SPIRAL:
AI for High Performance Code

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Joint work with the SPIRAL team at CMU and FFTX team at CMU and LBL

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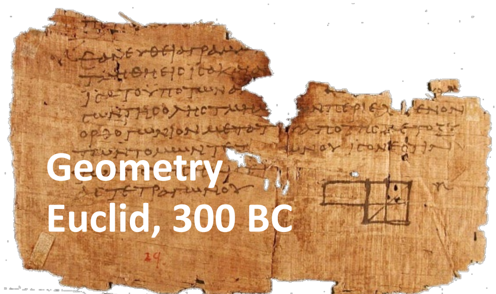
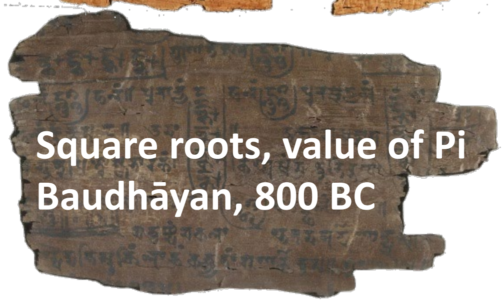
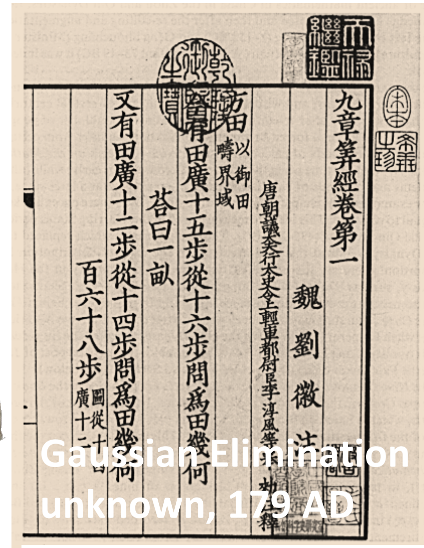
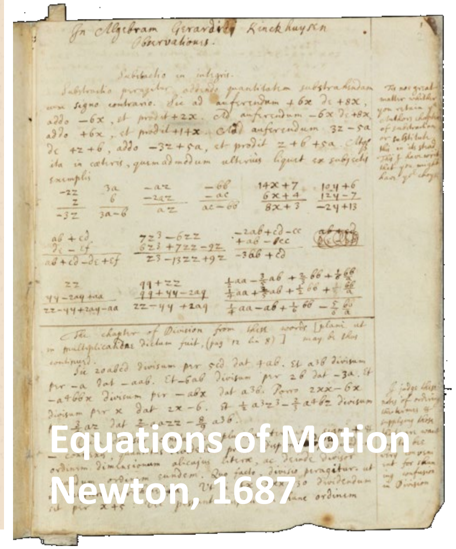
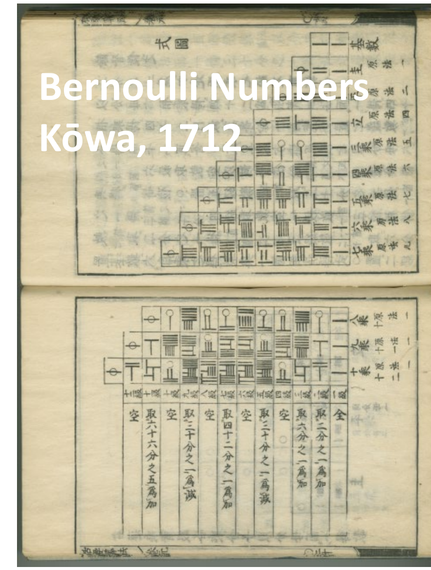
Algorithms and Mathematics: 2,500+ Years



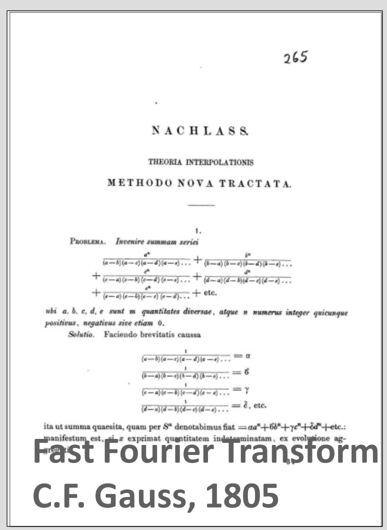


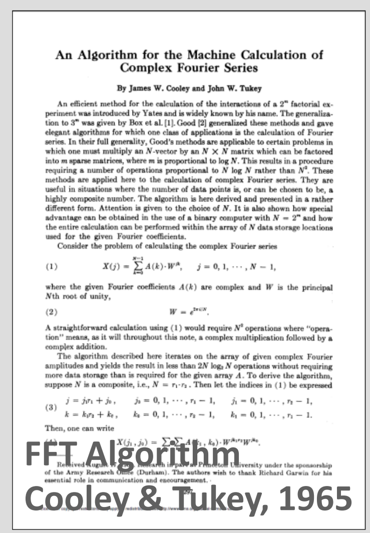


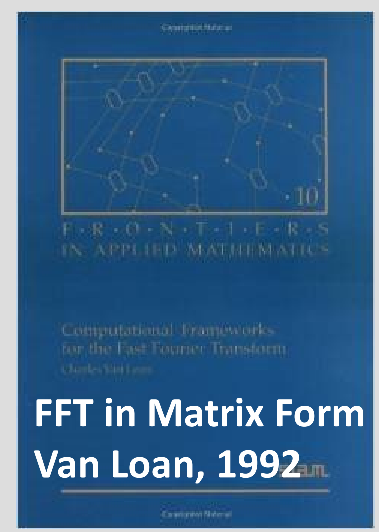




Fast Fourier Transform







Computing Platforms Over The Years

F-16A/B, C/D, E/F, IN, IQ, N, V: Flying since 1974

Compare: Desktop/workstation class CPUs/machines

Assembly code compatible !!

7

x86 binary compatible, but 500x parallelism ?!

1972
Intel 8008
0.2—0.8 MHz
Intelligent terminal

1989
IBM PC/XT compatible
8088 @ 8 MHz, 640kB RAM
360 kB FDD, 720x348 mono

1994
IBM RS/6000-390
256 MB RAM, 6GB HDD
67 MHz Power2+, AIX

2006
GeForce 8800
1.3 GHz, 128 shaders
16-way SIMT

2011
Xeon Phi
1.3 GHz, 60 cores
8/16-way SIMD

2018
Xeon Platinum 8180M
28 cores, 2.5-3.6 GHz
2/4/8/16-way SIMD

10^7 – 10^8 compounded performance gain over 45 years

Programming/Languages Libraries Timeline

Popular performance programming languages

- 1953: Fortran
- 1973: C
- 1985: C++
- 1997: OpenMP
- 2007: CUDA
- 2009: OpenCL

Popular performance libraries

- 1979: BLAS
- 1992: LAPACK
- 1994: MPI
- 1995: ScaLAPACK
- 1995: PETSc
- 1997: FFTW

Popular productivity/scripting languages

- 1987: Perl
- 1989: Python
- 1993: Ruby
- 1995: Java
- 2000: C#

2019: What $1M Can Buy You

Dell PowerEdge R940
4.5 Tflop/s, 6 TB, 850 W
4x 28 cores, 2.5 GHz

OSS FSAn-4
200 TB PCIe NVMe flash
80 GB/s throughput

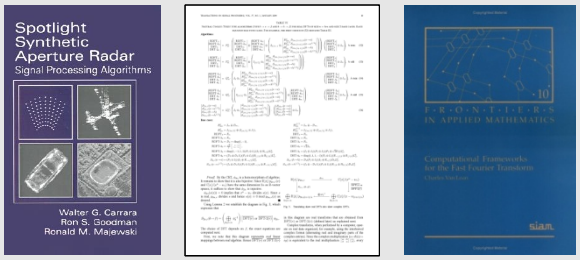
BittWare TeraBox
18M logic elements, 4.9 Tb/sec I/O
8 FPGA cards/16 FPGAs, 2 TB DDR4

AberSAN ZXP4
90x 12TB HDD, 1 kW
1PB raw

Nvidia DGX-1
8x Tesla V100, 3.2 kW
170 Tflop/s, 128 GB

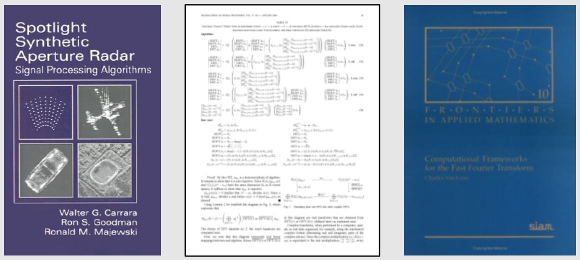
SPIRAL: AI for High Performance Code

Traditionally



High performance library optimized for given platform

SPIRAL Approach



SPIRAL

High performance library optimized for given platform

Comparable performance

Outline

- Introduction
- **Specifying computation**
- Achieving Performance Portability
- FFTX: A Library Frontend for SPIRAL
- Summary

F. Franchetti, T. M. Low, D. T. Popovici, R. M. Veras, D. G. Spampinato, J. R. Johnson, M. Püschel, J. C. Hoe, J. M. F. Moura:
SPIRAL: Extreme Performance Portability, Proceedings of the IEEE, Vol. 106, No. 11, 2018.
Special Issue on *From High Level Specification to High Performance Code*

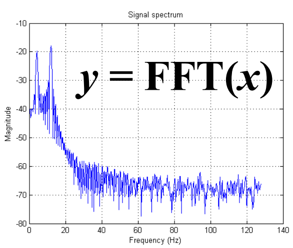
SPIRAL: AI for Performance Engineering

Given:

- **Mathematical problem specification**
  core mathematics does not change
- **Target computer platform**
  varies greatly, new platforms introduced often

Wanted:

- **Very good implementation of specification on platform**
- **Proof of correctness**



on

automatic

```
void fft64(double  *Y, double  *X) {
    ...
    s5674 = _mm256_permute2f128_pd(s5672, s5673, (0) | ((2) << 4));
    s5675 = _mm256_permute2f128_pd(s5672, s5673, (1) | ((3) << 4));
    s5676 = _mm256_unpacklo_pd(s5674, s5675);
    s5677 = _mm256_unpackhi_pd(s5674, s5675);
    s5678 = *((a3738 + 16));
    s5679 = *((a3738 + 17));
    s5680 = _mm256_permute2f128_pd(s5678, s5679, (0) | ((2) << 4));
    s5681 = _mm256_permute2f128_pd(s5678, s5679, (1) | ((3) << 4));
    s5682 = _mm256_unpacklo_pd(s5680, s5681);
    s5683 = _mm256_unpackhi_pd(s5680, s5681);
    t5735 = _mm256_add_pd(s5676, s5682);
    t5736 = _mm256_add_pd(s5677, s5683);
    t5737 = _mm256_add_pd(s5670, t5735);
    t5738 = _mm256_add_pd(s5671, t5736);
    t5739 = _mm256_sub_pd(s5670, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5735));
    t5740 = _mm256_sub_pd(s5671, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5736));
    t5741 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5677, s5683));
    t5742 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5676, s5682));
    ...
}
```

OL Operators

Definition

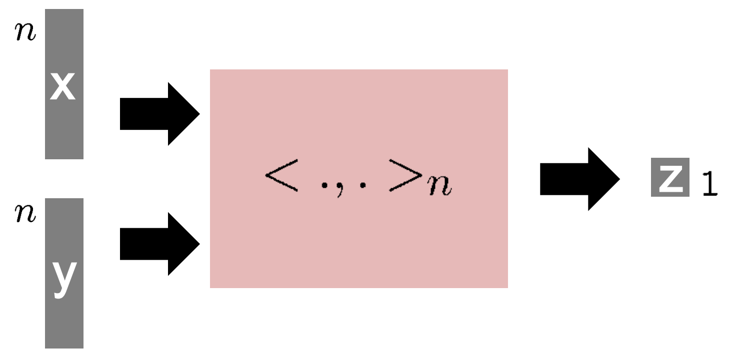
- Operator: Multiple vectors ! Multiple vectors
- Stateless
- Higher-dimensional data is linearized
- Operators are potentially nonlinear

$$\mathrm{M}: \begin{cases} \mathbb{C}^{n_0} \times \cdots \times \mathbb{C}^{n_{k-1}} \to \mathbb{C}^{N_0} \times \cdots \times \mathbb{C}^{N_{\ell-1}} \\ (\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}) \mapsto \mathrm{M}(\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}) \end{cases}$$

Example: Scalar product

$$< .,. >_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$

$$\big((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\big) \mapsto \sum_{i=0}^{n-1} x_i y_i$$



Example: Safety Distance as OL Operator

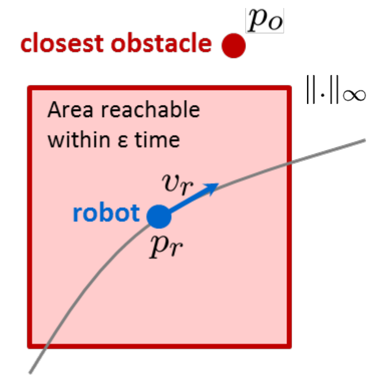
- **Passive Safety of Robots**

  p_o: Position of closest obstacle

  p_r: Position of robot

  v_r: Longitudinal velocity of robot

  A, b, V, ": constants



$$\|p_r - p_o\|_\infty > \frac{v_r^2}{2b} + V\frac{v_r}{b} + \left(\frac{A}{b} + 1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r + V)\right)$$

- **Definition as operator**

$$\begin{aligned}\mathsf{SafeDist}_{V,A,b,\varepsilon}: \quad & \mathbb{R} \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{Z}_2 \\ & (v_r, p_r, p_o) \mapsto \left(p(v_r) < d_\infty(p_r, p_o)\right)\end{aligned}$$

with

$$d_\infty(\vec{x}, \vec{y}) = \|\vec{x} - \vec{y}\|_\infty$$

$$p(x) = \alpha x^2 + \beta x + \gamma$$

$$\alpha = \frac{1}{2b}$$

$$\beta = \frac{V}{b} + \varepsilon\left(\frac{A}{b} + 1\right)$$

$$\gamma = \left(\frac{A}{b} + 1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon V\right)$$

