{p,n}$ is the bi-static delay consisting of the travel time from the transmitter to the p-th target and from this target to the n-th receiver, see example in Fig. 1. Defining positions in a two-dimensional plane, with the transmitter located at $\mathbf{x}_t$, the p-th target at $\mathbf{x}_p$, and the n-th receiver at $\mathbf{x}_{r,n}$, we have

$$\tau_{p,n} = \frac{1}{c}\|\mathbf{x}_p - \mathbf{x}_s\| + \frac{1}{c}\|\mathbf{x}_p - \mathbf{x}_{r,n}\|, \tag{2}$$

where $c = 3 \cdot 10^5$ km/s is the speed of light.

If the signal $s(t)$ has a finite bandwidth B, the received signal can be represented as discrete-time samples without loss:

$$r_n[k] = \int r_n(t) g_{\mathrm{LPF},B}\left(kT_s - t\right) dt, \quad k = 1, \ldots, K \tag{3}$$

We assume for simplicity that the low-pass filter $g_{\mathrm{LPF},B}(t)$ is ideal, and accordingly the signal is sampled at Nyquist rate $T_s = 1/B$. The number of samples K is chosen such that it can accommodate the maximum delay of interest, as determined by (2) and the "region of interest" in Fig. 1.

Since we assumed the low-pass filter as ideal, the transmitted waveform is not affected,

$$r_n[k] = \sum_p \xi_{p,n} s\left(kT_s - \tau_{p,n}\right) + w_n[k], \tag{4}$$

where the discrete Gaussian noise samples $w_n[k]$ have power N_0. We define the following length K vectors,

$$\mathbf{r}_n = \begin{bmatrix} r_n[1] \\ \vdots \\ r_n[K] \end{bmatrix} \quad \mathbf{s}(\tau) = \begin{bmatrix} s(T_s - \tau) \\ \vdots \\ s(KT_s - \tau) \end{bmatrix} \quad \mathbf{w}_n = \begin{bmatrix} w_n[1] \\ \vdots \\ w_n[K] \end{bmatrix} \tag{5}$$

The received signal is then

$$\mathbf{r}_n = \sum_p \xi_{p,n} \mathbf{s}(\tau_{p,n}) + \mathbf{w}_n. \tag{6}$$

B. Matched Filter Delay Estimates

The conventional approach consists of each receiver using matched filter processing to detect the number of targets and corresponding delays; which are then combined in a subsequent stage to localize the targets in the two-dimensional plane.

Assuming that the targets' delays can be approximately quantized $\tau_{p,n} \approx m_{p,n}\alpha T_s$, $m_{p,n} \in [1, M]$, we can reformulate (6) in the following way

$$\mathbf{r}_n = \mathbf{S}_\tau \boldsymbol{\xi}_n + \mathbf{w}_n, \tag{7}$$

$$\mathbf{S}_\tau = \begin{bmatrix} \mathbf{s}(\alpha T_s) & \cdots & \mathbf{s}(M\alpha T_s) \end{bmatrix} \tag{8}$$

where the length M vector $\boldsymbol{\xi}_n$ is equal to $\xi_{p,n}$ at its $m_{p,n}$ element and zero otherwise. Clearly the choice of α will affect the accuracy of this approximation significantly.

Estimating the delays can now be accomplished by detecting, which elements of the $\boldsymbol{\xi}_n$ are non-zero; where the elements are commonly referred to as "range bins". A common detection metric is the matched filter output,

$$\boldsymbol{\varphi}_{\tau,n} = \mathbf{S}_\tau^H \mathbf{r}_n = \mathbf{S}_\tau^H \mathbf{S}_\tau \boldsymbol{\xi}_n + \tilde{\mathbf{w}}_n, \tag{9}$$

where the filtered noise vector, $\tilde{\mathbf{w}}_n$, has covariance $N_0 \mathbf{S}_\tau^H \mathbf{S}_\tau$. The $M \times M$ matrix $\mathbf{S}_\tau^H \mathbf{S}_\tau$ contains the auto-correlation function of the transmit waveform $s(t)$,

$$\left[\mathbf{S}_\tau^H \mathbf{S}_\tau\right]_{m,k} = \int s^*(t) s\left(t - (k-m)\alpha T_s\right) dt. \tag{10}$$

