RECURSIVE TECHNIQUES FOR PASSIVE SOURCE LOCATION

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ABSTRACT

The paper is concerned with the location of passive sources. Conceptually, this is viewed as a time delay estimation followed by a geometry determination. Signal, noise, and channel modeling questions affect the first block of the processor, i.e., the delay estimator. The second block is sensitive to the geometry description, namely the hypotheses on the dynamics, the array shape, and the relative observer/ /source configuration. Commonly used assumptions lead to decoupled effects which simplify the receiver structure. For deterministic array and source dynamics, a finite parameter description results. The receiver is designed via Maximum-Likelihood techniques. These do not encompass more general situations. To treat the problem of uncertain sensor location, or of stochastic dynamics, a different geometry description is considered. This description represents line arrays and motions as curves in space. Recalling simple facts from Differential Geometry, one is naturally led to describe the array geometry and/or the motion dynamics by a set of differential equations. This casts the passive positioning problem in the context of recursive Kalman-Bucy filtering. The problem of sensor uncertainty location and stochastic dynamics can then be dealt with, without having to consider Taylor series type arguments or other unnatural approximations.

1. INTRODUCTION

Many detection and tracking problems are based on passive receiving systems where the available measurements are signals radiated by the target itself. Situations of significance arise for example within the sonar context, when a sea going observer receives acoustic waves generated by a noisy underwater platform. Passive measurements also occur when airborne receivers navigate by searching the location of radiowave sources (e.g. omega global positioning system). In another application, drifting buoys transmit narrowband signals (modulated by oceanographic data) to orbiting satellites. The available power at the geosynchronous satellite may be unsufficient for active range measurement. Demodulation of the buoys' signals by passive receivers, acquires the buoys' position and dynamics.

To formalize the problem, let the source be pointwise with radiated signature s(t). The receiver has a certain spatial extent ℓ and listens to the source during a given observations interval T. At each array element ℓ and at each time instant t, the receiver signal $r(t,\ell)$ is

$$r(t,\ell)= s(t)* h(t,\ell)+w(t,\ell) \quad ,t\epsilon T,\ell\epsilon L, \qquad (1)$$

where * stands for convolution, $h(t,\ell)$ is the propagating channel impulsive response and $w(t,\ell)$ is a broadband noise. The usual theory models the channel as a simple delay $\tau(t,\ell)$. This delay may be a complicated function of the geometry aspects of the problem. It is this propagation delay that conveys the information regarding the source location. In this paper, we do not deal with the questions of channel, signal, or noise modelling (1). We will follow the standard assumptions. The observation interval being sufficiently long, when compared to the signal and noise correlation times and to the travel time of the wavefield across the receiving array, leads to Fourier representations with uncorrelated coefficients for the signal process. The problem is reduced to a multitone problem. In what follows, a delay type channel, a

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single tone radiated signal and a broad noise are assumed.

In (2), the structure of the receiver for a common application where a linear discrete array tracks a moving source is studied. One distinguishes two main structures in the processor. The front-end of the processor is a beam former, focussing on the range beam and the bearing beam. The two beams are input to a combiner P and to a loop that accomplishes a geometry demodulation. A detailed analysis shows that the beamforming block is dependent on the signal structure, its performance being determined by the available signal to noise ratio. In contradistinction, the second block structure and behavior is critically affected by the geometry. In other words, the receiver, having a feedback structure, is the cascade of two blocks. The first one accomplishes a delay estimation. The second step processes these delay estimates to construct the geometry parameters. This paper concentrates on the discussion of the geometric aspects and its implications on the receiver structure and performance. Albeit the restrictive signal model adopted, this study remains relevant under much broader signal and channel conditions.

2. GEOMETRY

There are two aspects concerning the source/receiver geometry. The spatial extent of the receiver (the source is assumed pointwise) and the temporal diversity induced by the relative motions (synthetic). For example, the source/receiver configuration of figure 1 is parametrized

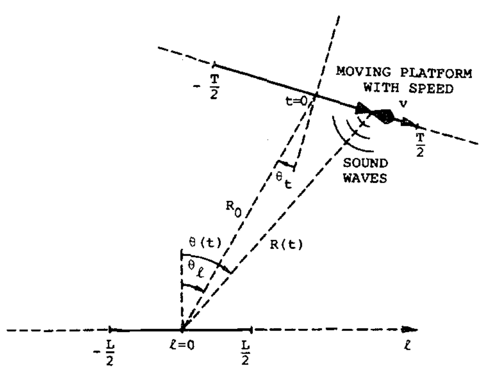

Figure 1: Linear Array/Constant Speed Track Configuration

by a four dimensional vector

$$A = [R_o \ \sin\theta_t \ v \ \sin\theta_\ell]^T \tag{2}$$

The delay is

$$\tau(t,\ell) \cong R(t,\ell)/c \tag{3}$$

where c is the propagation velocity and the range between the source and the antenna element ℓ is

$$R(t,\ell) = \{[R_o - \ell\sin\theta_\ell - vt\sin\theta_t]^2 + [\ell\cos\theta_\ell + vt\cos\theta_t]^2\}^{1/2} \tag{4}$$