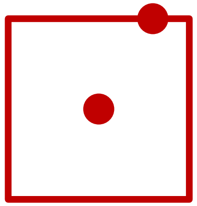
Formalizing Mathematical Objects in OL

- **Infinity norm**

$$\|.\|_\infty^n : \mathbb{R}^n \to \mathbb{R}$$
$$(x_i)_{i=0,\dots,n-1} \mapsto \max_{i=0,\dots,n-1} |x_i|$$

- **Chebyshev distance**

$$d_\infty^n(.,.) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$
$$(x, y) \mapsto \|x - y\|_\infty^n$$



- **Vector subtraction**

$$(-)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$$
$$(x, y) \mapsto x - y$$

- **Pointwise comparison**

$$(<)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{Z}_2^n$$
$$\left((x_i)_{i=0,\dots,n-1}, (y_i)_{i=0,\dots,n-1}\right) \mapsto (x_i < y_i)_{i=0,\dots,n-1}$$

- **Scalar product**

$$< ., . >_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$
$$\left((x_i)_{i=0,\dots,n-1}, (y_i)_{i=0,\dots,n-1}\right) \mapsto \sum_{i=0}^{n-1} x_i y_i$$

- **Monomial enumerator**

$$(x^i)_n : \mathbb{R} \to \mathbb{R}^{n+1}$$
$$x \mapsto (x^i)_{i=0,\dots,n}$$

- **Polynomial evaluation**

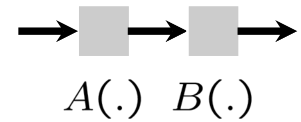
$$P[x, (a_0, \dots, a_n)] : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n$$

Beyond the textbook: explicit vector length, infix operators as prefix operators

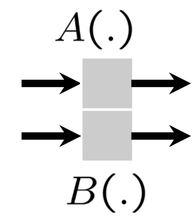
Operations and Operator Expressions

- **Operations (higher-order operators)**

$$\circ : (D \to S) \times (S \to R) \to (D \to R)$$
$$(A, B) \mapsto B \circ A$$

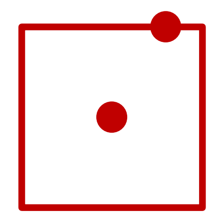


$$\times : (D \to R) \times (E \to S) \to (D \times E \to R \times S)$$
$$(A, B) \mapsto \big((x, y) \mapsto \big(A(x), B(y)\big)\big)$$



- **Operator expressions are operators**

$$\|.\|_\infty^n \circ (-)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$
$$\big((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\big) \mapsto \max_{i=0,\ldots,n-1} |x_i - y_i|$$

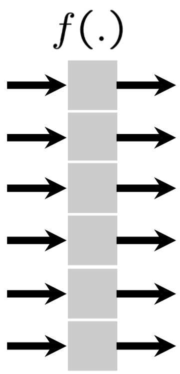


- **Short-hand notation: Infix notation**

$$A(.) - B(.) = \big(x \mapsto A(x) - B(x)\big)$$ can be expressed via $$(-)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$$
$$(x, y) \mapsto x - y$$

Basic OL Operators

- **Basic operators ≈ functional programming constructs**



map	$\mathsf{Pointwise}_{n,f_i} : \mathbb{R}^n \to \mathbb{R}^n$ $(x_i)_i \mapsto f_0(x_0) \oplus \cdots \oplus f_{n-1}(x_{n-1})$
binop	$\mathsf{Atomic}_{f(.,.)} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ $(x, y) \mapsto f(x, y)$
map + zip	$\mathsf{Pointwise}_{n \times n, f_i} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ $\big((x_i)_i, (y_i)_i\big) \mapsto f_0(x_0, y_0) \oplus \cdots \oplus f_{n-1}(x_{n-1}, y_{n-1})$
fold	$\mathsf{Reduction}_{n,f_i} : \mathbb{R}^n \to \mathbb{R}$ $(x_i)_i \mapsto f_{n-1}(x_{n-1}, f_{n-2}(x_{n-2}, f_{n-3}(\ldots f_0(x_0, \mathsf{id}())\ldots)$
unfold	$\mathsf{Induction}_{n,f_i} : \mathbb{R} \to \mathbb{R}^{n+1}$ $x \mapsto (f_n(x, f_{n-1}(\ldots)\ldots), \ldots, f_2(x, f_1(x, \mathsf{id})), f_1(x, \mathsf{id}), \mathsf{id}())$

- **Safety distance as (optimized) operator expression**

$$\begin{aligned}\mathsf{SafeDist}_{V,A,b,\varepsilon} = {} & \mathsf{Atomic}_{(x,y)\mapsto x<y} \\ & \circ \Big(\Big(\mathsf{Reduction}_{3,(x,y)\mapsto x+y} \circ \mathsf{Pointwise}_{3,x\mapsto a_i x} \circ \mathsf{Induction}_{3,(a,b)\mapsto ab,1}\Big) \\ & \quad \times \Big(\mathsf{Reduction}_{2,(x,y)\mapsto \max(|x|,|y|)} \circ \mathsf{Pointwise}_{2\times 2,(x,y)\mapsto x-y}\Big)\Big)\end{aligned}$$

Breaking Down Operators into Expressions

- **Application specific:** Safety Distance as Rewrite Rule

$$\mathsf{SafeDist}_{V,A,b,\varepsilon}(.,.,.) \rightarrow \Big(P[x,(a_0,a_1,a_2)](.) < d^2_\infty(.,.)\Big)(.,.,.)$$

$$\text{with} \quad a_0 = \tfrac{1}{2b},\ a_1 = \tfrac{V}{b} + \varepsilon\Big(\tfrac{A}{b}+1\Big),\ a_2 = \Big(\tfrac{A}{b}+1\Big)\Big(\tfrac{A}{2}\varepsilon^2 + \varepsilon V\Big)$$

Problem specification: hand-developed or automatically produced

- **One-time effort:** mathematical library

$$d^n_\infty(.,.) \rightarrow ||.||^n_\infty \circ (-)_n$$

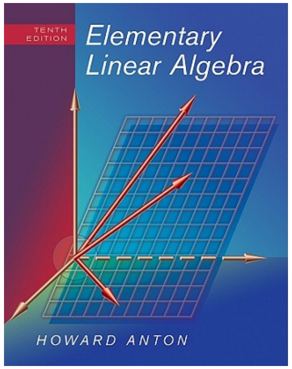
$$(\diamond)_n \rightarrow \mathsf{Pointwise}_{n\times n,(a,b)\mapsto a\diamond b}, \quad \diamond \in \{+,-\cdot,\wedge,\vee,\ldots\}$$

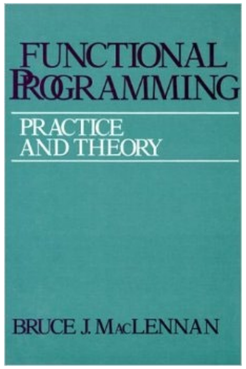
$$||.||^n_\infty \rightarrow \mathsf{Reduction}_{n,(a,b)\mapsto \mathsf{max}(|a|,|b|)}$$

$$<.,.>_n \rightarrow \mathsf{Reduction}_{n,(a,b)\mapsto a+b} \circ \mathsf{Pointwise}_{n\times n,(a,b)\mapsto ab}$$

$$P[x,(a_0,\ldots,a_n)] \rightarrow <(a_0,\ldots,a_n),.> \circ (x^i)_n$$

$$(x^i)_n \rightarrow \mathsf{Induction}_{n,(a,b)\mapsto ab,1}$$





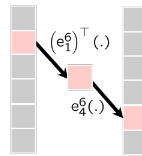
Library of well-known identities expressed in OL

Loop and Code Level Rule System

Mathematical Loop Abstraction

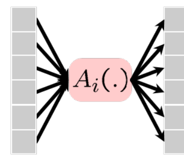
- **Selection and embedding operator:** *gather and scatter*

$$(e_i^n)^\top(.) : \mathbb{R}^n \to \mathbb{R}^1$$
$$(x_i)_{i=0,\dots,n-1} \mapsto x_i$$
$$e_i^n(.) : \mathbb{R}^1 \to \mathbb{R}^n$$
$$(x) \mapsto (0,\dots,0,\underbrace{x}_{i^{\text{th}}},0,\dots,0)$$



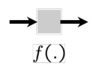
- **Iterative operations:** *loop*

$$\coprod_{i=0}^{n-1} : (D \to R)^n \to (D \to R)$$
$$A_i \mapsto (x \mapsto A_0(x) \sqcup \dots \sqcup A_{n-1}(x))$$
$$\text{with } \sqcup \in \left\{\Sigma, \vee, \wedge, \prod, \min, \max, \dots\right\}$$



- **Atomic operators:** *nonlinear scalar functions*

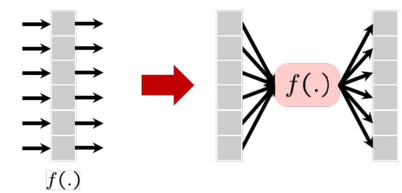
$$\text{Atomic}_f : \mathbb{R}^1 \to \mathbb{R}^1$$
$$(x) \mapsto (f(x))$$



Translation and Optimization

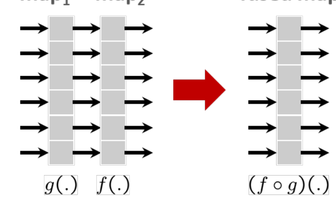
- **Translating Basic OL into Σ-OL**

$$\text{Pointwise}_{n,f_i} \to \sum_{i=0}^{n-1} \left(e_i^n \circ \text{Atomic}_{f_i} \circ (e_i^n)^\top\right)$$
$$\text{Reduction}_{n,(a,b)\mapsto a+b} \to \sum_{i=0}^{n-1} (e_i^n)^\top$$



- **Optimizing Basic OL/Σ-OL**

$$\text{Pointwise}_{n,f_i} \circ \text{Pointwise}_{n,g_i} \to \text{Pointwise}_{n,f_i \circ g_i}$$
$$\text{Pointwise}_{n,f_i} \circ e_n^j \to e_n^j \circ \text{Pointwise}_{1,f_j}$$



Abstract Code

Code objects

- Values and types
- Arithmetic operations
- Logic operations
- Constants, arrays and scalar variables
- Assignments and control flow

Properties: at the same time

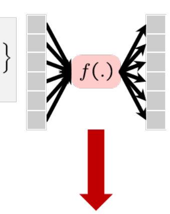
- Program = (abstract syntax) tree
- Represents program in restricted C
- OL operator over real numbers and machine numbers (floating-point)
- Pure functional interpretation
- Represents lambda expression

```
# Dynamic Window Monitor

let(
  i3 := var("i3", TInt), i5 := var("i5", TInt),
  w2 := var("w2", TBool), w1 := var("w1", T_Real(64)),
  s8 := var("s8", T_Real(64)), s7 := var("s7", T_Real(64)),
  s6 := var("s6", T_Real(64)), s5 := var("s5", T_Real(64)),
  s4 := var("s4", T_Real(64)), s1 := var("s1", T_Real(64)),
  q4 := var("q4", T_Real(64)), q3 := var("q3", T_Real(64)),
  D := var("D", TPtr(T_Real(64)).aligned([16, 0])),
  X := var("X", TPtr(T_Real(64)).aligned([16, 0])),

  func(TInt, "dwmonitor", [ X, D ],
     decl([q3, q4, s1, s4, s5, s6, s7, s8, w1, w2],
        chain(
           assign(s5, V(0.0)),
           assign(s8, nth(X, V(0))),
           assign(s7, V(1.0)),
           loop(i5, [0..2],
              chain(
                 assign(s4, mul(s7, nth(D, i5))),
                 assign(s5, add(s5, s4)),
                 assign(s7, mul(s7, s8))
              )
           ),
           assign(s1, V(0.0)),
           loop(i3, [0..1],
              chain(
                 assign(q3, nth(X, add(i3, V(1)))),
                 assign(q4, nth(X, add(V(3), i3))),
                 assign(w1, sub(q3, q4)),
                 assign(s6, cond(geq(w1, V(0)), w1, neg(w1))),
                 assign(s1, cond(geq(s1, s6), s1, s6))
              )
           ),
           assign(w2, geq(s1, s5)),
           creturn(w2)
        )
     )
  )
)
```

Rule Based Compiler

Compilation rules: recursive descent

$$\text{Code}\left(y = (A \circ B)(x)\right) \to \left\{\texttt{decl}(t), \text{Code}\left(t = B(x)\right), \text{Code}\left(y = A(t)\right)\right\}$$
$$\text{Code}\left(y = \left(\sum_{i=0}^{n-1} A_i\right)(x)\right) \to \left\{y := \vec{0}, \texttt{for}(i = 0..n-1)\ \text{Code}\left(y += A_i(x)\right)\right\}$$
$$\text{Code}\left(y = (e_i^n)^\top(x)\right) \to y[0] := x[i]$$
$$\text{Code}\left(y = e_i^n(x)\right) \to \left\{y = \vec{0}, y[i] := x[0]\right\}$$
$$\text{Code}\left(y = \text{Atomic}_f(x)\right) \to y[0] := f(x[i])$$