Each range bin is processed separately using an energy detector, a target is deemed present in bin m of receiver n if

$$\left| \left[\boldsymbol{\varphi}_{\tau,n} \right]_m \right|^2 > \Gamma_{\mathrm{th}}. \tag{11}$$

This simple detection scheme is optimal if $\mathbf{S}_\tau^H \mathbf{S}_\tau$ reduces to a diagonal matrix ($\mathbf{S}_\tau$ is a unitary matrix). Based on the signal bandwidth B, this is only possible for $\alpha \geq 1$, for which the accuracy of the approximation of the targets' delays is limited. In practice the waveform $s(t)$ is designed such that its auto-correlation function has strictly controlled side-lobes, making $\mathbf{S}_\tau^H \mathbf{S}_\tau$ a banded matrix. Then one target will lead to detections in neighboring bins, which can be addressed in a post-processing stage.

III. Coherent & Noncoherent Receiver Combining

To directly estimate the targets' positions in two dimensions, delays are not discretized uniformly as $\tau_m = m\alpha T_s$, but instead delays corresponding to two dimensional coordinates,

$$\tau_{m,n} = \frac{1}{c}\|\mathbf{x}_m - \mathbf{x}_s\| + \frac{1}{c}\|\mathbf{x}_m - \mathbf{x}_{r,n}\|, \tag{12}$$

where typically the tentative position estimates $\mathbf{x}_m$ are chosen as grid points within the region of interest as in Fig. 1. This means that the matched filter at each receiver will have to take into account the relative transmitter-receiver geometry,

$$\begin{aligned} \boldsymbol{\varphi}_{\mathbf{x},n} &= \begin{bmatrix} \mathbf{s}(\tau_{1,n}) & \ldots & \mathbf{s}(\tau_{M,n}) \end{bmatrix}^H \mathbf{r}_n = \mathbf{S}_{\mathbf{x},n}^H \mathbf{r}_n, & (13)\\ &= \mathbf{S}_{\mathbf{x},n}^H \mathbf{S}_{\mathbf{x},n} \boldsymbol{\xi}_n + \tilde{\mathbf{w}}_n. & (14) \end{aligned}$$

Where in the bi-static scenario the auto-correlation matrix $\mathbf{S}_{\mathbf{x},n}^H \mathbf{S}_{\mathbf{x},n}$ is rank-defficient, since all positions on a constant bi-static range ellipse correspond to the same delay. This is compensated using multiple receivers as will be shown next.

A. Coherent Combining

Coherent combining is based on the model that the targets' complex RCS coefficients are identical as observed by all receivers $\boldsymbol{\xi}_n = \boldsymbol{\xi}$. In this case, to estimate the targets' positions the individual receivers matched filter outputs are first added,

$$\sum_{n=1}^{N} \boldsymbol{\varphi}_{\mathbf{x},n} = \left(\sum_{n=1}^{N} \mathbf{S}_{\mathbf{x},n}^H \mathbf{S}_{\mathbf{x},n} \right) \boldsymbol{\xi} + \sum_{n=1}^{N} \tilde{\mathbf{w}}_n. \tag{15}$$

An energy detector is applied after combining, considering the m-th spatial target bin we have:

$$\left| \sum_{n=1}^{N} \left[\boldsymbol{\varphi}_{\mathbf{x},n} \right]_m \right|^2 > \Gamma_{\mathrm{th}} \tag{16}$$

Due to the variation in transmitter-receiver geometry, the joint auto-correlation matrix, given by the sum inside parenthesis in (15), will be much better behaved. Compared to delay estimation, to achieve a tightly banded structure, both the transmit waveform and the receiver positions need to be designed carefully (e.g. as in a receiver array).