Two points are to be mentioned with respect to equation (4). The first is that, being a joint description of the two curves (array geometry and source dynamics), it is hardly generalizable to different configurations. The second is that it casts the passive positioning problem in the context of finite parameter estimation theory. The receiver has been designed applying Maximum-Likelihood techniques (3). In practice, it requires the localization of a maximum in a 4-dimensional parameter space. This is not a trivial task. Using a search algorithm, the computational effort associated with this receiver is estimated by calculating the number of times the ambiguity function has to be evaluated.

This number is

$$M = V_\Omega \ (\det \mathcal{M})^{1/2}/k$$

where $\mathcal{M}$ is a matrix describing the quadratic structure of the receiver and k is a cell normalizing constant. Figure 2 shows the evolution of det $\mathcal{M}$ as a function of $X=L/2R_o$ for the simple context where the source is stationary (no dynamics) (the parameter space is two dimensional (R_o and $\sin\theta_\ell$)). Even so, the number M may become exceedingly large when the array length increases. Besides this computational complexity, the ML-approach exhibits two more drawbacks. The first is that it assumes a deterministic geometry. The second is its nonrecursiveness.

To encompass the stochastic nature of the motions (irregularities in the path) or of the sensors' location is not a simple matter with the ML-approach. In (2), we took a hybrid approach where the motions are assumed stochastic and the array is perfectly linear. In (4), the stationary source (no motions) problem is studied with random small displacements along the y-axis for a linear array along the x-axis. The analysis is based on linear structures and is carried out in terms of truncated Taylor series devellopments. Herein, we look at this problem in a more fundamental way, in order to establish a natural framework for it. It will turn out that, with this setting, one is led to the context of recursive filtering techniques.

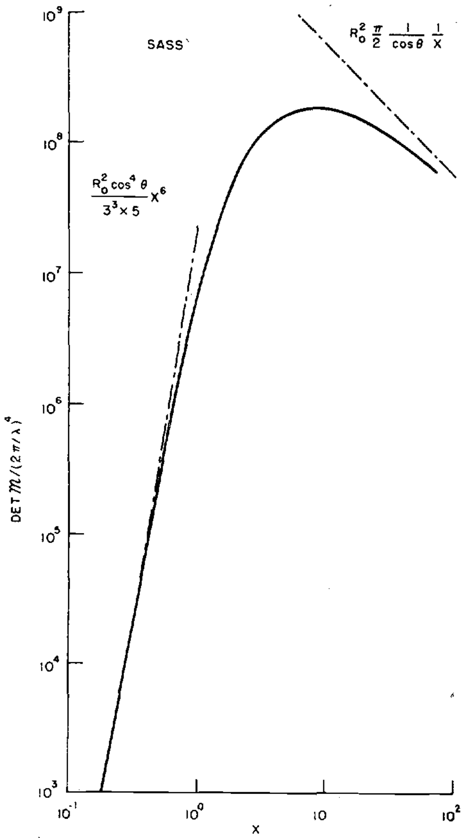

Figure 2: Variation of Det M with $X= L/2R_o$

3. DIFFERENTIAL DESCRIPTION

In this section, the discussion is concerned with planar curves. By Newton's law of motion, we can write an ordinary differential equation describing the point source trajectory and dynamics. In cartesian coordinates, for example, a linear nominal path with distributed accelerations, is described by

$$\ddot{x}(t)= u_x(t) \qquad \ddot{y}(t)= u_y(t) \tag{5}$$

where $u_x(t)$ and $u_y(t)$ are broadband noise processes (white noise). Integrating equations (5), with given initial conditions, a parametric representation of a curve on the plane results. The parameter is time. In Differential Geometry, a curve is represented by the vector $\tilde{r}(t)=[x(t)\ \ y(t)]^T$ defined by the generic point $P=(x(t),y(t))$ of the curve and the origin 0. Equivalent parametric representations may be obtained for the same curve. Under very natural conditions, regular curves admit a parametric representation, where the parameter is the arc length ℓ of the curve. At any point of the trajectory, the tangent vector is $\tilde{t}= d\tilde{r}/d\ell$. Leting $\tilde{n}(\ell)$ be the normal unit vector at point ℓ, the Frênet-Serret formulae are

$$\tilde{t}'= k\ \tilde{n} \quad , \quad \tilde{n}'= -k\ \tilde{t} \tag{6}$$

where the prime notation, according to the usual convention in Differential Geometry, stands for differentiation with respect to the arc length ℓ. In equation (6), $|k|$ is the curvature. The curvature is an intrinsic description of a curve*. Two curves with the same k are congruent, i.e., identical up to a translation and rotation. To fix a curve with a given curvature, we need further a point of the curve $\tilde{r}(\ell_o)$ and its orientation $\tilde{t}(\ell_o)$. The above equations, together with the initial conditions, describe in differential terms a trajectory (i.e. a parametric representation) of the curve. To write down specifically these equations, a reference coordinate system has to be chosen, e.g. cartesian or curvilinear (polar) coordinates.