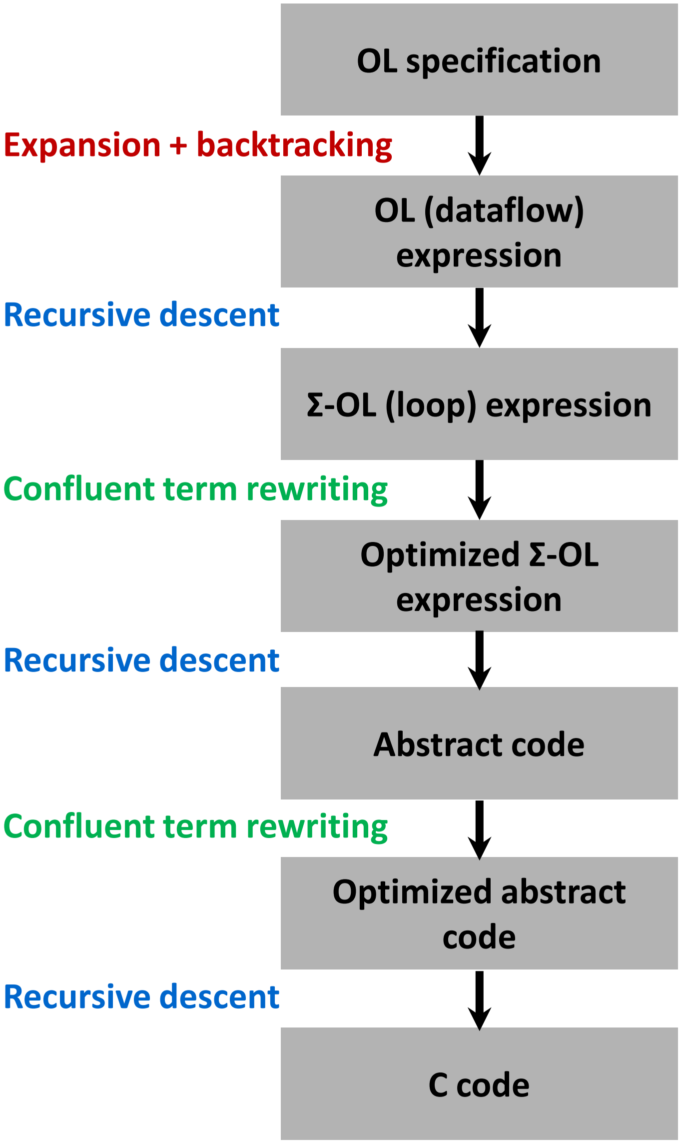



```
chain(
  assign(Y, V(0.0),
  loop(i1, [0..5],
    assign(nth(y, i1),
      f(nth(X, i1)))
  )
)
```

Cleanup rules: term rewriting

```
chain(a, chain(b))     → chain([a, b])
decl(D, decl(E, c))    → decl([D, E], c)
loop(i, decl(D, c))    → decl(D, loop(i, c))
chain(a, decl(D, b))   → decl(D, chain([a, b]))
```

Putting it Together: One Big Rule System



Mathematical specification

$$\mathrm{SafeDist}_{V,A,b,\varepsilon}(.,.,.) \to \Big(P[x,(a_0,a_1,a_2)](.) < d_\infty^2(.,.)\Big)(.,.,.)$$

with $a_0 = \frac{1}{2b},\ a_1 = \frac{V}{b} + \varepsilon\left(\frac{A}{b}+1\right),\ a_2 = \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon V\right)$

Final code

```
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8 , x1, x10, x13, x14, x17
    int w1;
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN + DBL_MIN)), _mm_lo
        x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
        x2 = _mm_mul_pd(x1, x6);
        x3 = _mm_mul_pd(_mm_shuffle_pd(x1, x1, _MM_SHUFFLE2(0, 1)),
        x4 = _mm_sub_pd(_mm_set1_pd(0.0), _mm_min_pd(x3, x2));
        u3 = _mm_add_pd(_mm_max_pd(_mm_shuffle_pd(x4, x4, _MM_SHUFF
```

Final Synthesized C Code

```
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8 , x1, x10, x13, x14, x17, x18, x19, x2, x3, x4, x6, x7, x8, x9;
    int w1;
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[0])));
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN + DBL_MIN)), _mm_loaddup_pd(&(D[i5])));
        x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
        x2 = _mm_mul_pd(x1, x6);
        x3 = _mm_mul_pd(_mm_shuffle_pd(x1, x1, _MM_SHUFFLE2(0, 1)), x6);
    }
    u6
    for
        u8 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[(i3 + 1)])));
        u7 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[(3 + i3)])));
        x14 = _mm_add_pd(u8, _mm_shuffle_pd(u7, u7, _MM_SHUFFLE2(0, 1)));
        x13 = _mm_shuffle_pd(x14, x14, _MM_SHUFFLE2(0, 1));
        u4 = _mm_shuffle_pd(_mm_min_pd(x14, x13), _mm_max_pd(x14, x13), _MM_SHUFFLE2(1, 0));
        u6 = _mm_shuffle_pd(_mm_min_pd(u6, u4), _mm_max_pd(u6, u4), _MM_SHUFFLE2(1, 0));
    }
    x17 = _mm_addsub_pd(_mm_set1_pd(0.0), u6);
    x18 = _mm_addsub_pd(_mm_set1_pd(0.0), u5);
    x19 = _mm_cmpge_pd(x17, _mm_shuffle_pd(x18, x18, _MM_SHUFFLE2(0, 1)));
    w1 = (_mm_testc_si128(_mm_castpd_si128(x19), _mm_set_epi32(0xffffffff, 0xffffffff, 0xffffffff, 0xffffffff)) –
         (_mm_testnzc_si128(_mm_castpd_si128(x19), _mm_set_epi32(0xffffffff, 0xffffffff, 0xffffffff, 0xffffffff))));
    __asm nop;
    if (_mm_getcsr() & 0x0d) {
        _mm_setcsr(_xm);
        return -1;
    }
    _mm_setcsr(_xm);
    return w1;
}
```

$$\begin{aligned}\mathsf{SafeDist}_{V,A,b,\varepsilon} = {} & \mathsf{Atomic}_{(x,y)\mapsto x<y} \\ & \circ \Big(\big(\mathsf{Reduction}_{3,(x,y)\mapsto x+y} \circ \mathsf{Pointwise}_{3,x\mapsto a_i x} \circ \mathsf{Induction}_{3,(a,b)\mapsto ab,1} \big) \\ & \quad \times \big(\mathsf{Reduction}_{2,(x,y)\mapsto \max(|x|,|y|)} \circ \mathsf{Pointwise}_{2\times 2,(x,y)\mapsto x-y} \big) \Big)\end{aligned}$$

Inspiration: Symbolic Integration

- **Rule based AI system**
  basic functions, substitution
- **May not succeed**
  not all expressions can be symbolically integrated
- **Arbitrarily extensible**
  define new functions as integrals
  Γ(.), distributions, Lebesgue integral
- **Semantics preserving**
  rule chain = formal proof
- **Automation**
  Mathematica, Maple

Table of Integrals

BASIC FORMS

(1) $\int x^n dx = \frac{1}{n+1} x^{n+1}$

(2) $\int \frac{1}{x} dx = \ln x$

(3) $\int u dv = uv - \int v du$

(4) $\int u(x) v'(x) dx = u(x)v(x) - \int v(x) u'(x) dx$

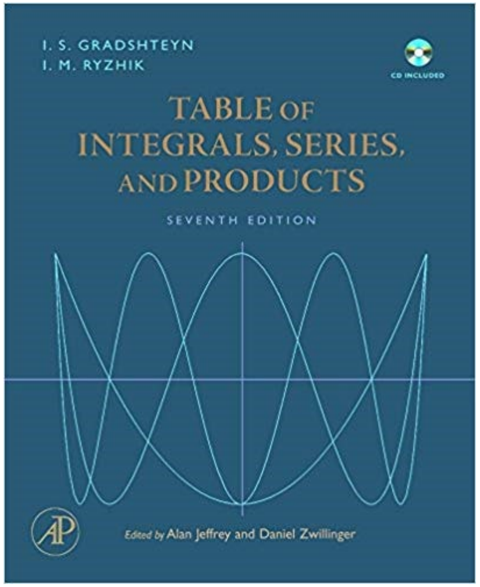
RATIONAL FUNCTIONS

(5) $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln(ax+b)$

(6) $\int \frac{1}{(x+a)^2} dx = \frac{-1}{x+a}$

(7) $\int (x+a)^n dx = (x+a)^n \left(\frac{a}{1+n} + \frac{x}{1+n} \right), \; n \neq -1$

(8) $\int x(x+a)^n dx = \frac{(x+a)^{1+n}(nx+x-a)}{(n+2)(n+1)}$



In[31]:= $\int_0^{2\pi} \frac{1}{a^2 \text{Cos}[t]^2 + b^2 \text{Sin}[t]^2} \, dt$

Out[31]= $\frac{2 \sqrt{\frac{b^2}{a^2}} \pi}{b^2}$

In[33]:= $\int_0^{2\pi} \frac{1}{a^2 \left(\frac{e^{i t} + e^{-i t}}{2}\right)^2 + b^2 \left(\frac{e^{i t} - e^{-i t}}{2 i}\right)^2} \, dt$

Out[33]= 0

Wolfram Mathematica

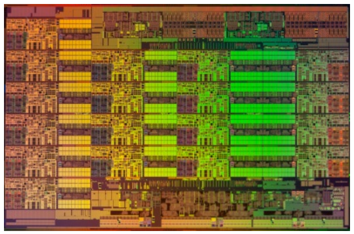
Outline

- Introduction
- Specifying computation
- **Achieving Performance Portability**
- FFTX: A Library Frontend for SPIRAL
- Summary

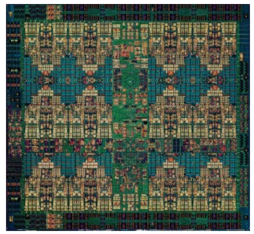
F. Franchetti, T. M. Low, D. T. Popovici, R. M. Veras, D. G. Spampinato, J. R. Johnson, M. Püschel, J. C. Hoe, J. M. F. Moura:
SPIRAL: Extreme Performance Portability, Proceedings of the IEEE, Vol. 106, No. 11, 2018.
Special Issue on *From High Level Specification to High Performance Code*

Today's Computing Landscape

1 Gflop/s = one billion floating-point operations (additions or multiplications) per second

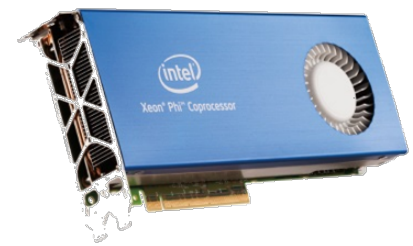


Intel Xeon 8180M
2.25 Tflop/s, 205 W
28 cores, 2.5—3.8 GHz
2-way—16-way AVX-512



IBM POWER9
768 Gflop/s, 300 W
24 cores, 4 GHz
4-way VSX-3

Nvidia Tesla V100
7.8 Tflop/s, 300 W
5120 cores, 1.2 GHz
32-way SIMT



Intel Xeon Phi 7290F
1.7 Tflop/s, 260 W
72 cores, 1.5 GHz
8-way/16-way LRBni

Snapdragon 835
15 Gflop/s, 2 W
8 cores, 2.3 GHz
A540 GPU, 682 DSP, NEON

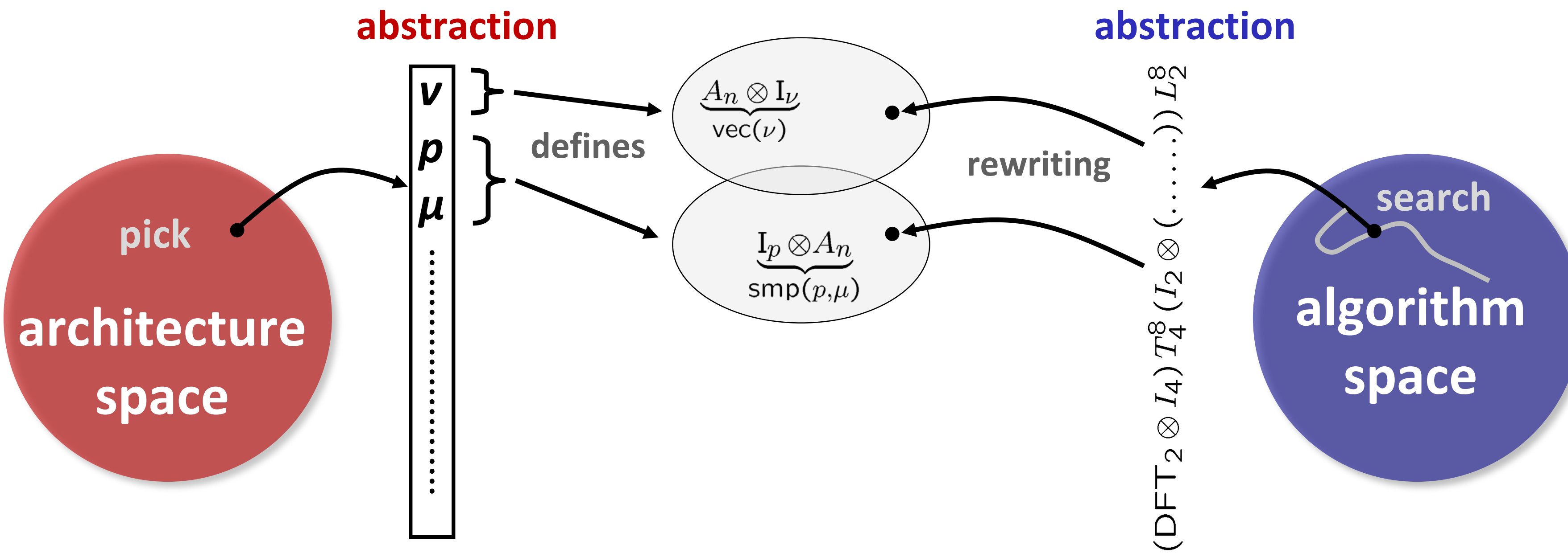
Intel Atom C3858
32 Gflop/s, 25 W
16 cores, 2.0 GHz
2-way/4-way SSSE3

Dell PowerEdge R940
3.2 Tflop/s, 6 TB, 850 W
4x 24 cores, 2.1 GHz
4-way/8-way AVX