B. Noncoherent Combining

Noncoherent combining assumes the worst-case, i.e., that the RCS coefficients in the $\boldsymbol{\xi}_n$ are fully uncorrelated across the receivers. This can be the case for widely spaced receivers and targets with a significantly varying RCS, but also for the case where the receivers do not have closely coupled oscillators and random phase rotations are introduced that prevent coherent combining (for more examples and discussion see e.g. [8]).

In noncoherent combining the output of the matched filters are combined in magnitude squared, as will be practical later we introduce the set of length N auxiliary vectors $\mathbf{u}_m$, $m = 1, \ldots, M$, using the following $M \times N$ matrix,

$$\begin{bmatrix} \boldsymbol{\varphi}_{\mathbf{x},1} & \ldots & \boldsymbol{\varphi}_{\mathbf{x},N} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 & \ldots & \mathbf{u}_M \end{bmatrix}^T. \tag{17}$$

A target is declared present at location $\mathbf{x}_m$ if,

$$\|\mathbf{u}_m\|^2 = \sum_{n=1}^{N} \left| \left[\boldsymbol{\varphi}_{\mathbf{x},n} \right]_m \right|^2 > \Gamma_{\mathrm{th}} \tag{18}$$

Compared to the coherent case, there is no simple linear relationship between the unknown vectors $\boldsymbol{\xi}_n$ and the detection metric. As is easy to see, since the various matched filter outputs are added as squared magnitude, there is no possibility that existing sidelobes will cancel each other as will be seen in the numerical example in Section V. This makes it near impossible to achieve an as tightly controlled relationship between the range bins as in the coherent case.

IV. Compressive Sensing Radar

The use of compressive sensing (CS) in radar processing has been suggested in several papers [5]–[7], but so far always a single receiver [5], [7] or coherent combining [6] has been considered. In a nutshell, since we would like to estimate the vector(s) $\boldsymbol{\xi}_n$, but the matrices $\mathbf{S}_{\mathbf{x},n}$ might not be invertible, CS radar processing makes use of the fact that the number of present targets is always much smaller than the number of range bins M. This can be used to "regularize" the problem formulation, leading to a well defined solution (see [5]).

Applying CS to the radar problem also makes use of the model in (7):

$$\mathbf{r}_n = \mathbf{S}_{\mathbf{x},n} \boldsymbol{\xi}_n + \mathbf{w}_n \tag{19}$$

We will first describe the case of coherent combining that is effectively equivalent to the model considered in [5]–[7]. Then we will explain the needed modifications for the case of noncoherent combining.

A. Coherent Case

Again, if we can assume that the amplitudes are identical between sensors (or deterministically related) $\boldsymbol{\xi}_n = \boldsymbol{\xi}$, then we can simply combine the various sensor measurements in one linear reconstruction problem, in standard CS notation we have

$$\mathbf{z} = \begin{bmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_N \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{S}_{\mathbf{x},1} \\ \vdots \\ \mathbf{S}_{\mathbf{x},N} \end{bmatrix}, \quad \mathbf{y} = \boldsymbol{\xi}. \tag{20}$$

CS solves sparse reconstruction using convex optimization algorithms like basis pursuit (BP) [4], or greedy algorithms like matching pursuits [10], [11].

