Error Coding

18-849b Dependable Embedded Systems

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Required Reading: *Applications of Error-Control Coding*; Costello, Daniel J., Jr.; Hagenauer, Joachim; Imal Hideki; Wicker, Stephen B.;

Best Tutorial: *Applied Coding and Information Theory for Engineers*; Wells, Richard B.

Authoritative Book: *Error Control Coding: Fundamentals and Applications*; Lin, Shu; Costello, Daniel J., Jr.;

Carnegie Mellon

Overview: Error Coding

- **Introduction**
  - What error coding is and how it is used
  - Applications
  - How it works
- **Key concepts**
  - Bandwidth versus Error Protection
  - Linear Block Codes
  - CRC Codes
  - Advanced Coding Techniques
  - Complexity
- **Conclusions**

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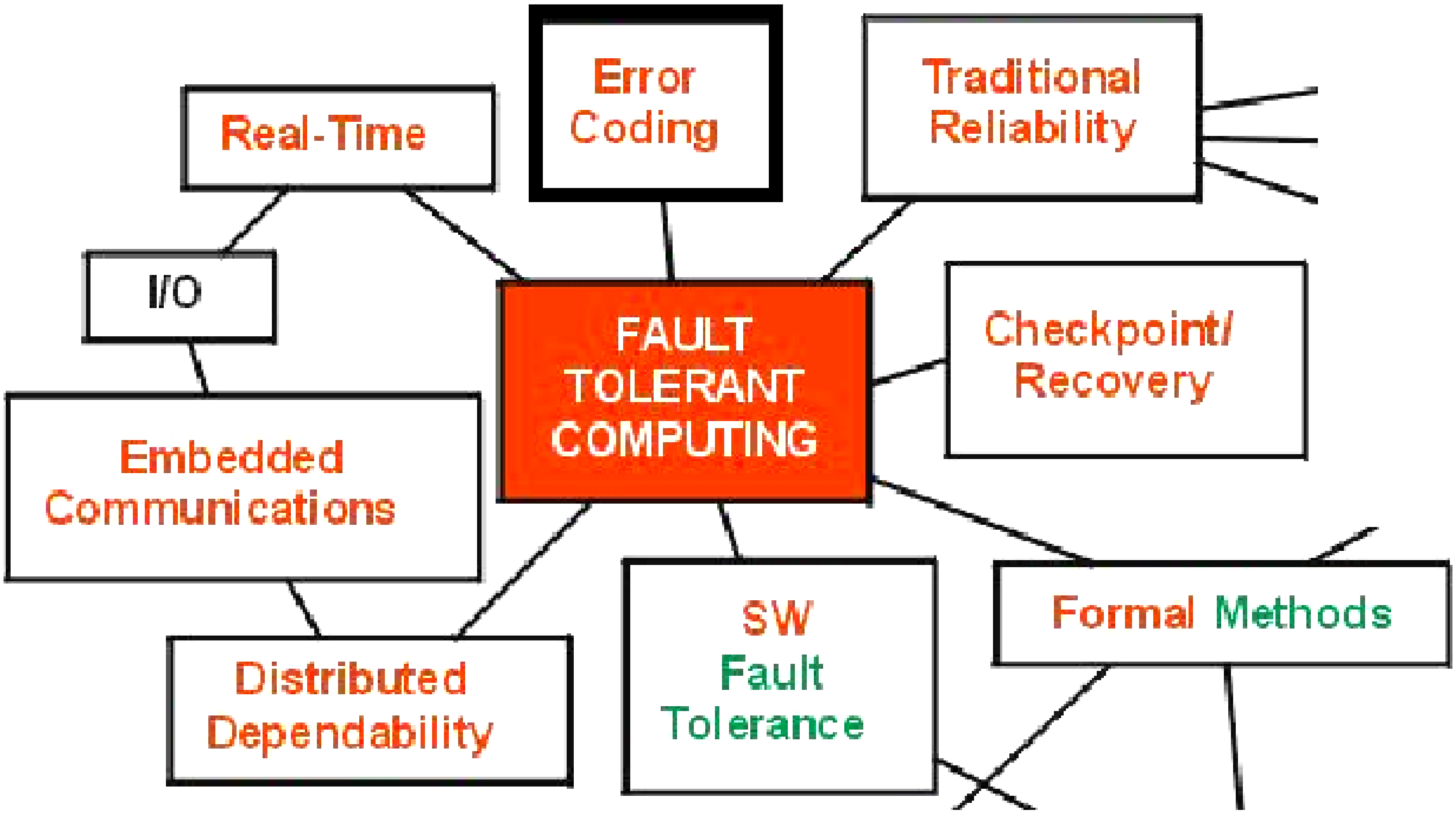
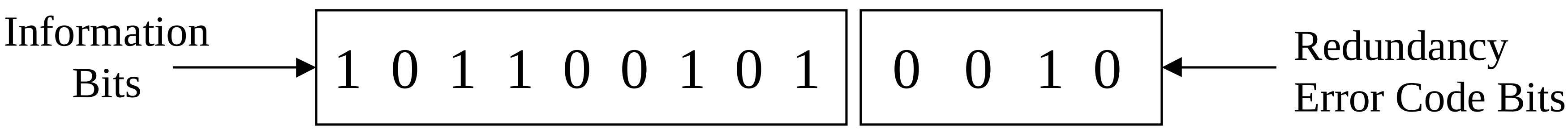
- **Error Coding is in the cluster topic of Fault Tolerant Computing:**