Summit
187.7 Pflop/s, 13 MW
9,216 x 22 cores POWER9
+ 27,648 V100 GPUs

Platform-Aware Formal Program Synthesis

Model: common abstraction
= spaces of matching formulas



Architectural parameter:
Vector length,
#processors, …

optimization

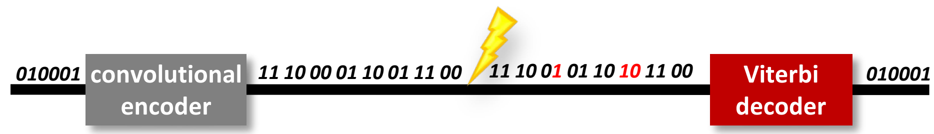
Kernel:
problem size,
algorithm choice

Some Application Domains in OL

Linear Transforms

$$
\begin{aligned}
\mathbf{DFT}_n &\rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathrm{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathrm{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\rightarrow P_n (\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \gcd(k,m) = 1 \\
\mathbf{DFT}_p &\rightarrow R^T_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) R_p, \quad p \text{ prime} \\
\mathbf{DCT\text{-}3}_n &\rightarrow (\mathrm{I}_m \oplus \mathrm{J}_m)\, \mathrm{L}^n_m (\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \\
&\quad \cdot (\mathrm{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, \quad n = 2m \\
\mathbf{DCT\text{-}4}_n &\rightarrow S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\rightarrow (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m) \left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \right) \mathrm{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m} \\
\mathbf{WHT}_{2^k} &\rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t \\
\mathbf{DFT}_2 &\rightarrow \mathrm{F}_2 \\
\mathbf{DCT\text{-}2}_2 &\rightarrow \mathrm{diag}(1, 1/\sqrt{2})\, \mathrm{F}_2 \\
\mathbf{DCT\text{-}4}_2 &\rightarrow \mathrm{J}_2\, \mathrm{R}_{13\pi/8}
\end{aligned}
$$

Software Defined Radio

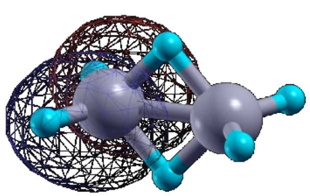


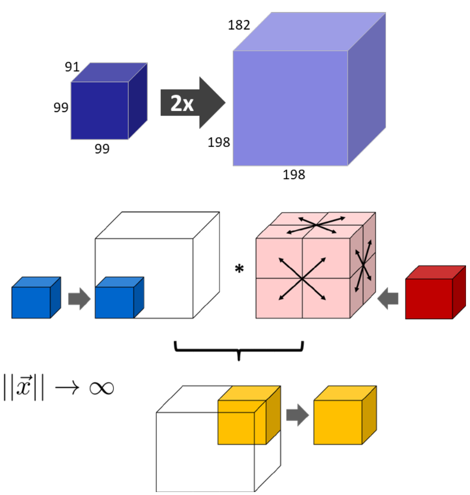
$$
F_{K,F} \rightarrow \prod_{i=1}^{F} \left((I_{2^{K-2}} \otimes_j B_{F-i,j}) L^{2^{K-1}}_{2^{K-2}} \right)
$$

$$
\underline{\mathbf{F}_{K,F}}\ \nu \rightarrow \prod_{i=1}^{F} \left(\left(\mathrm{I}_{2^{K-2}/\nu} \otimes_{j_1} \vec{\mathrm{L}}^{2\nu}_{\nu}\, \vec{B}^{\nu}_{F-i,j_1} \right) (\mathrm{L}^{2^{K-1}/\nu}_{2^{K-2}/\nu} \bar{\otimes}\, \mathrm{I}_\nu) \right)
$$

$$
B_{i,j} : \begin{cases} \pi_U = \min_{d_U}(\pi_A + \beta_{A\to U}, \pi_B + \beta_{B\to U}) \\ \pi_V = \min_{d_V}(\pi_A + \beta_{A\to V}, \pi_B + \beta_{B\to V}) \end{cases}
$$

PDEs/HPC Simulations



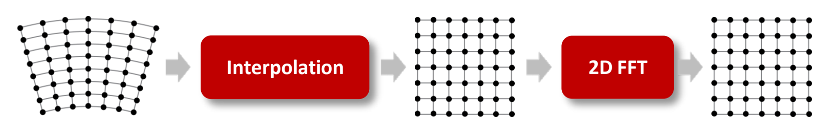


$$
\Phi : \mathbb{R}^3 \rightarrow \mathbb{R}
$$

$$
\Phi(\vec{x}) = \frac{Q}{4\pi ||\vec{x}||} + o\left(\frac{1}{||\vec{x}||}\right) \text{ as } ||\vec{x}|| \rightarrow \infty
$$

$$
Q = \int_D \rho\, d\vec{x}
$$

Synthetic Aperture Radar (SAR)



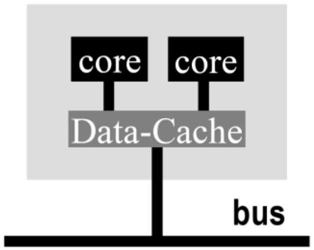
$$
\begin{aligned}
\mathsf{SAR}_{k\times m \rightarrow n\times n} &\rightarrow \mathsf{DFT}_{n\times n} \circ \mathsf{Interp}_{k\times m \rightarrow n\times n} \\
\mathsf{DFT}_{n\times n} &\rightarrow (\mathsf{DFT}_n \otimes \mathrm{I}_n) \circ (\mathrm{I}_n \otimes \mathsf{DFT}_n) \\
\mathsf{Interp}_{k\times m \rightarrow n\times n} &\rightarrow (\mathsf{Interp}_{k\rightarrow n} \otimes_i \mathrm{I}_n) \circ (\mathrm{I}_k \otimes_i \mathsf{Interp}_{m\rightarrow n}) \\
\mathsf{Interp}_{r\rightarrow s} &\rightarrow \left(\bigoplus_{i=0}^{n-2} \mathsf{InterpSeg}_k \right) \oplus \mathsf{InterpSegPruned}_{k,\ell} \\
\mathsf{InterpSeg}_k &\rightarrow \mathrm{G}^{u\cdot n \rightarrow k}_f \circ \mathsf{iPrunedDFT}_{n\rightarrow u\cdot n} \circ \left(\frac{1}{n}\right) \circ \mathsf{DFT}_n
\end{aligned}
$$

Formal Approach for all Types of Parallelism

- **Multithreading** (Multicore) $\mathrm{I}_p \otimes_{\|} A_{\mu n}, \quad \mathrm{L}_m^{mn} \bar{\otimes} \mathrm{I}_\mu$
- **Vector SIMD** (SSE, VMX/Altivec,…) $A \hat{\otimes} \mathrm{I}_\nu \quad \underbrace{\mathrm{L}_2^{2\nu}}_{\text{isa}}, \quad \underbrace{\mathrm{L}_\nu^{2\nu}}_{\text{isa}}, \quad \underbrace{\mathrm{L}_\nu^{\nu^2}}_{\text{isa}}$
- **Message Passing** (Clusters, MPP) $\mathrm{I}_p \otimes_{\|} A_n, \quad \underbrace{\mathrm{L}_p^{p^2} \bar{\otimes} \mathrm{I}_{n/p^2}}_{\text{all-to-all}}$
- **Streaming/multibuffering** (Cell) $\mathrm{I}_n \otimes_2 A_{\mu n}, \quad \mathrm{L}_m^{mn} \bar{\otimes} \mathrm{I}_\mu$
- **Graphics Processors** (GPUs) $\prod_{i=0}^{n-1} A_i, \quad A_n \hat{\otimes} \mathrm{I}_w, \quad P_n \otimes Q_w$
- **Gate-level parallelism** (FPGA) $\prod_{i=0}^{n-1}{}^{\text{ir}} A, \quad \mathrm{I}_s \tilde{\otimes} A, \quad \underbrace{\mathrm{L}_n^m}_{\text{bram}}$
- **HW/SW partitioning** (CPU + FPGA) $\underbrace{A_1}_{\text{fpga}}, \quad \underbrace{A_2}_{\text{fpga}}, \quad \underbrace{A_3}_{\text{fpga}}, \quad \underbrace{A_4}_{\text{fpga}}$

Modeling Hardware: Base Cases

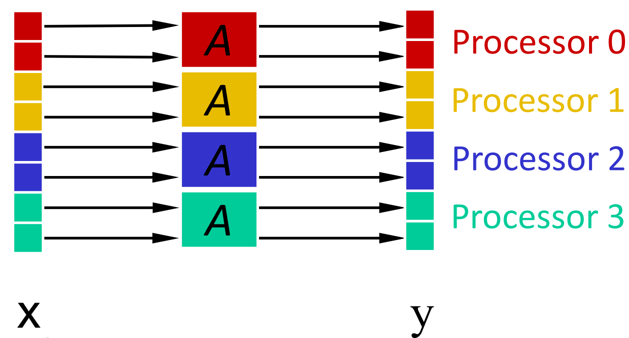
- **Hardware abstraction: shared cache with cache lines**



$$\underbrace{A_{k \times m \to n}}_{\mathsf{smp}(p,\mu)}$$

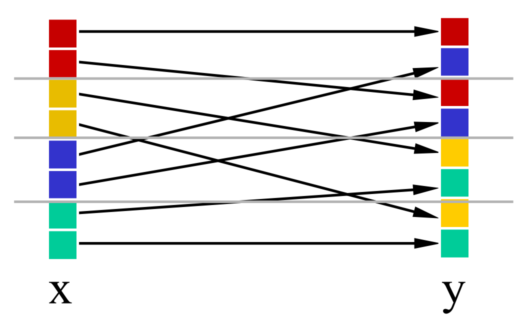
- **Tensor product: embarrassingly parallel operator**

$$y = \big(\mathrm{I}_p \otimes A\big)(x)$$



- **Permutation: problematic; may produce false sharing**

$$y = \mathrm{L}^8_4(x)$$



Example Program Transformation Rule Set

$$\underbrace{AB}_{\mathsf{smp}(p,\mu)} \rightarrow \underbrace{A}_{\mathsf{smp}(p,\mu)} \underbrace{B}_{\mathsf{smp}(p,\mu)}$$

$$\underbrace{A_m \otimes \mathrm{I}_n}_{\mathsf{smp}(p,\mu)} \rightarrow \underbrace{\left(\mathsf{L}_m^{mp} \otimes \mathrm{I}_{n/p}\right)\left(\mathrm{I}_p \otimes (A_m \otimes \mathrm{I}_{n/p})\right)\left(\mathsf{L}_p^{mp} \otimes \mathrm{I}_{n/p}\right)}_{\mathsf{smp}(p,\mu)}$$

$$\underbrace{\mathsf{L}_m^{mn}}_{\mathsf{smp}(p,\mu)} \rightarrow \begin{cases} \underbrace{\left(\mathrm{I}_p \otimes \mathsf{L}_{m/p}^{mn/p}\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\left(\mathsf{L}_p^{pn} \otimes \mathrm{I}_{m/p}\right)}_{\mathsf{smp}(p,\mu)} \\ \underbrace{\left(\mathsf{L}_m^{pm} \otimes \mathrm{I}_{n/p}\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\left(\mathrm{I}_p \otimes \mathsf{L}_m^{mn/p}\right)}_{\mathsf{smp}(p,\mu)} \end{cases}$$

Recursive rules

$$\underbrace{\mathrm{I}_m \otimes A_n}_{\mathsf{smp}(p,\mu)} \rightarrow \mathrm{I}_p \otimes_{\|} \left(\mathrm{I}_{m/p} \otimes A_n\right)$$

$$\underbrace{(P \otimes \mathrm{I}_n)}_{\mathsf{smp}(p,\mu)} \rightarrow \left(P \otimes \mathrm{I}_{n/\mu}\right) \overline{\otimes}\, \mathrm{I}_\mu$$

Base case rules

Autotuning in Constraint Solution Space

$$\overbrace{\mathrm{DFT}_8}^{}_{\mathrm{AVX}(2\text{-way }\mathbb{C})}$$

AVX 2-way `_Complex double`

DFT_8

Base cases

$A^{n\times n} \vec{\otimes} \mathrm{I}_2$

$\underbrace{\mathrm{L}_2^4}_{\mathrm{vec}(2)}$

$\underbrace{\mathrm{T}_n^{mn}}_{\mathrm{vec}(2)}$

Transformation rules

$$(\mathrm{I}_m \otimes A^{n\times n})\mathrm{L}_m^{mn} \to \left(\mathrm{I}_{m/\nu} \otimes \mathrm{L}_\nu^{n\nu}(A^{n\times n} \otimes \mathrm{I}_\nu)\right)(\mathrm{L}_{m/\nu}^{mn/\nu} \otimes \mathrm{I}_\nu)$$

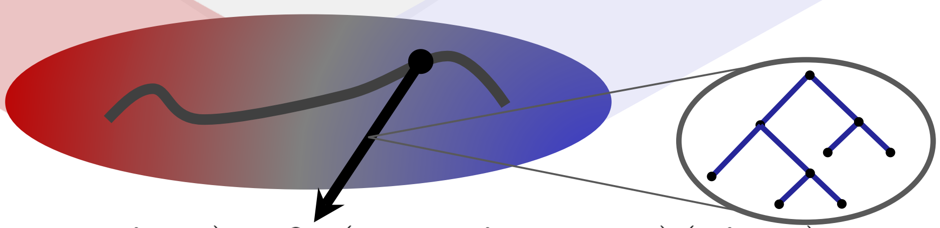
$$\mathrm{L}_\nu^{n\nu} \to (\mathrm{L}_\nu^n \otimes \mathrm{I}_\nu)(\mathrm{I}_{n/\nu} \otimes \mathrm{L}_\nu^{\nu^2})$$

$$A^{m\times m} \otimes \mathrm{I}_n \to (A^{m\times m} \otimes \mathrm{I}_{n/\nu}) \otimes \mathrm{I}_\nu$$