As an example the BP formulation for the coherent case would be,

$$\min_{\mathbf{y}} \|\mathbf{z}-\mathbf{A}\mathbf{y}\|^2+\lambda\|\mathbf{y}\|_1 = \min_{\boldsymbol{\xi}} \sum_{n=1}^{N} \|\mathbf{r}_n-\mathbf{S}_{\mathbf{x},n}\boldsymbol{\xi}\|^2+\lambda\|\boldsymbol{\xi}\|_1, \quad (21)$$

where the parameter λ controls sparsity, and the ℓ_1-norm is the sum of the magnitudes of its (complex) elements,

$$\|\boldsymbol{\xi}\|_1 = \sum_{m=1}^{M} |\xi_m|. \quad (22)$$

B. Noncoherent Case

In the noncoherent case, we have to use separate variables for each target *and* sensor, but if a target is present at location $\mathbf{x}_m$ then all variables $\xi_{m,n}$ will take non-zero values. As before we group the cross-correlation values corresponding to one location seen by different receivers in the auxiliary vectors $\mathbf{u}_m$, but also use the new length N auxiliary vectors:

$$\mathbf{v}_m = \begin{bmatrix}\xi_{m,1} & \dots & \xi_{m,N}\end{bmatrix}^T, \quad (23)$$

containing the target RCS coefficients.

We explain the needed modifications to the popular algorithms basis pursuit (BP) and (orthogonal) matching pursuit (MP/OMP) in the following.

1) Noncoherent Basis Pursuit: The formulation is similar to (21), but the ℓ_1 norm is now replaced by the sum of the ℓ_2 norms of the vectors $\mathbf{v}_m$,

$$\min_{\boldsymbol{\xi}_n} \sum_{n=1}^{N} \|\mathbf{r}_n-\mathbf{S}_{\mathbf{x},n}\boldsymbol{\xi}_n\|^2+\lambda\sum_{m=1}^{M}\|\mathbf{v}_m\| \quad (24)$$

$$\begin{bmatrix}\boldsymbol{\xi}_1 & \cdots & \boldsymbol{\xi}_N\end{bmatrix} = \begin{bmatrix}\mathbf{v}_1 & \cdots & \mathbf{v}_M\end{bmatrix}^T \quad (25)$$

Similar problems have been studied as “jointly-sparse” or “block-sparse” signals, see [9] and reference therein. The formulation is also a convex problem and can be solved with available algorithms, e.g., in [12] this formulation fits the case of “Group-Separable Regularizers”.

2) Noncoherent Matching Pursuit:

- Start first iteration $i=1$, residuals are $\mathbf{y}_{0,n}=\mathbf{r}_n$, set of target locations is empty $\mathcal{S}_0=\emptyset$
- Calculate cross-correlation

  $$\boldsymbol{\varphi}_{\mathbf{x},n} = \mathbf{S}_{\mathbf{x},n}^H\mathbf{y}_{i-1,n}$$

  and arrange by range bin

  $$\begin{bmatrix}\boldsymbol{\varphi}_{\mathbf{x},1} & \dots & \boldsymbol{\varphi}_{\mathbf{x},N}\end{bmatrix} = \begin{bmatrix}\mathbf{u}_1 & \dots & \mathbf{u}_M\end{bmatrix}^T$$

  Detect new target based on noncoherent metric

  $$m_i = \arg\max_m \|\mathbf{u}_m\|$$
- Add detection to set of target locations $\mathcal{S}_i=\mathcal{S}_{i-1}\cup m_i$
- Amplitude of target at nth sensor is $\xi_{m_i,n}=[\mathbf{u}_{m_i}]_n$
- Update residuals as,

  $$\mathbf{y}_{i,n} = \mathbf{r}_n - \sum_{m\in\mathcal{S}_i}\xi_{m,n}\mathbf{s}(\tau_{m,n})$$

The algorithm terminates either when a certain number of targets have been identified or if the fitting error decreases beyond a threshold

$$\mathcal{E}_i = \sum_{n=1}^{N}\|\mathbf{y}_{i,n}\|^2. \quad (26)$$

In case of orthogonal matching pursuit (OMP), simply the amplitudes $\xi_{m,n}$ are updated jointly each iteration by minimizing the fitting error as in [10], [11].