Error Coding: Introduction

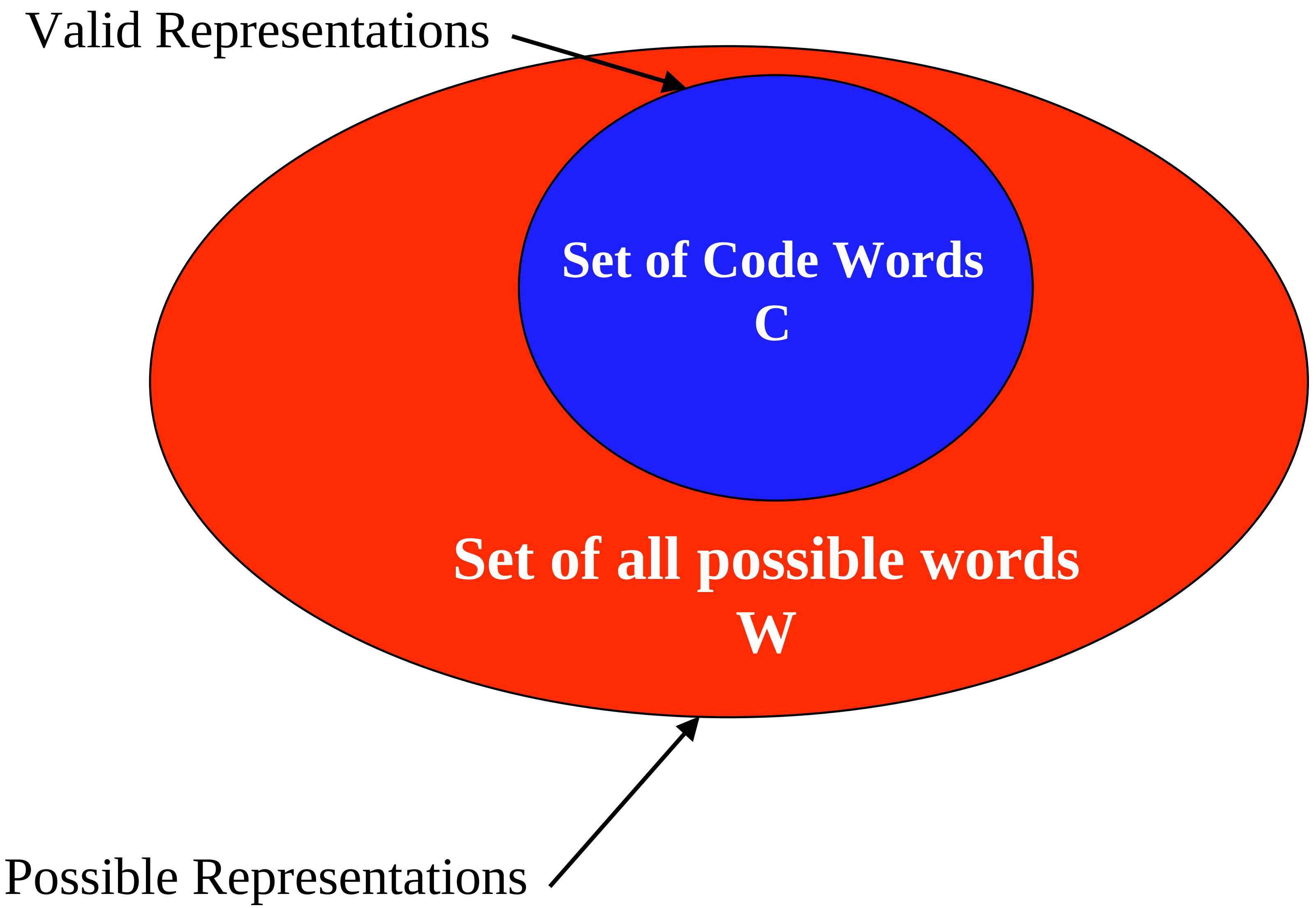
- **Subtopic of Information Coding Theory developed from work by Claude Shannon in 1948**
  - Any transmission of information digitally is susceptible to errors from noise and/or interference