Breakdown rules

$$\mathbf{DFT}_{mn} \to (\mathbf{DFT}_m \otimes \mathrm{I}_n)\mathrm{T}_n^{mn}(\mathrm{I}_m \otimes \mathbf{DFT}_n)\mathrm{L}_m^{mn}$$

$$\mathbf{DFT}_2 \to \mathrm{F}_2$$



$$\left((\mathrm{F}_2 \otimes \mathrm{I}_2)\mathrm{T}_2^4(\mathrm{I}_2 \otimes \mathrm{F}_2)\mathrm{L}_2^4 \vec{\otimes} \mathrm{I}_2\right) \underbrace{\mathrm{T}_2^8}_{\mathrm{vec}(2)} \left(\mathrm{I}_2 \otimes \underbrace{\mathrm{L}_2^4}_{\mathrm{vec}(2)} (\mathrm{F}_2 \vec{\otimes} \mathrm{I}_2)\right)\left(\mathrm{L}_2^4 \vec{\otimes} \mathrm{I}_2\right)$$

OL specification

Expansion + backtracking

OL (dataflow) expression

Recursive descent

Σ-OL (loop) expression

Confluent term rewriting

Optimized Σ-OL expression

Recursive descent

Abstract code

Confluent term rewriting

Optimized abstract code

Recursive descent

C code

Translating an OL Expression Into Code

Constraint Solver Input: $\underbrace{\mathrm{DFT}_8}_{\text{AVX(2-way } \mathbb{C})}$

Output =

Ruletree, expanded into

OL Expression:

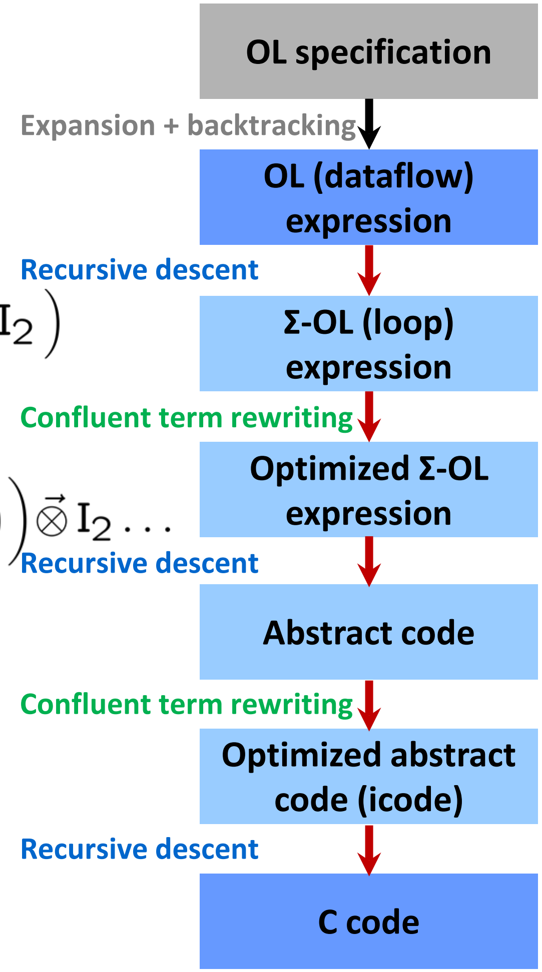
$$\left((\mathrm{F}_2 \otimes \mathrm{I}_2)\mathrm{T}^4_2(\mathrm{I}_2 \otimes \mathrm{F}_2)\mathrm{L}^4_2 \vec{\otimes}\, \mathrm{I}_2\right) \underbrace{\mathrm{T}^8_2}_{\text{vec}(2)} \left(\mathrm{I}_2 \otimes \underbrace{\mathrm{L}^4_2}_{\text{vec}(2)} (\mathrm{F}_2 \vec{\otimes}\, \mathrm{I}_2)\right)\left(\mathrm{L}^4_2 \vec{\otimes}\, \mathrm{I}_2\right)$$

∑-OL:

$$\left(\sum_{j=0}^{1}\left(\mathrm{S}_{\imath_2 \otimes (j)_2} \mathrm{F}_2 \mathrm{Map}^2_{x \mapsto \omega_4^{2i+j} x} \mathrm{G}_{\imath_2 \otimes (j)_2}\right) \sum_{j=0}^{1}\left(\mathrm{S}_{(j)_2 \otimes \imath_2} \mathrm{F}_2 \mathrm{G}_{\imath_2 \otimes (j)_2}\right)\right) \vec{\otimes}\, \mathrm{I}_2 \ldots$$

C Code:

```
void dft8(_Complex double *Y, _Complex double *X) {
    __m256d s38, s39, s40, s41,...
    __m256d  *a17, *a18;
    a17 = ((__m256d  *) X);
    s38 = *(a17);
    s39 = *((a17 + 2));
    t38 = _mm256_add_pd(s38, s39);
    t39 = _mm256_sub_pd(s38, s39);
    ...
    s52 = _mm256_sub_pd(s45, s50);
    *((a18 + 3)) = s52;
}
```



Symbolic Verification for Linear Operators

- **Linear operator = matrix-vector product**
  Algorithm = matrix factorization

$$\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}=\begin{bmatrix}1&\cdot&1&\cdot\\\cdot&1&\cdot&1\\1&\cdot&-1&\cdot\\\cdot&1&\cdot&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&\cdot&\cdot&j\end{bmatrix}\begin{bmatrix}1&1&\cdot&\cdot\\1&-1&\cdot&\cdot\\\cdot&\cdot&1&1\\\cdot&\cdot&1&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&\cdot&1\end{bmatrix}\quad = ?$$

$$\mathrm{DFT}_4 = (\mathrm{DFT}_2 \otimes \mathrm{I}_2)\,\mathrm{T}^4_2\,(\mathrm{I}_2 \otimes \mathrm{DFT}_2)\,\mathrm{L}^4_2$$

- **Linear operator = matrix-vector product**
  Program = matrix-vector product

$$\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\\0\end{bmatrix} = ?$$

```
DFT4([0,1,0,0])
```

Symbolic evaluation and symbolic execution establishes correctness

Outline

- Introduction
- Specifying computation
- Achieving Performance Portability
- **FFTX: A Library Frontend for SPIRAL**
- Summary

F. Franchetti, D. G. Spampinato, A. Kulkarni, D. T. Popovici, T. M. Low, M. Franusich, A. Canning, P. McCorquodale, B. Van Straalen, P. Colella: **FFTX and SpectralPack: A First Look**, Workshop on Parallel Fast Fourier Transforms (PFFT).

FFTX and SpectralPACK

Numerical Linear Algebra		Spectral Algorithms
LAPACK LU factorization Eigensolves SVD …	≈	**SpectralPACK** Convolution Correlation Upsampling Poisson solver …
BLAS BLAS-1 BLAS-2 BLAS-3		**FFTX** DFT, RDFT 1D, 2D, 3D,… batch

Define the LAPACK equivalent for spectral algorithms

- **Define FFTX as the BLAS equivalent**
  provide user FFT functionality as well as algorithm building blocks
- **Define class of numerical algorithms to be supported by SpectralPACK**
  PDE solver classes (Green's function, sparse in normal/k space,…), signal processing,…
- **Library front-end, code generation and vendor library back-end**
  mirror concepts from FFTX layer

FFTX and SpectralPACK solve the "spectral motif" long term

Example: Poisson's Equation in Free Space

Partial differential equation (PDE)

$$\Delta(\Phi) = \rho$$

$$\rho : \mathbb{R}^3 \to \mathbb{R}$$

$$D = \text{supp}(\rho) \subset \mathbb{R}^3$$

Poisson's equation. Δ is the Laplace operator

Solution characterization

$$\Phi : \mathbb{R}^3 \to \mathbb{R}$$

$$\Phi(\vec{x}) = \frac{Q}{4\pi||\vec{x}||} + o\left(\frac{1}{||\vec{x}||}\right) \text{ as } ||\vec{x}|| \to \infty$$

$$Q = \int_D \rho d\vec{x}$$

Approach: Green's function

$$\Phi(\vec{x}) = \int_D G(\vec{x} - \vec{y})\rho(\vec{y})d\vec{y} \equiv (G * \rho)(\vec{x}), \quad G(\vec{x}) = \frac{1}{4\pi||\vec{x}||_2}$$

Solution: ϕ(.) = convolution of RHS *ρ*(.) with Green's function *G*(.). Efficient through FFTs (frequency domain)

Method of Local Corrections (MLC)

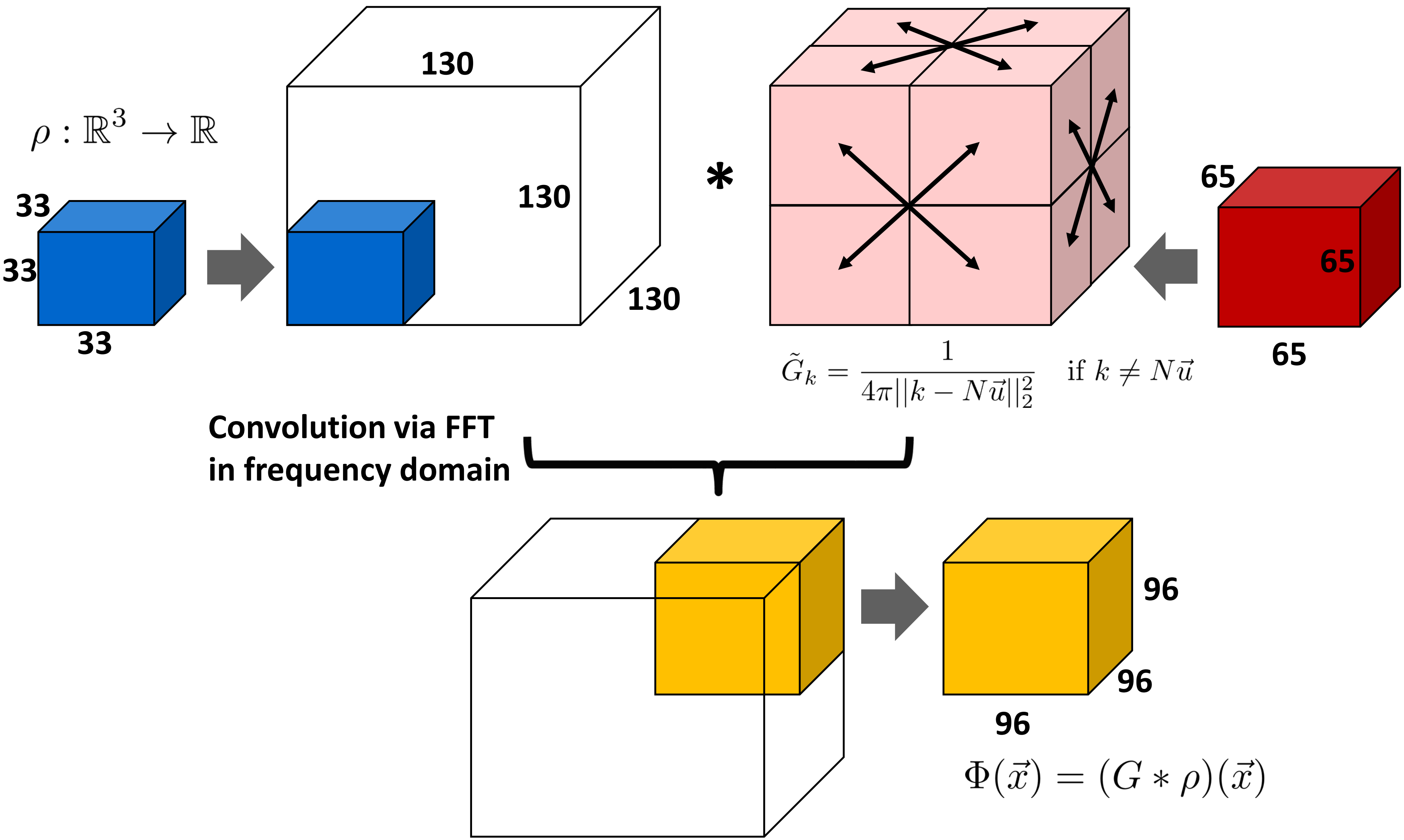
$$\tilde{G}_k = \frac{1}{4\pi||k - N\vec{u}||_2^2} \quad \text{if } k \neq N\vec{u}$$

Green's function kernel in frequency domain

P. McCorquodale, P. Colella, G. T. Balls, and S. B. Baden: **A Local Corrections Algorithm for Solving Poisson's Equation in Three Dimensions.** Communications in Applied Mathematics and Computational Science Vol. 2, No. 1 (2007), pp. 57-81., 2007.

C. R. Anderson: **A method of local corrections for computing the velocity field due to a distribution of vortex blobs.** Journal of Computational Physics, vol. 62, no. 1, pp. 111–123, 1986.