We see that in both cases N parallel reconstruction problems are conducted that are linked only through their sparsity pattern (ℓ_1 norm of groups of variables in BP, joint selection of non-zero variables in MP/OMP).

C. Performance Metric & Discretization

When applying CS to radar processing, we should be aware that conventional CS performance metrics like support recovery or mean-square error (MSE) might not be pertinent. This is closely related to the way we fit the problem at hand, here radar processing, to the CS framework. Obviously the MSE in CS is based on estimating the complex amplitudes $\xi_{p,n}$, while the accuracy of these estimates is totally irrelevant in the radar setting. Since the goal of radar processing is to detect and locate targets, we might be inclined to match this to the recovery support problem, and if the discretization of the target space was perfect – meaning targets only appear on discrete positions *and* discrete positions have low mutual coherence – this might be a reasonable metric (for detection performance).

Unfortunately the target space is continuous and when discretizing, targets that fall between discrete values will “bleed” into multiple neighboring locations. This in some ways brings back the side-lobe problem, as a strong target might mask a weaker one. The solution to this is to sample the target space finely relative to the ambiguity function, which corresponds to the mutual coherence in CS. This in turn will make the support recovery highly unlikely as we introduce many dictionary entries with high mutual coherence, but this only means that if we recover the “wrong” dictionary entry, we are placing the target in a location close by.

As a result, we want to look at the location estimation performance or its MSE, where now the error is measured in terms of the estimated target location. As the localization accuracy is always also limited by the chosen discretization, a finer discretization will most likely lead to a smaller position MSE. We will now look at a simple numerical example to verify this hypothesis.

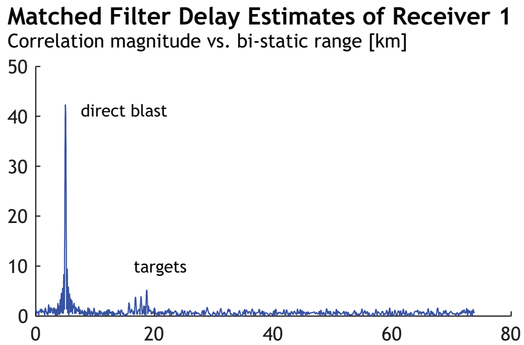


Fig. 2. Cross-correlation output at receiver 1, $\boldsymbol{\varphi}_{\tau,1}$, delay plotted as bi-static range $\tau \cdot c$, the direct blast is simulated at 20 dB above the target returns.

V. Numerical Example

A. Scenario

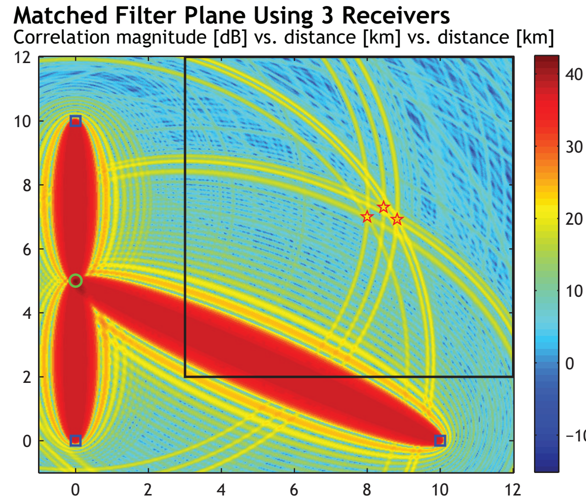
We consider the multi-static setup in Fig. 1; there are $N = 3$ receivers present, and 3 targets. The considered transmit waveform is a digital communications broadcast signal as in [7], of bandwidth $B = 1.537$ MHz and center frequency $f_c = 227.36$ MHz. Each sensor (individually) can only detect target delay information, as an example we plot one delay cross-correlation function in Fig. 2. The transmitted signal shape is only important as its bandwidth defines the spatial resolution as $c/B = 195$ m. Furthermore due to the communication type signal, the auto-correlation function follows a "sinc" shape that is not well suited for matched filter processing; see comparison between matched filter and CS delay estimation in [7].