We note the following natural choice of parameters to represent the curves described by the motions and by the geometry of the array:

Moving source trajectory: parameter =t= time
Static extended array geometry: parameter =ℓ= arc length
Moving point array: parameter =t= time

If the array is rigid and presents a certain line extension, we can still couple in possible motions of the array. The more general problem of representation of non rigid arrays with motions involve the intricacies and technicalities associated with the theory of multiparameter Markov processes. In this paper, this general situation is not considered. In what follows, we will distinguish quantities reffering to the array observer and to the source with indices o and s respectively. For the differential descriptions, we need the state vector $x_i=[x\ \dot{x}\ y\ \dot{y}]$, i= o or s, and where the . represents differentiation with respect to the parameter. Also t takes values on the observation interval T and ℓ on the array extent L.

Example 1: Point source with linear perturbed motions/nominal linear array

*- For general spatial curves, besides the curvature, we have the torsion, and the Frênet-Serret formulae relate, besides $\tilde{t}$ and $\tilde{n}$, the binormal $\tilde{b}$.

Let $A=\begin{bmatrix}0100\\0000\\0001\\0000\end{bmatrix}$, $B=\begin{bmatrix}00\\10\\00\\01\end{bmatrix}$

Source trajectory:

$$\dot{x}_s(t)= A x_s (t)+ B u_s (t) , x_s(t_o) \quad (7)$$

Array geometry:

$$\dot{x}_o(\ell)= A x_o (\ell)+ B u_o(\ell) , x_o(\ell_o) \quad (8)$$

In equations (7) and (8), $u_s(t)$ and $u_o(\ell)$ are bidimensional white noise processes, $x_s(t_o)$ and $x_o(\ell_o)$ are initial conditions. If the array is oriented along the x-axis, $x(\ell_o)=[0100]^T$.

Equations (7) and (8) describe in terms of a linear system of ordinary differential equations the source motions and the geometry of the array. In the state variable framework, we say the state dynamics are linear.

For a given coordinate system, let x_i, i= o or s, be the state vector collecting the position coordinates and their derivatives of the source at time t and of the array element ℓ. Generalizing equations (7) and (8), we can then model the source trajectory and the array geometry by the diffusion equations:

Source:

$$d x_s (t)= f_s (x_s,t)dt + g_s(x_s,t)d\beta_s (t), x_s(t_o) \quad (9)$$

Array:

$$d x_o(\ell)= f_o(x_o,\ell)d\ell + g_o(x_o,\ell)d\beta_o(\ell), x_o(\ell_o) \quad (10)$$

Equation (10) represents a static array with uncertain elements location. If this is an array which experiences a translational motion (no rotation), x_o is also time dependent. We only need to describe the array center's motion. We have

$$dx_o(\ell_o,t)= \alpha(x_o(\ell_o,t),t)\ dt + \sigma(x_o(\ell_o,t),t)d\gamma_o(t), x(\ell_o,t_o) \quad (11)$$

For the general array element ℓ at time t,

$$\tilde{r}_o(\ell,t)=\tilde{r}_o(\ell,\hat{t}_o)+\tilde{r}_o(\ell_o,t)-\tilde{r}_o(\ell_o,t_o) \quad (12)$$

where $\tilde{r}_o(\ell,t)$ is the position vector of the point with coordinates $x_o(\ell,t)$, etc.. Notice that the array description requires now two diffusion equations (10) and (11).

As reffered above the model is not the more general one. However, with no added conceptual difficulty, the model just described has encompassed stochastic motions for the source, uncertain sensor location for the receiver coupled with rigid translational stochastic motions.

4. RECEIVED SIGNAL: DELAY MODEL

As noted at the begining of the paper, at the receiver, the geometric aspects are imbeded in the delay $\tau(t,\ell)$ of the signal replica available at the sensor element ℓ at time t. We have

$$\tau(t,\ell)= \frac{1}{c}||\tilde{r}_o(t,\ell)- r_s(t)|| \quad (13)$$

where $||.||$ stands for the norm.

5. CONCLUSION

Equations (9)- (13) describe in a recursive framework the passive positioning problem. By particularizing their form, we may generalize in a trivial way the usual models of the passive positioning literature (e.g. point stationary source with linear array). They are in a suitable format to apply recursive filtering techniques. Preliminary results for a linear model have been presented in (5). A non optimal hybrid strategy has been discussed in (2).

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