Algorithm: Hockney Free Space Convolution



Hockney: Convolution + problem specific zero padding and output subset

FFTX C++ Code: Hockney Free Space Convolution

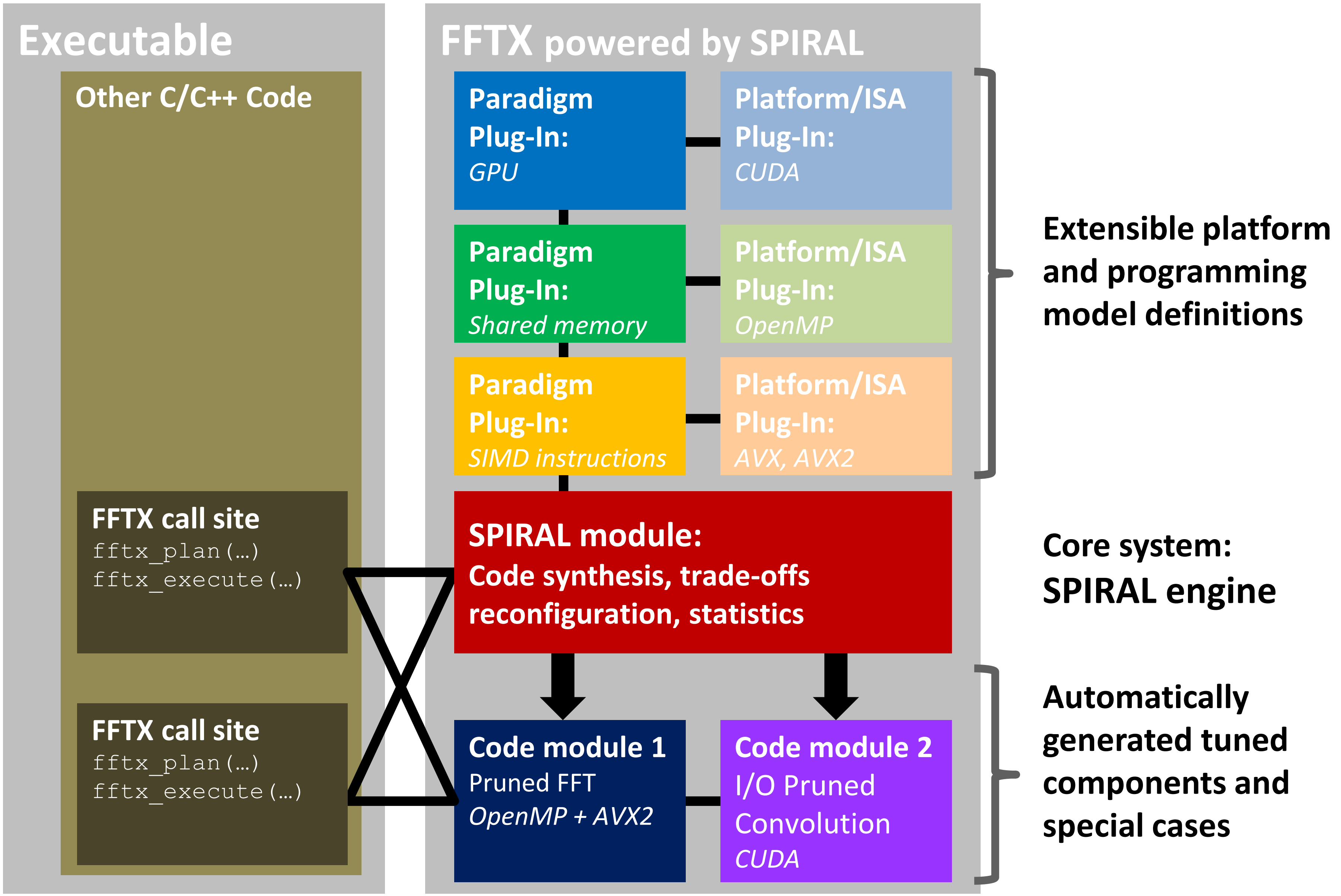
```
box_t<3> inputBox(point_t<3>({{0,0,0}}),point_t<3>({32,32,32}));
array_t<3, double> rho(inputBox);
// ... set input values.

box_t<3> transformBox(point_t<3>({{0,0,0}}),point_t<3>({{129,129,129}})));
box_t<3> outputBox(point_t<3>({33,33,33}),point_t<3>({129,129,129}));

point_t<3> kindF({{DFT,DFT,DFT}});

size_t

auto fo
    pla

auto ke                                                                    lize);

point_t
auto in

auto so

context
context
```

This is a specification dressed as a program

- Needs to be clean and concise
- No code level optimizations and tricks
- Don’t think “performance” but “correctness”
- *For production code and software engineering*

```
std::ofstream splFile("hockney.spl");
export_spl(context, solver, splFile, "hockney33_97_130");
splFile.close();
// Offline codegen.
auto fptr = import_spl<3, double, double>("hockney33_97_130");
array_t<3, double> Phi(inputBox);
fptr(&rho, &Phi, 1);
```

FFTX Backend: SPIRAL



C/C++ FFTX Program Trace

```
fftx_session := [
  rec(op := "fftx_init", flags := IntHexString("8000000")),
  rec(op := "fftx_create_data_real", rank := 1, dims := [ rec(n := 4, is := 1, os := 1) ],
    ptr := IntHexString("000000D236DD2460")),
  ...
  rec(op := "fftx_create_zero_temp_real", rank := 1,
    dim
  rec(o                                                                               1,
    dat
      i
      o
  rec(o
    ran
    how
    inp
    fla
  rec(o
    ran
    ds
    dof
    inp
    dat
    callback := [
      rec(op := "call", inp := IntHexString("A000000000000001"),
        outp := IntHexString("A000000000000002"), data := IntHexString("A000000000000003")),
      rec(op := "FFTX_COMPLEX_VAR", var := IntHexString("000000D236A0FA30"),
        re := 0.000000e+00, im := 0.000000e+00),
      rec(op := "FFTX_COMPLEX_MOV", target := IntHexString("000000D236A0FA30"),
        source := IntHexString("A000000000000001")),
      rec(op := "FFTX_COMPLEX_MUL", target := IntHexString("000000D236A0FA30"),
        source := IntHexString("A000000000000003")),
      ...
```

The whole convolution kernel is captured

- DAG with all dependencies
- User-defined call-backs
- Captures prunining, zero-padding and symmetries
- *Lifts sequence of C++ library calls to a specification*

SPIRAL Script Captures Performance Engineering

```
# Pruned 3D Real Convolution Pattern
Import(realdft);
Import(filtering);

# set up algorithms needed for multi-dimensional pruned real convolution
```

Recognizes pattern and applies code generation

- Developed by performance engineer + application specialist
- Casts FFTX call sequence as SPIRAL non-terminal
- Does code generation and autotuning
- *Clear separation of concerns frontend/backend*

```
sym := var.fresh_t("S", TArray(TReal, 2*n_freq));
t := IOPrunedRConv(N, sym, 1, [minout..N-1], 1, [0..maxin], true);

# generate code and autotune
rt := DP(t, opts)[1].ruletree;
c := CodeRuleTree(rt, opts);

# create files
PrintTo(name::".c", PrintCode(name, c, opts));
```

Backend: SPIRAL Code Generation

```
__global__ void ker_code0(int *D48, double *D49, double *D50, double *D51, int *D52, double  *X) {
    __shared__ double T235[260];
    ...
    if (((threadIdx.x < 13))) {
        for(int i96 = 0; i96 <= 4; i96++) {
            int a31, a32, a33, a34;
            a31 = (2*i96);
            a32 = (threadIdx.x + (13*a31));
            a33 = (threadIdx.x + (13*((a31 + 5) % 10)));
            a34 = (4*i96);
            *((((T235 + 0) + a34) + (20*threadIdx.x))) = (*((T6 + a32)) + *((T6 + a33)));
            *(((1 + (T235 + 0) + a34) + (20*threadIdx.x))) = 0.0;
            *(((2 + (T235 + 0) + a34) + (20*threadIdx.x))) = (*((T6 + a32)) - *((T6 + a33)));
            *(((3 + (T235 + 0) + a34) + (20*threadIdx.x))) = 0.0;
        }
        double t261, t262, t263, t264, t265, t266, t267, t268;
        int a129;
        t263 = (*((((T235+0)+12)+(20*threadIdx.x)))+*((((T235+0)+8)+(20*threadIdx.x))));
        t264 = (*((((T235+0)+12)+(20*threadIdx.x)))-*((((T235+0)+8)+(20*threadIdx.x))));
        ...
        *((3 + T5 + a129)) = ((0.5877852522924731*t268) - (0.9510565162951535*t266));
    }
    __syncwarp();
    if (((threadIdx.x < 1))) {
        double t305, t306, t307, t308, t309, t310, t311, t312, t313, t314, t315, t316;
        int a387;
        t305 = (*((T5 + 12)) + *((T5 + 144)));
        ...
```

FFTX/SPIRAL with CUDA backend

Early result:
130 Gflop/s
on par with cuFFT

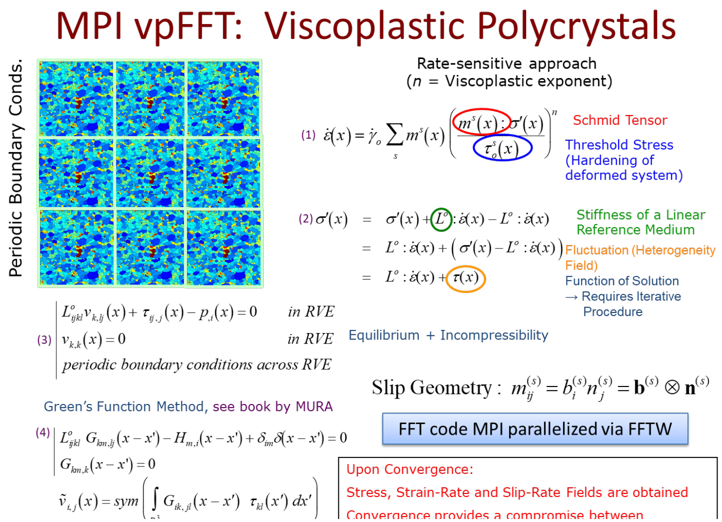
3,000 lines of code, kernel fusion, cross call data layout transforms

Outline

- Introduction
- Specifying computation
- Achieving Performance Portability
- FFTX: A Library Frontend for SPIRAL
- **Summary**

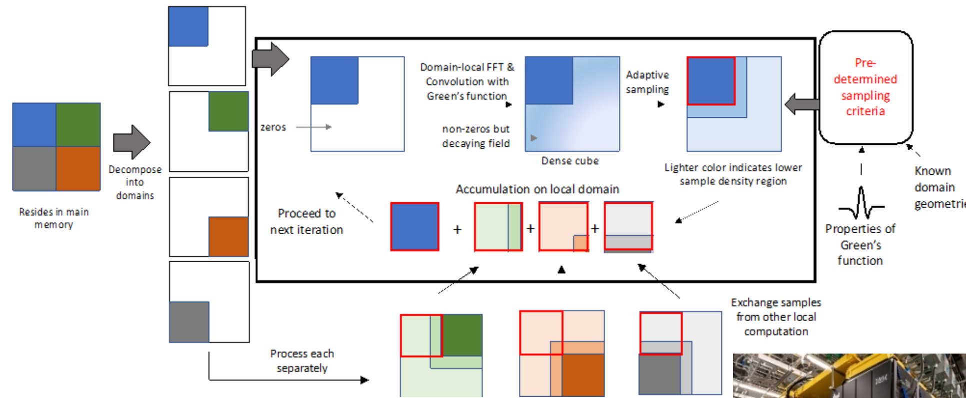
FFTX Extension For MASSIF/LANL

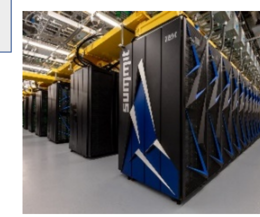
Convolution with Rank-4 Tensor



R. Lebensohn (LANL), A.D. Rollett (CMU)

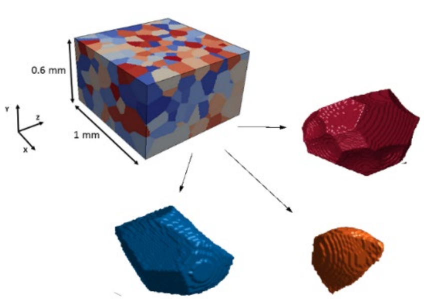
Challenge: Fitting Into GPU Memory

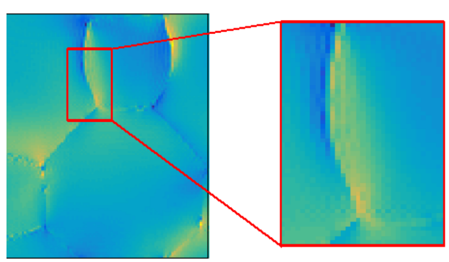




FORTRAN + MATLAB,
reduce MPI traffic
towards FFTX/SpectralPACK

Irregular Domain Decomposition





Signal processing + PDE tricks to compress

Performance Model

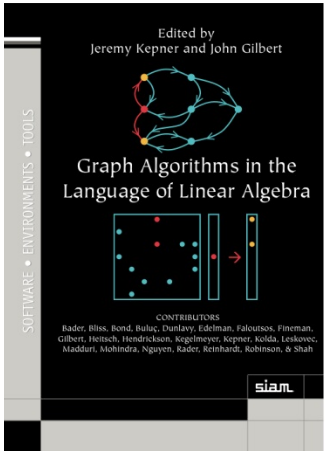
90224.994	32
11301.902	64

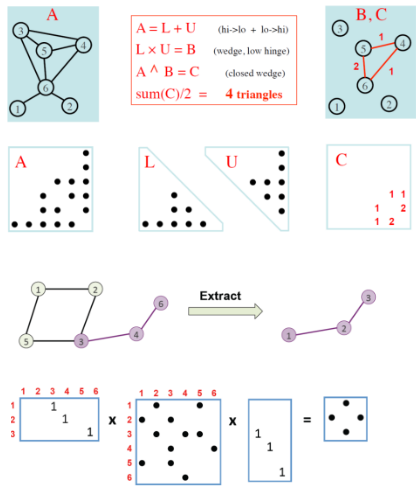
Problem size	Domain size	Compute time/domain [s]	No. of domains	Total compute time (single GPU) [Hrs]	Memory for Domain-local FFT [GB]
1024 × 1024 × 1024	128 × 128 × 128	0.292	512	0.041	1
1024 × 1024 × 1024	512 × 512 × 512	0.329	8	0.0007	4
2048 × 2048 × 2048	128 × 128 × 128	2.352	4096	2.676	4
2048 × 2048 × 2048	512 × 512 × 512	2.485	64	0.044	16
4096 × 4096 × 4096	128 × 128 × 128	19.079	32,768	173.662	16
4096 × 4096 × 4096	512 × 512 × 512	19.589	512	2.786	64
4096 × 4096 × 4096	1024 × 1024 × 1024	20.399	64	0.362	128
8192 × 8192 × 8192	64 × 64 × 64	154.882	2,097,152	90224.994	32
8192 × 8192 × 8192	128 × 128 × 128	155.208	2,62,144	11301.902	64
8192 × 8192 × 8192	512 × 512 × 512	157.272	4096	178.940	256
8192 × 8192 × 8192	1024 × 1024 × 1024	160.303	512	22.798	512