We assume that the transmitter and receiver positions are known with sufficient accuracy relative to the spatial resolution $c/B = 195$ m. (note that for coherent processing the required accuracy would be relative to the carrier frequency's wavelength $c/f_c = 1.3$ m). Correlating in the spatial domain with a single sensor measurement each maximum in delay will lead to an ellipse with the transmitter and corresponding receiver in its points of focus. When combining all three sensors the target positions emerge at the intersection of the ellipses, see Fig. 3. It is also interesting to point out that due to the bi-static setup the measurement accuracy or width of the ambiguity functions will vary with position relative to the ellipses' baseline. Due to the strong direct blast, the first detected target will always be the transmitter, so its (known) position has to be included as a possible target in addition to the region of interest.

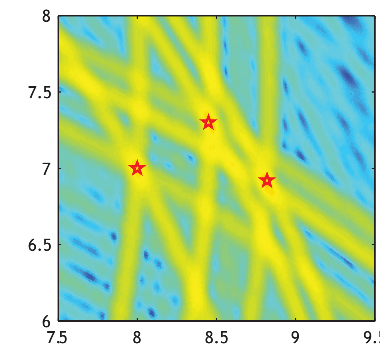
To focus on the position MSE, we choose the scenario such that detection performance is close to ideal; the targets' RCS are assumed constant at a signal-to-noise ratio (SNR) of about 18 dB (but the phases are different at each receiver). The Cramér-Rao lower bound (CRLB) on the position estimation accuracy can be easily determined for this additive Gaussian noise model [13], [14].

B. Results

Based on the spatial resolution of about 0.2 km the targets should be well separated with distances of about half a



(a) Overview



(b) Zoom

Fig. 3. Example of noncoherent cross-correlation function in spatial domain; for each sensor there is an ellipse with the transmitter and the sensor in the points of foci; the targets are found at the intersections.

kilometer, see Fig. 3(a), but in the zoom Fig. 3(b) we can observe many local maxima when various ellipses intersect due to the noncoherent combining. Generally three ellipses will only intersect at target positions, but using a matched filter any intersection of two ellipses could easily exceed any set threshold, especially if some targets' RCS are significantly stronger than others.

To evaluate the localization performance, independent of the the chosen discretization of the search space relative to the target positions, we randomize the grid position by adding a common offset to the grid points (uniformly distributed between zero and one grid space in both the x and y dimension). The detection results of fifty Monte-Carlo simulations are super-imposed in Fig. 4, where we consider both the OMP and BP algorithm and two grid resolutions of 50 m and 25 m. There are a few spurious detections or "false alarms", which are (mostly) caused by a close intersection of three ellipses, c.f. Fig. 3(b). It is easy to see that the estimation error is much smaller than the nominally expected 0.2 km predicted by matched filter processing.

To investigate further, we plot the detection results for target 1 (the "left-most") in Fig. 5, and as comparison the CRLB,