- **By encapsulating data from digital communications in "code words" with extra bits in each transmission, we can detect and correct a large portion of these errors**



- **Error codes developed mathematically using algebra, geometry and statistics**

Code Space



Valid Representations
Set of Code Words
C
Set of all possible words
W
Possible Representations

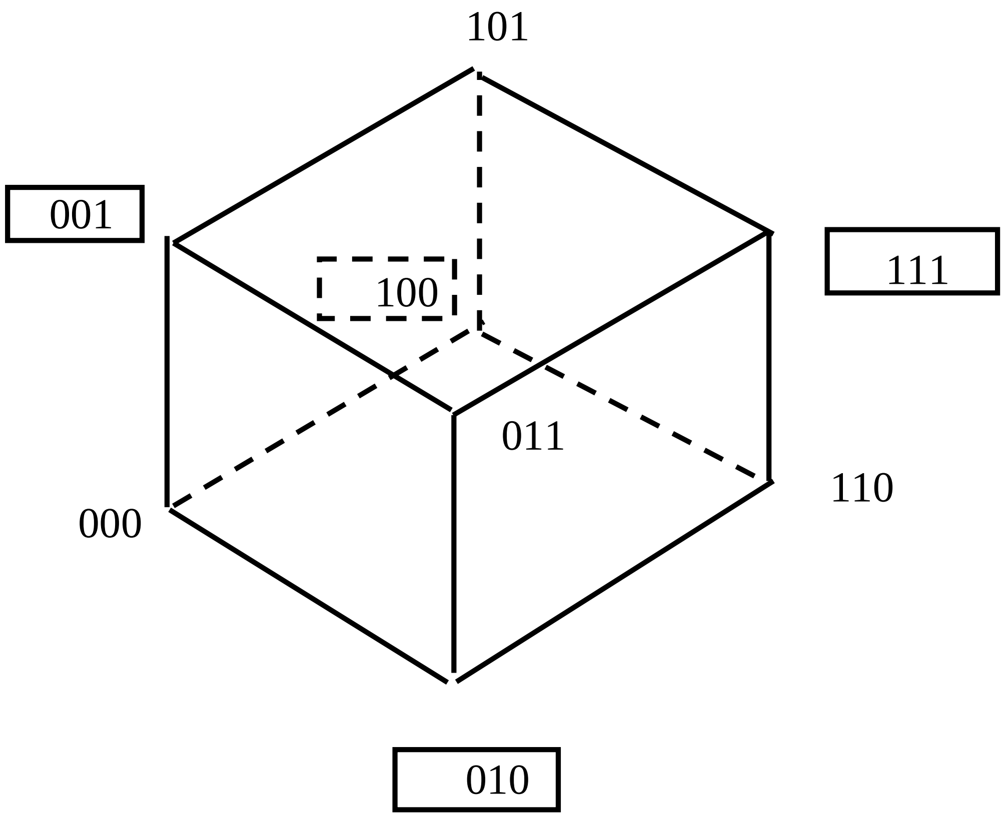
How Coding Works

- **Information sent in a fixed number of k bits over a noisy channel is mapped to a space of “code words” that are strings of n > k bits**
  - Information is random so there are 2^k possible messages from a space of 2^n possible bit patterns
  - Coding scheme is generated mathematically so destination can decode only valid code words and reject other bit strings

- **Minimum Hamming distance d_{min}**
  - Minimum number of bits two code words differ by
  - Error code can detect d_{min} - 1 errors and correct d_{min}/2 -1 errors

- **Simple example: adding a parity check bit to data string**

Simple 3-bit Error Detecting Code Space



Space of Possible words:

000
001
010
011
100
101
110
111

Boxed words = odd parity; blue words are valid code words; d_{min} = 2

Applications of Error Coding

- **Storage**
  - Computer Memory (RAM)
  - Magnetic and Optical Data Storage (hard disks, CD-ROM's)