Model: 8k x 8k x 8k possible on Summit

Graph Algorithms in SPIRAL

Foundation





Triangle Counting in SPIRAL

```
TriangleCount()
```

$$\Delta = \Delta + \frac{1}{2}\alpha_{10} A_{00} \alpha_{01}$$

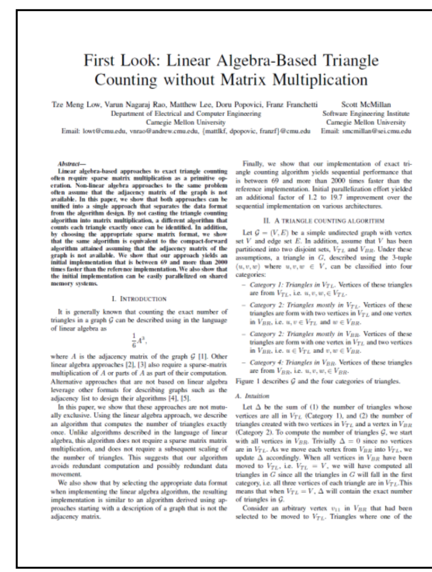
```
BB(
   Accum(i4, 1, X.N-1,
      Accum_X(i6, [ i4, 0 ], i4,
         Dot([ i6, add(i4, V(1)) ],
               [ i4, add(i4, V(1)) ],
                sub(sub(X.N, i4), V(1)))
)))
```

Formalization

Operation	Mathematical Description	Output	Inputs
mxm	C⟨¬M, z⟩ = C ⊙ (A^T ⊕.⊗ B^T)	C	¬, M, z, ⊙, A, T, ⊕.⊗, B, T
mxv, (vxm)	c⟨¬m, z⟩ = c ⊙ (A^T ⊕.⊗ b)	c	¬, m, z, ⊙, A, T, ⊕.⊗, b
eWiseMult	C⟨¬M, z⟩ = C ⊙ (A^T ⊗ B^T)	C	¬, M, z, ⊙, A, T, ⊗, B, T
eWiseAdd	C⟨¬M, z⟩ = C ⊙ (A^T ⊕ B^T)	C	¬, M, z, ⊙, A, T, ⊕, B, T
reduce (row)	c⟨¬m, z⟩ = c ⊙ [$\oplus_j$ A^T(:,j)]	c	¬, m, z, ⊙, A, T, ⊕
apply	C⟨¬M, z⟩ = C ⊙ f(A^T)	C	¬, M, z, ⊙, A, T, f
transpose	C⟨¬M, z⟩ = C ⊙ A^T	C	¬, M, z, ⊙, A (T)
extract	C⟨¬M, z⟩ = C ⊙ A^T(i,j)	C	¬, M, z, ⊙, A, T, i, j
assign	C⟨¬M, z⟩ (i,j) = C(i,j) ⊙ A^T	C	¬, M, z, ⊙, A, T, i, j
build (meth.)	C = $\mathbb{S}^{m \times n}$(i,j,v,⊙)	C	⊙, m, n, i, j, v
extractTuples (meth.)	(i,j,v) = A	i,j,v	A

Notation: **i**,**j** – index arrays, **v** – scalar array, **m** – 1D mask, other bold-lower – vector (column), **M** – 2D mask, other bold-caps – matrix, **T** – transpose, ¬ - structural complement, z – clear output, ⊕ monoid/binary function, ⊕.⊗ semiring, blue – optional parameters, red – optional modifiers

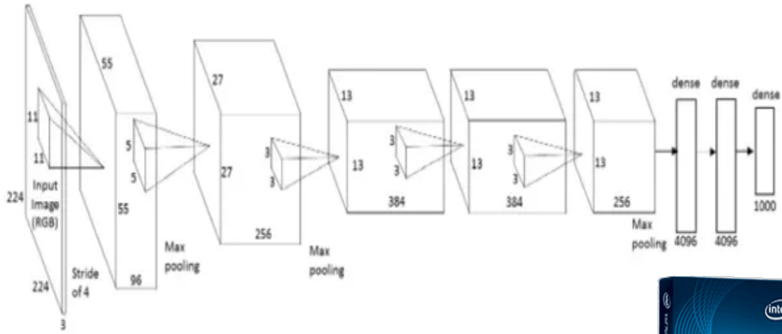
HPEC Graph Challenge



In collaboration with CMU-SEI

Towards Deep Learning in SPIRAL

Standard: Use GEMM

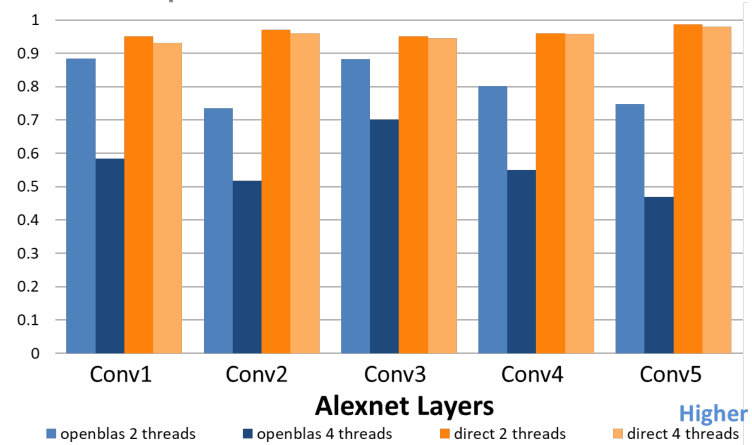



```
dgemm(transa, transb, m, n, k, alpha,
      a, lda, b, ldb, beta, c, ldc)
```

Direct CNN – More efficient

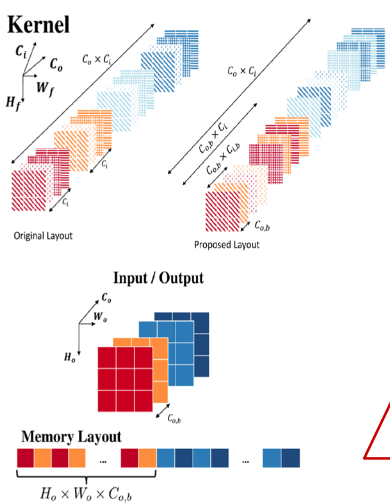
Scalability on AMD Piledriver

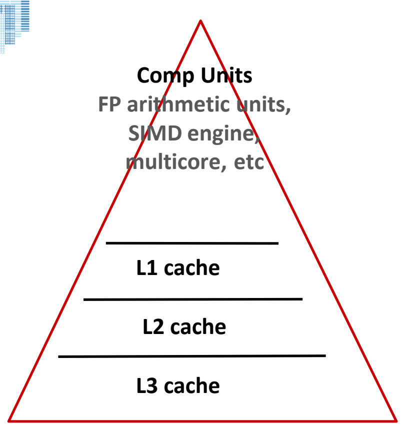
Normalized performance to 1 thread



Jiyuan Zhang, Franz Franchetti, Tze Meng Low, "High Performance Zero-Memory Overhead Direct Convolutions", 2018, International Conference of Machine Learning

CNN/System Friendly Layout





Towards CNNs in SPIRAL

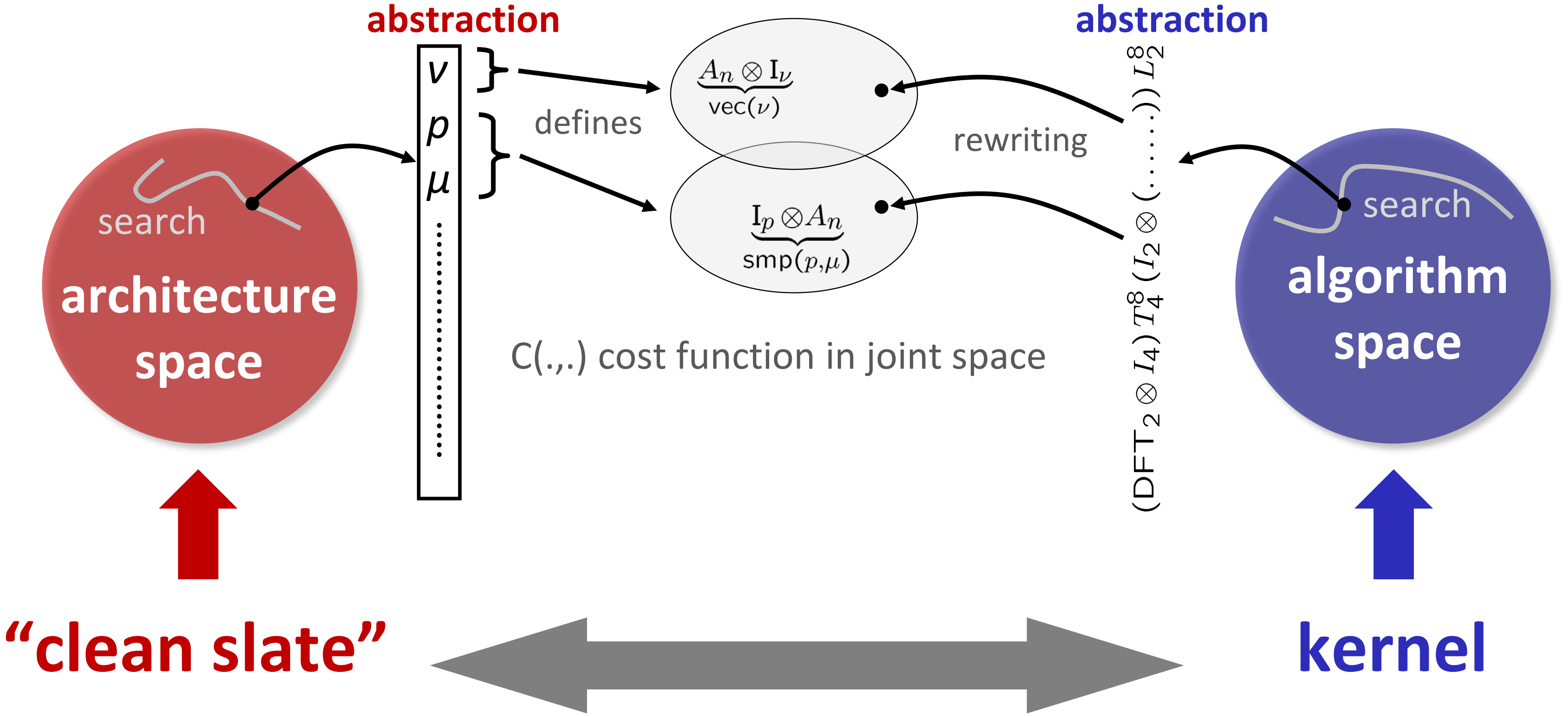
- **Level 0: simple C program**
  implements the algorithm cleanly
- **Level 1: C macros plus search script**
  use C preprocessor for meta-programming
- **Level 2: scripting for code specialization**
  text-based program generation, e.g., ATLAS
- **Level 3: add compiler technology**
  internal code representation, e.g., FFTW's genfft
- **Level 4: synthesize the program from scratch**
  high level representation, e.g., TCE and Spiral

Co-Optimizing Architecture and Kernel

Architectural parameter:
Vector length,
#processors, …

Model: common abstraction
= spaces of matching formulas

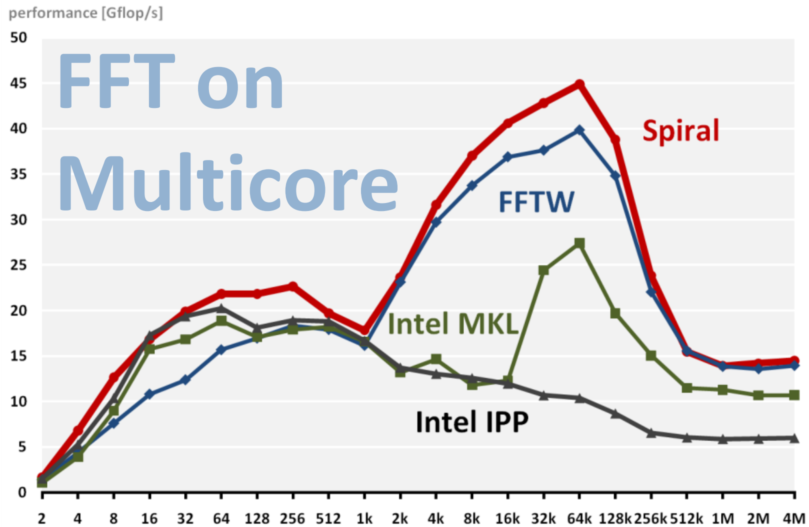
Kernel:
problem size,
algorithm choice



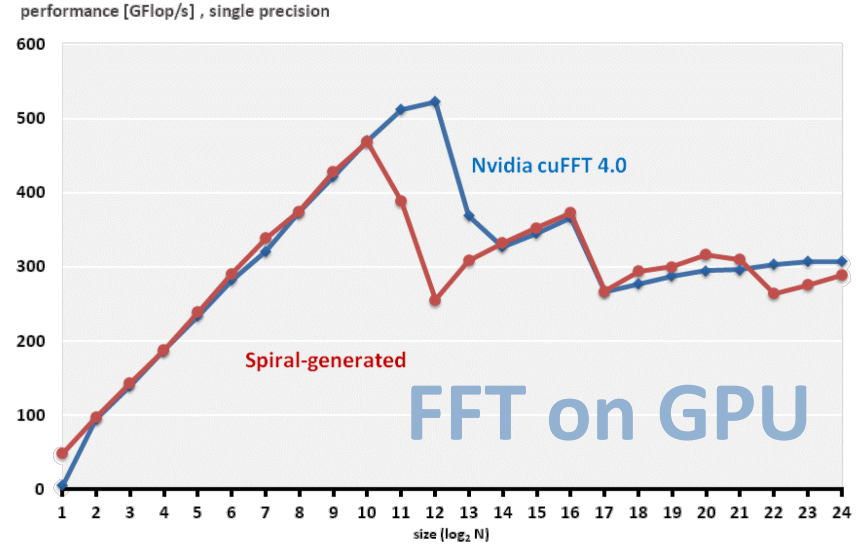
Goal: SPIRAL co-designed RISC-V accelerator chip, taped out