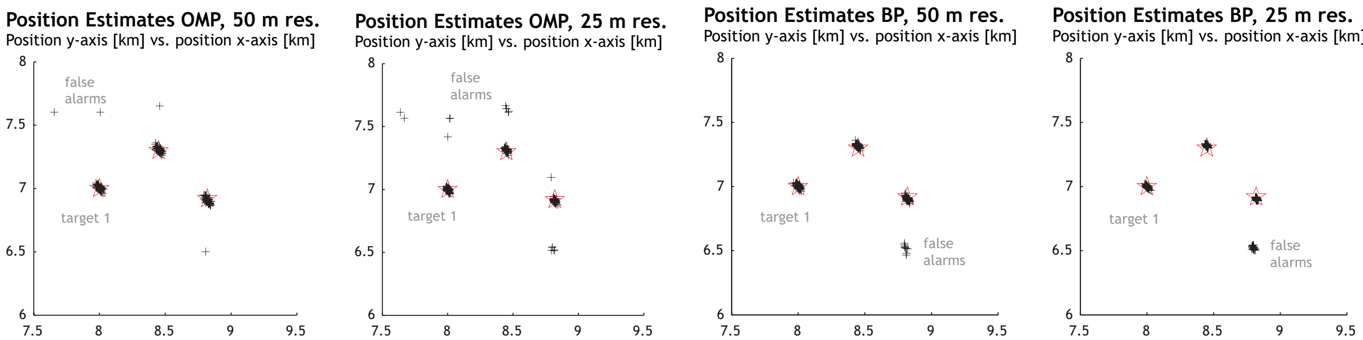


Fig. 4. Localization performance for the scenario depicted in Fig. 3, where we consider both OMP and BP as well as two possible grid resolutions of 25 m and 50 m; clearly the accuracy is much better than the commonly used range resolution of $c/B \approx 0.2$ km; some false alarms can be seen.

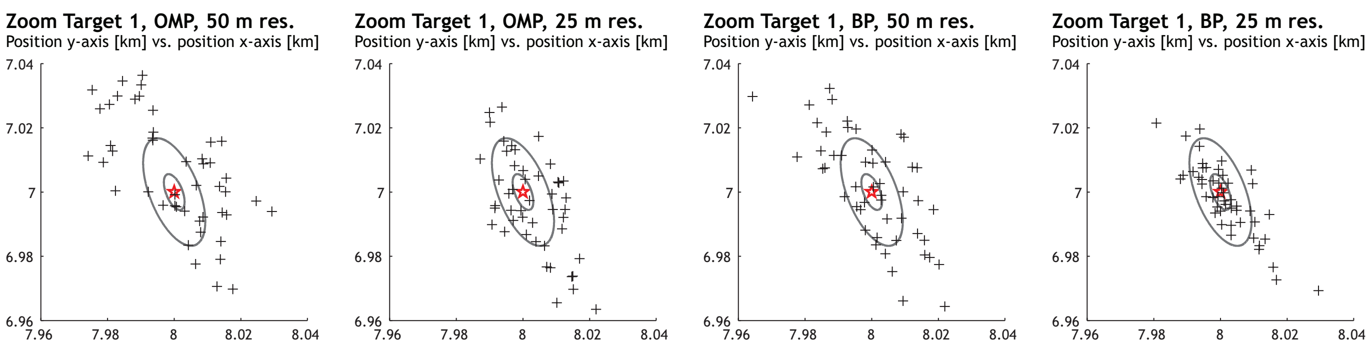


Fig. 5. Example of position estimation accuracy for target 1 in Fig. 4, ellipses are one and three standard deviation of Cramér-Rao bound covariance; the finer grid resolution of 25 m significantly reduces localization error; BP seems to slightly outperform OMP.

specifically the one and three standard deviation ellipses. We first note that at the given noise level the expected localization error in x-direction is half that of the y-direction due to the target-receiver geometry. Furthermore the error predicted by the CRLB is much smaller than the range resolution assumed in matched filter processing (independent of the noise level). The observed error seems to be roughly as predicted by the CRLB, where BP performs slightly better (although more false alarms in Fig. 4). Finally we note that the estimation accuracy clearly improves with the finer grid resolution; in theory we would expect that the estimation error due to noise and due to grid "quantization" should add in terms of their variance.

VI. Conclusion

Compressive sensing is a promising technique for detection and localization of targets in radar processing. Especially in scenarios where the waveform and/or receiver locations can not be controlled, compressive sensing seems to lead to much improved target resolution. We extend the application of compressive sensing to distributed radar processing where receivers have separate linear reconstruction problems that share a common sparsity pattern.

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