- **Communications**
  - Satellite and Deep Space Communications
  - Network Communications (TCP/IP Protocol Suite)
  - Cellular Telephone Networks
  - Digital Audio and Video Transmissions

Bandwidth versus Error Protection

- **Code Rate - Ratio of data bits to total bits transmitted in code**
- **Shannon's Noisy Channel Coding Theorem**
  - Given a code rate R that is less than the communication channel C, a code can be constructed that will have an arbitrarily small decoding error probability.
- **Tradeoff of bandwidth for data transmission reliability.**
  - The more bits used for coding and not data, the more errors can be detected and corrected.
  - At a constant bit rate, noisier channel means less real data sent (higher error coding overhead)

Linear Block Codes

- **Data stream is divided into several blocks of fixed length k.**
  - Each block is encoded into a code word of length n > k
  - Very high code rates, usually above 0.95, high information content but limited error-correction capabilities
  - Useful for channels with low raw error rate probabilities, less bandwidth

- **Cyclic Redundancy Check (CRC) codes are a subset**

CRC Codes

- **One of the most common coding schemes used in digital communications**
  - Very easy to implement in electronic hardware
  - Efficient encoding and decoding schemes
  - Only error-detecting - must be concatenated with another code for error correcting capabilities
- **All CRC codes have the cyclic shift property - when any code word is rotated left or right, the resulting bit string is also a code word**
  - Example Cyclic Code: {[000000], [010101], [101010], [111111]}

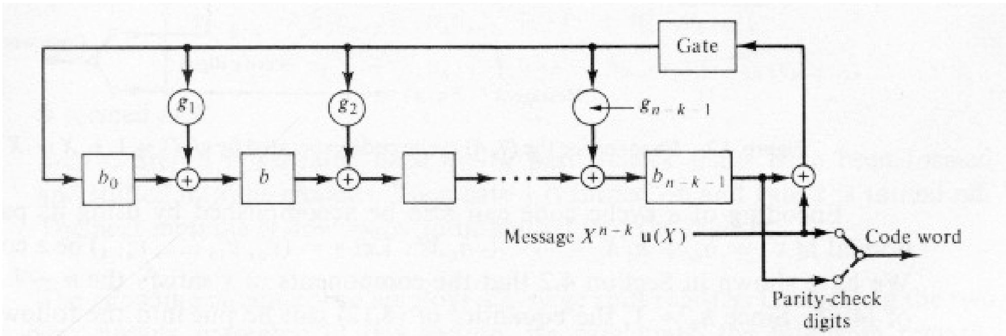


Figure 4.1 Encoding circuit for an (n, k) cyclic code with generator polynomial $g(X) = 1 + g_1X + g_2X^2 + \cdots + g_{n-k-1}X^{n-k-1} + X^{n-k}$.

Advanced Coding Techniques

- **Convolutional Codes**
  - Entire data stream is encoded into one code word
  - Code rates usually below 0.90, but very powerful error-correcting capabilities
  - Useful for channels with high raw error rate probabilities, need more bandwidth to achieve similar transmission rate
  - Viterbi Codes used in satellite communication
- **Burst-Correcting Codes**
  - Used for channels where errors occur in bursts and not random bit errors
  - Interleaving codes useful technique for burst-correcting codes

Coding Complexity

- **More complex coding schemes provide better error protection**
  - Higher error detection and recovery
  - But require more time to encode and decode information from source to destination
- **Real-time systems may not tolerate delay associated with sophisticated coding of data transmissions**
  - But cannot tolerate corrupted messages either
  - So, what are you going to do about it?

Conclusions

- **Choose coding scheme based on types of errors expected**
  - Burst errors vs random bit errors
  - Ability to retransmit (detect only)
  - Expected error rate
- **Error coding can protect information transmission over an error-prone communication medium**
  - Must trade bandwidth for error protection
  - More complex coding schemes will provide more error protection at the expense of delay encoding/decoding at source/destination
  - Real-time systems must balance error protection with tolerable coding delay
- **No one has ever found a code that satisfies Shannon's theorem of arbitrarily low error rates**

Paper:Applications of Error-Control Coding

- **Nice summary of several places error coding is used**
- **Coding schemes becoming more complex with more processor power**
  - Faster processors allow for sophisticated coding/decoding techniques to provide higher error protection without sacrificing speed
  - Parallel and serial concatenated coding and iterative coding can be done with faster processors
- **Error Coding is a mature field but there is still much work to be done**
  - Coding originally driven by military/government applications but later by commercial interests
  - Coding schemes moving away from algebraic codes and towards more probabilistic codes for better error protection