Some Results: FFTs and Spectral Algorithms

1D DFT on 3.3 GHz Sandy Bridge (4 Cores, AVX)

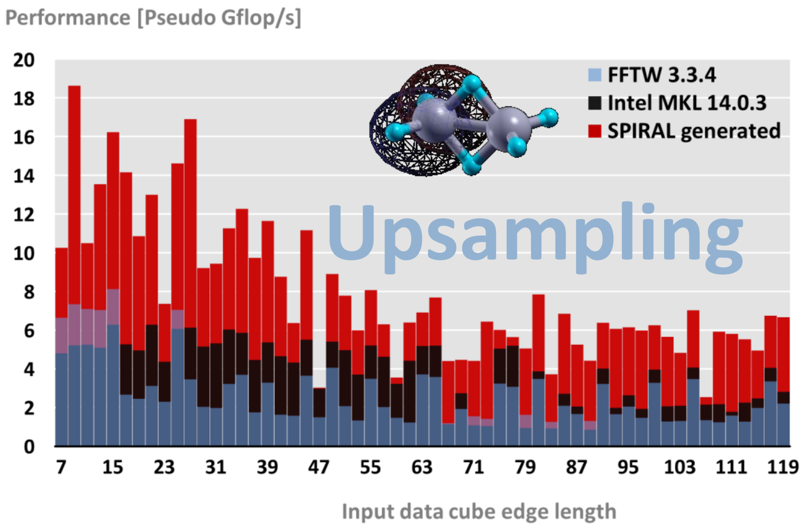


1D Batch DFT (Nvidia GTX 480)

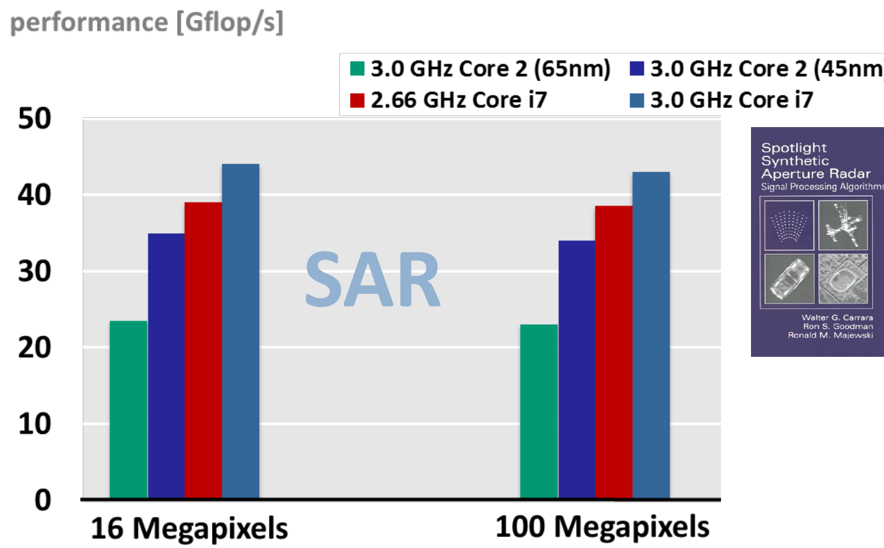


Performance of 2x2x2 Upsampling on Haswell

3.5 GHz, AVX, double precision, interleaved input, single core

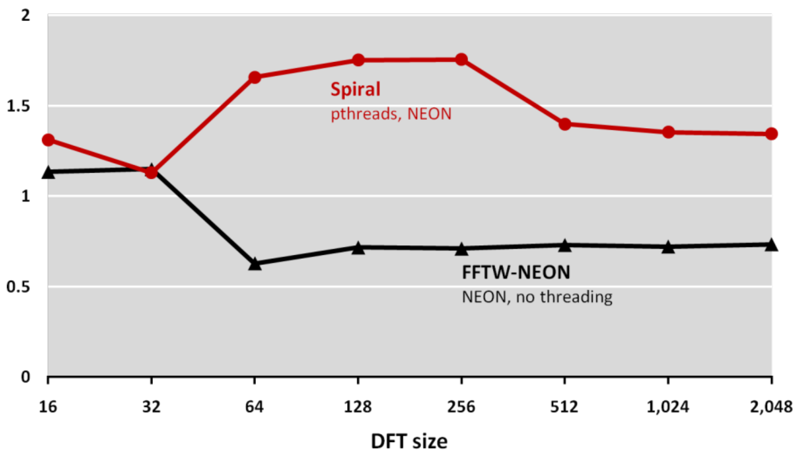


PFA SAR Image Formation on Intel platforms

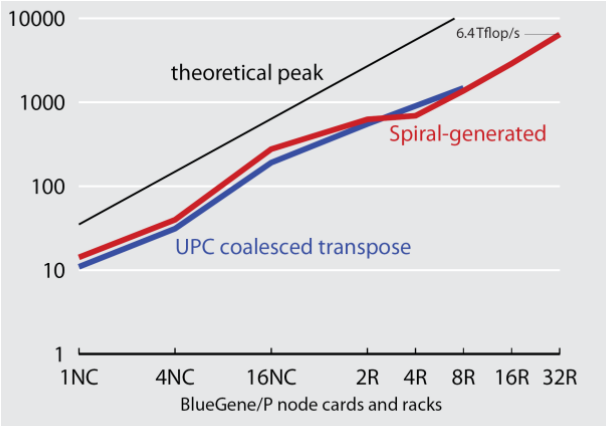


From Cell Phone To Supercomputer

DFT on Samsung Galaxy S II
Dual-core 1.2 GHz Cortex-A9 with NEON ISA



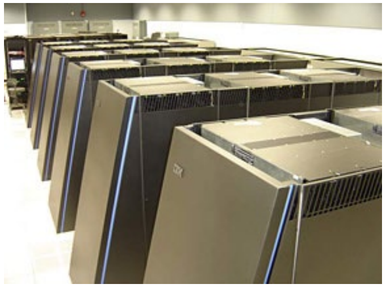
Global FFT (1D FFT, HPC Challenge)
performance [Gflop/s]



6.4 Tflop/s on BlueGene/P

Samsung i9100 Galaxy S II
Dual-core ARM at 1.2GHz with NEON ISA

BlueGene/P at Argonne National Laboratory
128k cores (quad-core CPUs) at 850 MHz



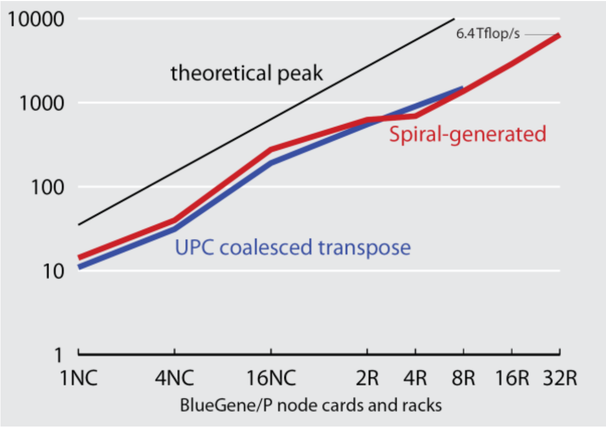
F. Gygi, E. W. Draeger, M. Schulz, B. R. de Supinski, J. A. Gunnels, V. Austel, J. C. Sexton, F. Franchetti, S. Kral, C. W. Ueberhuber, J. Lorenz, **"Large-Scale Electronic Structure Calculations of High-Z Metals on the BlueGene/L Platform,"** In Proceedings of Supercomputing, 2006. ***2006 Gordon Bell Prize (Peak Performance Award).***

G. Almási, B. Dalton, L. L. Hu, F. Franchetti, Y. Liu, A. Sidelnik, T. Spelce, I. G. Tānase, E. Tiotto, Y. Voronenko, X. Xue, **"2010 IBM HPC Challenge Class II Submission,"** ***2010 HPC Challenge Class II Award (Most Productive System).***

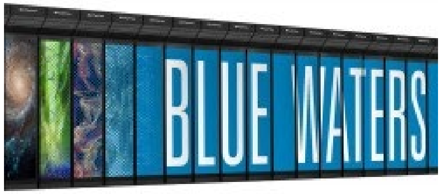
SPIRAL: Success in HPC/Supercomputing

- **NCSA Blue Waters**
  PAID Program, FFTs for Blue Waters
- **RIKEN K computer**
  FFTs for the HPC-ACE ISA
- **LANL RoadRunner**
  FFTs for the Cell processor
- **PSC/XSEDE Bridges**
  Large size FFTs
- **LLNL BlueGene/L and P**
  FFTW for BlueGene/L's Double FPU
- **ANL BlueGene/Q Mira**
  Early Science Program, FFTW for BGQ QPX

Global FFT (1D FFT, HPC Challenge)
performance [Gflop/s]



BlueGene/P at Argonne National Laboratory
128k cores (quad-core CPUs) at 850 MHz

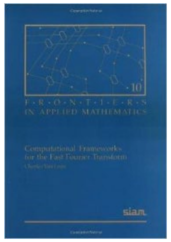


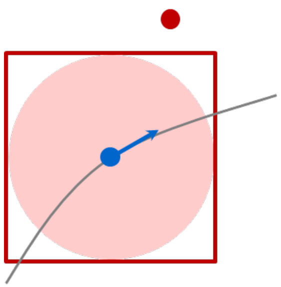
2006 Gordon Bell Prize (Peak Performance Award) with LLNL and IBM

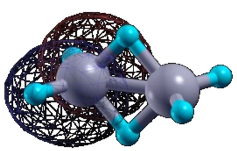
2010 HPC Challenge Class II Award (Most Productive System) with ANL and IBM

SPIRAL: AI for High Performance Code

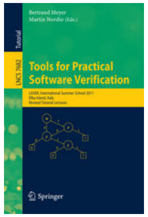
Algorithms

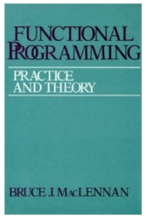


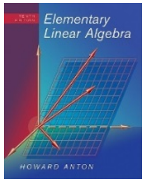




Correctness







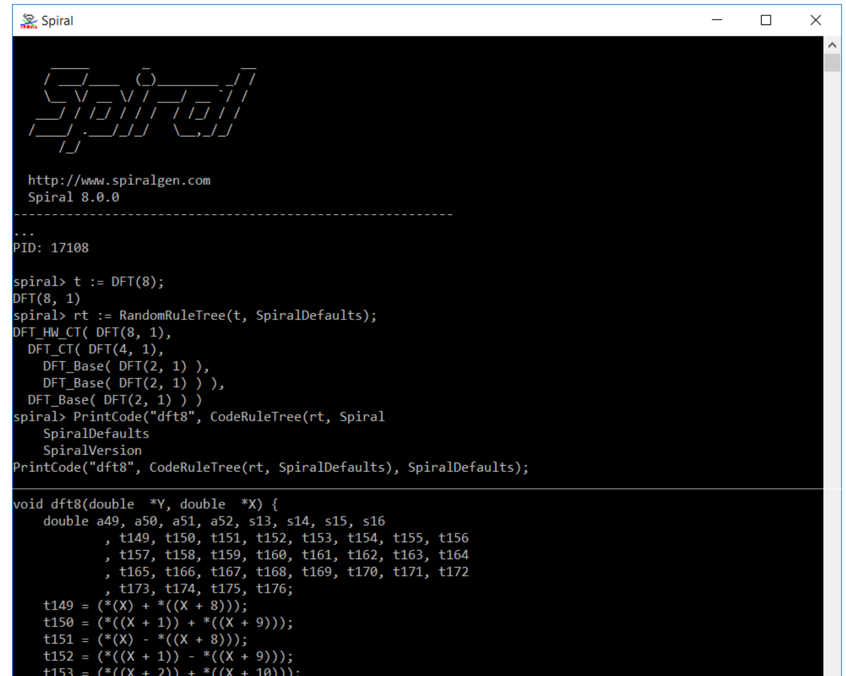

```
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8,...
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(
    _mm_set1_ps(FLT_MIN), _mm_set1_ps(X[0])));
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN
           +DBL_MIN)), _mm_loaddup_pd(&(D[i5])));
        x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
        x2 = _mm_mul_pd(x1, x6);
    ...
```

Hardware

SPIRAL 8.1.0: Available Under Open Source

- **Open Source SPIRAL available**
  - non-viral license (BSD)
  - Initial version, effort ongoing to open source whole system
  - Commercial support via SpiralGen, Inc.
- **Developed over 20 years**
  - Funding: DARPA (OPAL, DESA, HACMS, PERFECT, BRASS), NSF, ONR, DoD HPC, JPL, DOE, CMU SEI, Intel, Nvidia, Mercury
- **Open sourced under DARPA PERFECT, continuing under DOE ECP**
- **Tutorial material available online**

www.spiral.net



F. Franchetti, T. M. Low, D. T. Popovici, R. M. Veras, D. G. Spampinato, J. R. Johnson, M. Püschel, J. C. Hoe, J. M. F. Moura: SPIRAL: Extreme Performance Portability, **Proceedings of the IEEE, Vol. 106, No. 11, 2018.**
Special Issue on *From High Level Specification to High Performance Code*